Applied Energy 226 (2018) 905–915

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Cooperative game theory and last addition method in the allocation of firm energy rights

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HIGHLIGHTS

- We propose a faster method to compute core constraints for the firm energy problem.
- Cooperative game theory is applied together with a traditional allocation method.
- We propose an efficient way to allocate firm energy rights.
- Our proposed firm energy allocation framework is applied to real-sized instances.
- Benders has a slower performance than MILP to compute core constraints of the game.

ARTICLE INFO

Keywords:
Cooperative game theory
Mixed integer linear programming
Firm energy rights
Electric power systems
Hydro power
Benders decomposition

ABSTRACT

The firm energy rights of a hydro plant is a parameter used in some electricity markets to define the maximum amount of energy that a power plant can trade through contracts. In a centralized dispatch scheme, the coordinated operation of the hydro plants generates a synergetic gain in the system firm energy, in this setting, a question that often arises is how to fairly allocate this energy among each hydro plant. This work proposes a formulation to compute the firm energy rights of hydro plants using cooperative game theory and the last addition allocation method. The main goal is to integrate the interests of hydro agents with the needs of the regulatory agencies, searching in the core of the game for solutions that give the right incentives to the optimal system development. In order to make simulations of real instances possible, it is proposed a reformulation of the traditional mixed integer linear programming model that computes the core constraints, which induces a significant speed-up of the algorithm solution time. It is shown an application of the proposed methodology to a real instance representing the Brazilian electric power system.

Nomenclature

The main notation used throughout this paper is listed below. Subscripts k and ℓ are used to indicate the value of a parameter or variable at a specific stage k or ℓ.

Abbreviations

AE	assured energy
AFE	Allocation of firm energy. Represented by Eqs. (20)–(22)
APCP	Average production in the critical period. Represented by Eq. (11)
CGM	Cooperative game model. Represented by Eqs. (14)–(19)
FE	firm energy
FEMILP	algorithm that allocates firm energy rights using cooperative game theory and the last addition method. Presented in Fig. 1
LA	Last addition. Represented by Eq. (12)
LB	lower bound
MGFE(I_s)	model that computes the global firm energy associated with subset $I_s \subseteq I$. Represented by Eqs. (1)–(10)
MP	master problem
MP^R	reformulated master problem
MILP	mixed integer linear programming
SP	sub-problem

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https://doi.org/10.1016/j.apenergy.2018.06.065
Received 7 February 2018; Received in revised form 16 May 2018; Accepted 10 June 2018
0306-2619/

SP^R reformulated sub-problem
UB upper bound

Indices and Sets

$CP \subseteq T$ subset of months that define the critical period of the hydro power system inflow
$i \in I$ set of hydro power plants
$I_s \subseteq I$ subset of hydro power plants
$I_{LA}^i \subseteq I_s$ subset of all hydro plants in I excluding plant i
$m \in M_i$ set composed by hydro plants located immediately upstream of hydro plant i
$m \in \widehat{M}_i$ set composed by all hydro plants located upstream of hydro plant i
$t \in T$ set of monthly time stages

Functions

$\phi_i(\cdot)$ 4-th order polynomial to represent the reservoir head and volume of plant i

Deterministic parameters

A_i^t incremental water inflow in the river that supplies hydro plant i, at stage t, expressed in [m^3/month]
AE_i individual assured energy of hydro plant i, expressed in average MW, or [MW·month]
$\widehat{FE}_i$ individual firm energy of hydro plant i, expressed in average MW, or [MW·month]
$\widehat{GFE}_{I_s}$ firm energy of the subset of hydro plants I_s, expressed in average MW, or [MW·month]
h_{eq}^i equivalent net head of hydro plant i, expressed in [m]
HE hydro power energy fraction of the system assured energy, expressed in average MW, or [MW·month]
HL_i hydraulic losses at hydro plant i due to the water flow though pipelines, expressed in [m]
N_i^t natural water inflow in the river that supplies hydro plant i, at stage t, expressed in [m^3/month]
N_{CP} number of months in the critical period
NP number of hydro plants in the subset I
$\overline{Q_i}$ maximum turbine outflow of plant i, expressed in [m^3/month]
$\overline{V_i}$ maximum storage volume of hydro plant i, expressed in [m^3]
$\underline{V_i}$ minimum storage volume of hydro plant i, expressed in [m^3]
$\widehat{Y}_i$ binary coefficient that defines if the plant i belongs to the subset I_s ($\widehat{Y}_i = 1$) or not ($\widehat{Y}_i = 0$)
$\overline{\theta_i}$ hydro plant i average tailrace level, expressed in [m]
ρ_{eq}^i equivalent productivity of hydro plant i, expressed in$\left[\frac{\text{MW}\cdot\text{month}}{\text{m}^3/\text{month}}\right]$
ρ_{sp}^i specific productivity of hydro plant i, expressed in$\left[\frac{\text{MW}\cdot\text{month}}{\frac{\text{m}^3}{\text{month}}\cdot\text{m}}\right]$
$\gamma_{k,i}$ k-th polynomial coefficient that represent reservoir head and volume of hydro plant i

Decision variables

FE_i individual firm energy of the hydro plant i, measured in average MW, or [MW·month]
GFE_{I_s} firm energy of the subset of hydro plants I_s, measured in average MW, or [MW·month]
PG_i^t average power generated by hydro plant i, at stage t, measured in average MW
Q_i^t turbined outflow of hydro plant i, at stage t, expressed in [m^3/month]
S_i^t water spillage outflow of hydro plant i, at stage t, expressed in [m^3/month]
V_i^t available water volume stored in the reservoir of hydro plant i, at stage t, expressed in [m^3]
Y_i binary variable that defines if the plant i belongs to the subset I_s ($Y_i = 1$) or not ($Y_i = 0$)
π^α dual variable associated with each constraint ($\alpha = 0, \ldots, 4$) in the sub-problem

1. Introduction

Renewable energy sources are currently playing a key role in the energy matrix of many countries around the world [1]. In 2016, the renewable energy production represented approximately 24% of the total electricity generated worldwide [2]. This amount is likely to increase in the next few years/decades as the investments in solar, wind and other renewable sources are ramping up. Another important renewable energy source is hydro power, which is considered by many as a conventional form of electricity production [3]. Nowadays, hydro power alone represents the largest share in renewable energy production, e.g. in 2016 hydro power alone corresponded to 67% of the total renewable electricity production [2].

In hydroelectric generating systems the optimal operation of the hydro power plants depends on the wise use of the water available at the reservoirs. Upstream hydro plants have to coordinate their operation with downstream plants in order to minimize spillages and maximize the total electricity production [4]. Sometimes the optimal operation of a hydro power system is even more complex and involves coordination of plants that are not connected hydrologically and that are owned by different agents.

In some countries, with predominance of hydro generation, such as Brazil [5], Canada [6], and Norway [7], the coordination of the hydro power generation [8] is an essential task related to the security of supply and the power system stability [9]. In this work, we consider a centralized coordination of energy resources, where a synergetic energy gain is achieved by the optimal dispatch of a set of hydro power plants. As a result from the optimization process, the total energy production for the system composed by the set of hydro plants is obtained. However, due to the synergetic energy gains obtained from the coordinated operation, it is necessary to properly allocate each hydro plant share from the total system production. In this context, the problem of firm energy (FE) rights allocation [10,11], and its associated models are the key to find satisfactory answers.

For example, in Brazil, where a centralized dispatch of energy sources is performed, the total hydro-thermal energy production that a system can guarantee for a safe and reliable operation is determined according to a procedure similar to the one described in [12]. This energy measure is known as assured energy (AE) and represents a hypothetical amount of energy that the system is capable of generate under a determined level of supply risk. After dividing the system AE into a hydro power energy fraction (HE) and a thermal energy fraction, the individual assured energy (AE_i) is determined according to the firm energy rights allocation method, that basically uses an individual firm energy variable (FE_i) to divide the HE among each hydro plant of the system (this procedure will be described in detail in the Section 2). The larger is the FE_i, the bigger will be the portion of the HE allocated to the plant and its AE_i. The AE_i works as ballast for energy sales in the electricity market, this way, if a particular hydro plant has more AE_i, it can sell more energy in the market and achieve larger profits.

For hydro power systems operating in a centralized dispatch scheme, it is possible to attribute a desirable property in the allocation models called fairness. The concept of fairness was first proposed by Von Neumann [13] and was recently applied in the FE computation [11]. According to the cooperative game theory, an allocation is fair, if and only if, none of its participants have interest in leaving the grand coalition to form sub-coalitions. In other words, the benefit of participating in the grand coalition overcomes the benefit of forming any sub-coalitions [14]. When an allocation satisfies the condition of fairness of a cooperative game, it is said that this allocation is in the core of the game. The core concept is a stability property of the grand coalition, since if an allocation is in the core of the game its participants have no benefit in leaving the grand coalition.

Due to its properties, the cooperative game theory is gradually getting the attention of the power and energy system community. Ref. [15] uses cooperative game theory to evaluate the potential of cooperative behaviors of multiple grid-connected microgrids to achieve higher energy efficiency. Ref. [16] uses the Aumann-Shapley allocation model, a well-known cooperative game solution concept, to compute the benefit obtained by individual transmission network users from each transmission projects within an expansion plan. Ref. [17] presents an economic analysis of power system flexibility considering a range of variable renewable energy sources capacity levels, also using the cooperative game theory in its studies. Ref. [18] allocates the utility cost of electric vehicle service equipment also using the cooperative game theory, more specifically, a Shapley allocation method. Finally, the work presented in [11] uses the concepts of cooperative game theory in the FE allocation problem.

In an allocation problem, it can happen that the core of the game is empty or that for a specific method sometimes the allocation is in the core, and sometimes is not [13]. When the FE problem is modeled as a deterministic linear programming, it is possible to prove that the core of the game is not empty as numerically showed in [11]. Knowing that the core of the game is not empty for the problem considered here, one may wonder if a specific allocation is in the core or not, i.e. if the allocation obtained is fair or not. In some allocation models, as the marginal benefit method [10], it is possible to prove that, under specific conditions, we always obtain an allocation inside the core of the game for the FE allocation problem. However, this is not always the case for several other allocation methods.

Given the importance of the fairness concept in a cooperative game, there is an obvious interest of the hydro agents in knowing if an allocation belongs to the core or not. Finding whether or not an allocation is in the core can be highly complex when dealing with real instances since the number of constraints to be verified is of the order 2^{NP}, where *NP* is the number of hydro power plants that compose the system. To circumvent the challenges of verifying the 2^{NP} constraints, it is possible to solve a mixed integer linear programming (MILP) model that finds the most violated constraint [11,19,20]. The MILP model however has limitations when dealing with medium- to large-sized instances since as it requires an expressive computational time for the algorithm to reach convergence, for more details one should refer to [11].

Among the methods to allocate FE rights, the last addition (LA) or incremental allocation method has gained considerable attention in Brazil [21], Canada, Chile, Western U.S., and other places as well as mentioned in [10]. The LA method allocates energy between hydro plants proportionally to the incremental benefit that exists when the system is simulated without and with this plant. One advantage of this approach is the proper indication of new investments entry since there is an incentive for companies to invest in new hydro plants that will contribute to increase the system energy. It is important to understand that a plant can increase the system energy generation not necessarily generating more energy but also storing water to regularize downstream flows.

As far as the authors know, no other allocation method carries the same incentives of the LA method in terms of proper indication of new investments entry. The LA method encourages the entry of new plants proportionally to the increase that this plant generates to an existent system (this increase is computed by the difference in results from simulations with and without the representation of this plant) [10]. It is not hard to see that methods such as Aumann-Shapley [22], Shapley [23], average production in the critical period (APCP), and marginal benefits [10] can provide incentives to the entry of new hydro plants that are not necessarily going to increase the total system energy. Despite its positive qualities, the LA method does not guarantee an allocation inside the core of the cooperative game.

The main goal of this work is to propose a new scheme of allocation that integrates the concepts of cooperative game theory and LA allocation method. The idea is to explore the positive advantages of both methods and search in the core of the game for allocations that are closest to the LA allocation. Also, we propose a formulation capable of overcoming the previous limitations, discussed in [10,11], of the traditional MILP model in finding the most violated constraints of the cooperative game. The proposed methodology can solve real-sized instances that were not yet explored in the literature due to the computational burden faced by other existent models. Moreover, we propose a Benders Decomposition approach over the MILP model that finds the most violated constraint of the cooperative game. The proposed methodology is computationally tested using data from a real benchmark case composed by more than 80 years of monthly hydrological data and 170 hydro plants representing the Brazilian hydro power system. The framework proposed here is general and can be further applied to any other hydro power system in order to properly compute optimal allocations of firm energy among multiple plants.

The remainder of this paper is divided as follows: Section 2 presents the considerations about the FE computation and model formulation using the LA method. This section explains the concept of cooperative game theory applied to the FE computation and presents the proposed methodology to integrate the LA method with cooperative game theory. Section 3 presents a Benders formulation for the MILP model that finds the cooperative game constraints. This Section also proposes a reformulation of the MILP approach from [11] to compute the constraints of the cooperative game. Section 4 presents cases studies simulations and results. Section 5 concludes the paper.

2. Firm energy and cooperative game theory in hydro power systems

There are several models in the literature to deal with the FE allocation problem. However, all of them have strengths and weaknesses, depending on the situation analyzed. Therefore, there is a constant need for improving existent models and adapting them to the present reality of the hydro power systems, overcoming some limitations of the past and understanding the new challenges of the present.

A widely used linear optimization model used to represent the FE rights allocation problem [10,11] is presented in (1)–(10). Basically, it computes the maximum energy that a specific system can generate continuously given the repetition of the historical flows, this energy is denoted as global FE (GFE_I) for a specific power system. The GFE_I is mainly limited by the system critical period, that is the most severe drought registered in the hydrological history of the system, where the conditions to produce hydro power are extremely harsh. Usually, the stored energy curve, which is obtained indirectly from the solution of the model (1)–(10) is used to compute the critical period in FE applications. By analyzing this curve, it is possible to find the largest period where the system stored energy goes from its maximum to its minimum value without intermediaries refills, this period is defined as the critical period [24,25].

Firm energy rights allocation models basically propose ways to divide the GFE_I among each hydro plant of the system. As a result of this allocation, the individual firm energy (FE_i) is obtained. At this point, it is important to understand that despite the fact that the FE_i comes as an allocation of the GFE_I among each hydro plant, this measure may not be used as the total hydro energy available for trades in long-term energy markets. For example, in Brazil, stochastic optimization models [5,12] are used to define the total system *AE* (considering an aggregate representation of hydro plants), after that, the results from firm energy allocation models (GFE_I and FE_i) are used to separate the system *AE* into individual energy values that each hydro plant is allowed to trade in the long-term.

Another important point is that GFE_I is one of the parameters used to investigate the reliability of the system in terms of generation capacity, since it represents the maximum amount of energy that the system can generate continuously in the repetition of the worst sequence of historical inflows. This means that, a larger GFE_I represents a more robust system.

$$\max GFE_I \tag{1}$$

$$\text{s.t. } V_i^{t+1} = V_i^t - Q_i^t - S_i^t + A_i^t + \sum_{m \in M_i} (Q_m^t + S_m^t) \quad \forall\, i \in I, t \in T \tag{2}$$

$$GFE_I = \sum_{i \in I} PG_i^t \quad \forall\, t \in T \tag{3}$$

$$0 \leqslant Q_i^t \leqslant \overline{Q_i} \quad \forall\, i \in I, t \in T \tag{4}$$

$$0 \leqslant V_i^t \leqslant (\overline{V_i} - \underline{V_i}) \quad \forall\, i \in I, t \in T \tag{5}$$

$$S_i^t \geqslant 0 \quad \forall\, i \in I, t \in T \tag{6}$$

where

$$PG_i^t = \rho_{eq}^i Q_i^t \quad \forall\, i \in I, t \in T \tag{7}$$

$$\rho_{eq}^i = \rho_{sp}^i h_{eq}^i \quad \forall\, i \in I \tag{8}$$

$$h_{eq}^i = \int_{\underline{V_i}}^{\overline{V_i}} \phi_i dV_i / (\overline{V_i} - \underline{V_i}) - (\overline{\theta_i} + HL_i) \quad \forall\, i \in I \tag{9}$$

$$\phi_i = \sum_{j=1}^{5} \gamma_{j,i} (V_i)^{j-1} \quad \forall\, i \in I \tag{10}$$

In model (1)–(10), constraint (2) represents the hydro balance equation, constraint (3) establishes that the total energy generated by the system in each month of the simulation period should be the same and equal to GFE_I, i.e., constraint (3) ensures that the energy GFE_I can be supplied during all the monthly time stages. Constraints (4) and (5) limit the turbined outflow, and the storage volume, respectively. Constraint (6) establishes that the volume of spillage should be positive. Finally, the objective function defined in (1) aims to maximize the total energy production in each month. Eq. (7) comes as consequence of the definitions in (8)–(10), Eq. (10) relates the reservoir head with the water stored volume at the reservoirs. Eq. (9) computes an equivalent net head for each hydro plant i, this step is necessary since we are modeling the FE problem as a linear model, as previously represented in [10,11]. Eq. (8) is an intermediary step to compute an equivalent productivity for each hydro plant. Eq. (7) describes the power generated by the plant i as a function of Q_i^t.

It is important to notice that the model (1)–(10) is described in month time stages, this way, units such as [m^3/month] or [m^3] can be used interchangeably when describing, for example, the total turbined volume or the turbined outflow in a specific time stage t. This observation should be take in consideration while analyzing all models described in this work.

To avoid confusion in the next sections, when the model (1)–(10) is going to be used to compute the total system FE for the set of hydro plants I (GFE_I) and for the subset $I_s \subseteq I$ (GFE_{I_s}), the model (1)–(10) is going to be called MGFE(I_s) in the latter case. This way, the model MGFE(I_s) is the model (1)–(10) substituting the set I by I_s.

2.1. Firm energy allocation methods

With the model MGFE(I_s), it is possible to use different allocation methods such as Aumann-Shapley [22], Shapley [23], APCP, LA, marginal benefits, and nucleolus [11,13], to compute the FE rights of each hydro plant, that is, to divide GFE_I among each hydro plant of the system. Ref. [10] provides a review of the methods described above in the context of FE rights. Two methods of special interest are the APCP and the LA method, since these are the most used in the literature. The APCP method solves only one time the model MGFE(I) and computes the FE_i as the average generation of each hydro plant during the critical period using Eq. (11), where N_{CP} is the number of months in the critical period.

$$FE_i = \frac{\sum_{t \in CP} PG_i^t}{N_{CP}} \tag{11}$$

The APCP method is of easy implementation and its associated results provide intuitive interpretation. However, the method also has some negative points that prevent it from being used in several cases. For example, it is not capable of properly signalize the benefit that hydro plants with reservoirs promote to the system FE. In an extreme case, a plant without generation but with a large reservoir can regularize flows and increase the system FE, however, this plant would have an FE_i equals to zero. Usually this method is implemented together with other methods to circumvent some of its weakness [26].

The LA method requires one simulation of the model MGFE(I_s) for each hydro plant i considering in such simulation scheme all hydro plants except i, we denote this energy amount as $GFE_{I_{LA}^i}$, where I_{LA}^i is the subset of all hydro plants in I excluding plant i. The FE attributed to hydro plant i is determined by (12). Note that, to obtain GFE_I all the hydro plants are considered in the analysis.

$$FE_i = \frac{(GFE_I - GFE_{I_{LA}^i})}{\sum_{i \in I} (GFE_I - GFE_{I_{LA}^i})} GFE_I \tag{12}$$

As it is possible to notice, the LA method computes the FE_i considering only the benefit that plant i generates to the system when it is the last plant to enter, that is why the method is called last addition (LA). In the case of a set of plants candidates of being built, the ones that increase most the system FE will receive a higher FE_i. Knowing that a higher FE_i indicates a higher profit, we can say that the LA incentivizes properly the entry of the new plants in the system.

2.2. Cooperative game theory for firm energy computation

A FE allocation satisfies the conditions of fairness of a cooperative game if the inequalities represented by (13) are satisfied [11,14]. The basic idea is that the summation of the FE energy allocated to any subset $I_s \subseteq I$ should be greater than or equal to the energy amount that subset I_s could generate when operating alone, i.e. maximizing only its own energy (GFE_{I_s}).

$$\sum_{i \in I_s} FE_i \geqslant \widehat{GFE}_{I_s} \text{ and } \sum_{i \in I} FE_i = \widehat{GFE}_I \tag{13}$$

It is not obvious that there exists an allocation that satisfies constraint (13). Also, it is also not easy to verify if a determined allocation, $FE_i\ \forall\, i \in I$, satisfies (13), since there are 2^{NP} constraints to be checked, where NP is the number of hydro plants in the set I, even for a small system it could be a challenge to check all these constraints as argued in [10,11]. An alternative to this issue is to solve the MILP described below that is capable to find the most violated constraint of the set represented by (13). The most violated constraint of (13), when it exists, is the one where $GFE_{I_s} - \sum_{i \in I_s} \widehat{FE}_i$ is positive and the largest.

It is important to understand that for the constraints represented by (13) and the model represented by (14)–(20), $\widehat{FE}_i$ is a parameter and not a decision variable.

$$\max GFE_{I_s} - \sum_{i \in I} (\widehat{FE}_i Y_i) \tag{14}$$

$$\text{s.t. } V_i^{t+1} = V_i^t - Q_i^t - S_i^t + A_i^t + \sum_{m \in M_i} (Q_m^t + S_m^t) \quad \forall\, i \in I, t \in T \tag{15}$$

$$GFE_{I_s} = \sum_{i \in I} PG_i^t \quad \forall\, t \in T \tag{16}$$

$$0 \leqslant Q_i^t \leqslant \overline{Q_i} Y_i \quad \forall\, i \in I, t \in T \tag{17}$$

$$0 \leqslant V_i^t \leqslant (\overline{V_i} - \underline{V_i}) Y_i \quad \forall\, i \in I, t \in T \tag{18}$$

$$S_i^t \geqslant 0,\ Y_i \in \{0, 1\} \quad \forall\, i \in I, t \in T \tag{19}$$

In the model represented by (14)–(19), the binary variable Y_i defines if a specific plant i is part of the most violated coalition (I_s) $Y_i = 1$ or not $Y_i = 0$, and the objective quantifies the violation, being GFE_{I_s} the minimum amount of energy that should be allocated to the found coalition (I_s). When $Y_i = 0$, constraints (17) and (18) will enforce $Q_i^t = V_i^t = 0$, this means that all the water arriving at hydro plant i in time stage t will be spilled. This behavior is equivalent as taking the hydro plant i out of the simulation, since this plant will not generate energy or regularize flows.

Looking at the model (14)–(19) it is possible to notice some similarities with the MGFE(I_s). Actually, model (14)–(19) can compute the GFE_{I_s} of any subset I_s and compare this parameter with the sum of the FE allocations that was proposed to the correspondent subset. Different subsets can be selected by the MILP model changing the Y_i variables.

2.3. Cooperative game theory with last addition method

Let's denote model (14)–(19) as CGM (Cooperative Game Model), for a given FE allocation ($\widehat{FE}_i^0$), that can be obtained using the LA method for example. It is possible to solve the CGM and find the most violated constraint of the set (13), that can be defined by the subset of hydro plants (I_s^1) that belongs to this constraint and the minimum sum of FE that the subset I_s^1 should receive ($GFE_{I_s^1}$). With this first constraint in hands, it would be possible to solve model (20)–(22) that determines another allocation FE_i^k ($k = 1$) that is the closest from the original allocation $\widehat{FE}_i^0$ and satisfies the constraint found in the first CGM simulation. This new allocation FE_i^k ($k = 1$) could be used as input for the CGM that determines the next most violated constraint (I_s^2 and $GFE_{I_s^2}$), this cycle continues until the violation determined by the CGM assumes a value smaller than a pre-set ε. Fig. 1 presents a flow diagram describing the algorithm.

In model (20)–(22), denoted here as AFE (Allocation of FE), FE_i^k is a decision variable that corresponds to the new FE allocated to each plant i at stage k, I_s^ℓ is the subset of hydro plants that belong to the constraint determined by the CGM at stage ℓ, and $\widehat{GFE}_{I_s^\ell}$ is the right hand side of the constraint determined by the CGM at stage ℓ.

$$\min \sum_{i \in I} (FE_i^k / \widehat{FE}_i^0 - 1)^2 \tag{20}$$

$$\text{s.t.} \sum_{i \in I} FE_i^k = \widehat{GFE}_I \tag{21}$$

$$\sum_{i \in I_s^\ell} FE_i^k \geqslant \widehat{GFE}_{I_s^\ell} \quad \forall\, \ell = 1, \ldots, k \tag{22}$$

The model AFE searches for an allocation FE_i^k that is the closest one to $\widehat{FE}_i^0$ and satisfies constraints (21) and (22). Constraints (21) and (22) are a subset of the constraints presented in (13) and are chosen iteratively by the algorithm of Fig. 1.

There are several ways to compute how close one allocation is from another, in this paper we decided to use the sum of the percentage difference between the new allocation FE_i^k and the allocation $\widehat{FE}_i^0$ (the first allocation) squared. This approach uncouples the influence that the plant size has in the optimization model. For example, if the objective function was defined as $\min \sum_{i \in I} (FE_i^k - \widehat{FE}_i^0)^2$, the optimization model would prioritize the minimization of the term $(FE_i^k - \widehat{FE}_i^0)^2$ only for plants with high FE_i, usually large hydro plants, and the small plants would have their percentage differences $(FE_i^k / \widehat{FE}_i^0 - 1)$ extremely high.

It is important to understand that in the algorithm presented in Fig. 1 the CGM computes at each iteration k the most violated constraint from (13) given a FE_i^k that comes from the AFE model. The constraints determined by the CGM feed the AFE, which now can propose allocations that are not going to violate the previous constraints found by the CGM. Also, the AFE tries to propose allocations as close as possible from the initial allocation $\widehat{FE}_i^0$, which in this case, is the LA allocation.

The algorithm stops when the maximum violation found by the CGM is less than an error ε, i.e., the AFE proposes an allocation that satisfies the set of constraints (13) with a maximum error tolerance ε. This error can be chosen based on the system GFE_I, for example $\varepsilon = GFE_I \cdot 0.1\%$, the smaller is ε the better is the solution. The algorithm presented in Fig. 1 converges to an allocation that is in the core of the game, if and only if, $\varepsilon = 0$, since, if an allocation satisfies all the constraints in (13), by definition, the maximum value possible for (14) is zero.

For the remainder of this paper, the algorithm presented in Fig. 1 will be called FE_MILP, and the $\widehat{FE}_i^0$ allocation used will be the LA allocation (12).

The LA allocation will be used in the FE_MILP model because it properly incentives the entry of new hydro plants. In an example, suppose that there are two plants candidates to be built, each one of

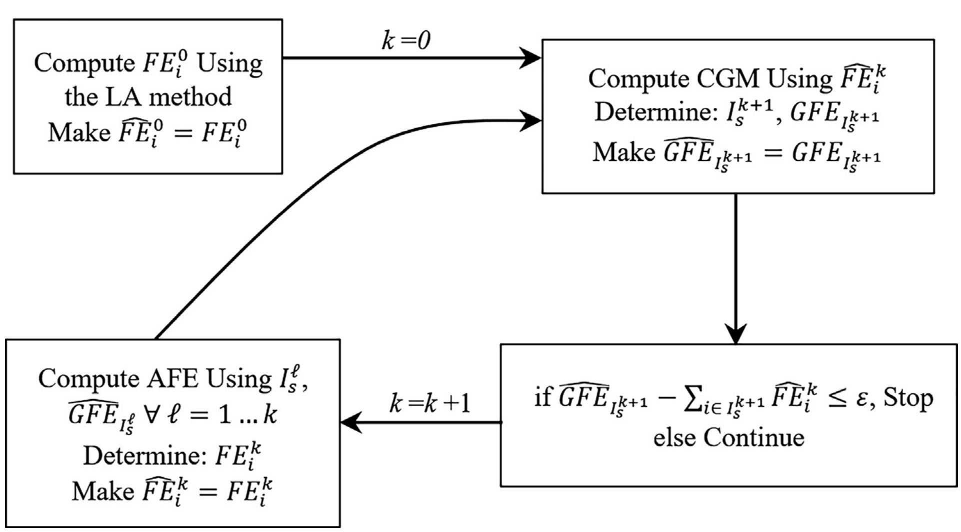

Fig. 1. Flow diagram of the FE allocation algorithm.

these plants increase the system FE differently. In the case that plant 1 increases more the system FE it is natural to attribute to this plant a higher benefit, however, this is not what necessarily happens in the Aumann-Shapley, nucleolus, marginal benefits or average production in the critical period methods [10,11]. The LA allocation will always give more benefits to plant 1 in the example presented here because of its formulation (12) However, similarly to the majority of the allocation methods previously described, solutions from the LA allocation method are not necessary in the core of the game.

For the cooperative game theory applied together with the LA allocation method, as in the formulation described in Fig. 1, the idea is to choose from the core allocations the one that carries more properties of the LA allocation. In this case, for the example of two plants given above, the algorithm can behave similar to the LA method and benefits more plant 1 than plant 2, or in an exceptional case, it can happen that the FE_MILP model gives more incentives to plant 2 than plant 1. Nevertheless, if the last case happens it is because in order to benefit plant 1 more than plant 2 the cooperative game constraints should be violated by the AFE model. The benefits that the core allocations bring to the FE problem are much more relevant than the benefits that the LA method can bring. In this way, even in this exceptional scenario, the model developed here will give the right incentives to the entry of new plants.

3. Benders decomposition to compute cooperative game constraints

3.1. A Benders reformulation of the problem

It is well known that many solvers such as CPLEX [27], Gurobi [28], and Xpress [29], to name a few, are capable of solving MILP problems very efficiently by choosing different algorithm strategies to accelerate convergence. However, the Benders decomposition algorithm is still widely used in literature to decompose MILP problems and help improving solution times in specific applications where the size of the model and associated number of constraints are large. For example, the Benders decomposition algorithm has been successfully applied with the cooperative game theory in the coordination of multi-microgrid operation [15]. Also, the Benders algorithm has been applied in network design problems [30,31], scheduling problems [32,33] and logistics facility location problems [34].

Since the existent literature [11] associated with FE rights allocation models that compute the core constraints (13) for the problem still show limitations while using MILP algorithms, this work explored the idea of using Benders decomposition aiming to reduce the computational time required to find the most violated constraint of (13).

The Benders decomposition algorithm [35,36], when applied to MILP problems basically divides the model in a sub-problem, that only have continuous variables, and a master problem, that contains the integer variables. In the sub-problem integer variables are arbitrarily fixed, the model is solved, a lower bound (LB) is obtained (for the CGM) and the dual variables obtained are used in the master problem to create planes (Benders cuts) that map the feasible region of the original MILP problem. The master problem searches in the feasible region for an upper bound (UB) and gives to the sub-problem the integer variables values decided, then the sub-problem is solved again. The process continues until a pre-set stopping criteria is reached.

For the CGM a sub-problem (SP) and a master problem (MP) are described by (23)–(29) and (30)–(32) respectively.

$$\max SP \tag{23}$$

$$\text{s.t. } V_i^{t+1} - V_i^t + Q_i^t + S_i^t - \sum_{m \in M_i} (Q_m^t + S_m^t) = A_i^t \quad \forall\, i \in I,\, t \in T \quad (\pi_{i,t}^0) \tag{24}$$

$$GFE_{I_s} - \sum_{i \in I_s} PG_i^t = 0 \quad \forall\, t \in T \quad (\pi_t^1) \tag{25}$$

$$SP - GFE_{I_s} = -\sum_{i \in I} (FE_i \hat{Y}_i) \quad (\pi^2) \tag{26}$$

$$Q_i^t \leqslant \overline{Q}_i \hat{Y}_i \quad \forall\, i \in I,\, t \in T \quad (\pi_{i,t}^3) \tag{27}$$

$$V_i^t \leqslant (\overline{V}_i - \underline{V}_i)\hat{Y}_i \quad \forall\, i \in I,\, t \in T \quad (\pi_{i,t}^4) \tag{28}$$

$$Q_i^t \geqslant 0,\ V_i^t \geqslant 0,\\ S_i^t \geqslant 0,\ GFE_{I_s} \geqslant 0 \tag{29}$$

The model (23)–(29) is similar to the CGM (14)–(19), however, the equations have been rearranged in order to make easier the construction and interpretation of the Benders master problem. Also, in the sub-problem, $\hat{Y}_i$ is a pre-defined constant defined by the master problem, and not a variable as in the CGM.

$$\max MP \tag{30}$$

$$\text{s.t. } MP \leqslant \sum_{i \in I} \sum_{t \in T} Y_i [\overline{Q}_i \pi_{i,t}^{3,\ell} + (\overline{V}_i - \underline{V}_i)\pi_{i,t}^{4,\ell}] + \sum_{i \in I} \sum_{t \in T} (A_i^t \pi_{i,t}^{0,\ell}) \\ -\pi^{2,\ell} \sum_{i \in I} (\widehat{FE}_i Y_i) \quad \forall\, \ell = 1, \ldots, k \tag{31}$$

$$MP \geqslant 0,\ Y_i \in \{0, 1\} \quad \forall\, i \in I \tag{32}$$

In the master problem, the dual variable represented by π_t^1 is not relevant in the Benders cuts computation, since the right-hand side of inequality (25) is zero.

Fig. 2 depicts a flow diagram representing the Benders algorithm. When the difference between the UB and the LB is less than a preset ε, the algorithm stops and the most violated coalition of (13) is obtained.

3.2. Improving the Benders algorithm convergence

The Benders decomposition algorithm does not always guarantee a smaller computational time when dealing with MILPs [37]. Usually, changes in the model formulation and application of enhancement techniques such as Papadakos [38] and Tang [34], are needed. Other enhancement techniques for the Benders algorithm can be found in [39,40], and [41], but they are not explored in this work.

The algorithms proposed by Papadakos and Tang basically explore the degeneracy of the SP. If degeneracy exists, these approaches choose between different Benders cuts (different dual variables), the one that will contribute the most to the convergence of the Benders algorithm. While Papadakos uses a reference point to evaluate the strengths of different planes, Tang uses a procedure to generate denser cuts, that is, planes with less non-zero coefficients and with the same scale order.

As mentioned before, changes in the model formulation can contribute significantly to the speed of the Benders algorithm [34,42]. With Benders cuts represented in (31), only dual variables $\pi_{i,t}^0$ are not multiplied by an integer variable (Y_i) because A_i^t in Eq. (24) is also not multiplied by $\hat{Y}_i$. The constant A_i^t is the incremental flow of the hydro plant i, in the case that this plant is taken off a specific coalition ($\hat{Y}_i = 0$), the downstream hydro plants still need to have access to their natural flows. If A_i^t is multiplied by $\hat{Y}_i$ it would change the natural water flow of the plants that are downstream of plant i, when this plant is taken off the coalition. This is not desired, since the water is a resource of the system and not a resource of a single hydro power plant or agent.

If in the water balance constraint (24) the coefficient that appears on the right-hand side was the natural flow it would be possible to multiply this coefficient by $\hat{Y}_i$ without changing the natural flow of the downstream plants. This could improve the quality of the Benders cuts, since the dynamics of the system introduced by the dual variable $\pi_{i,t}^0$ would be related to the decision variables Y_i and not to a constant.

If the water balance constraint uses the incremental flow as a parameter, the model is going to be called a series representation of the

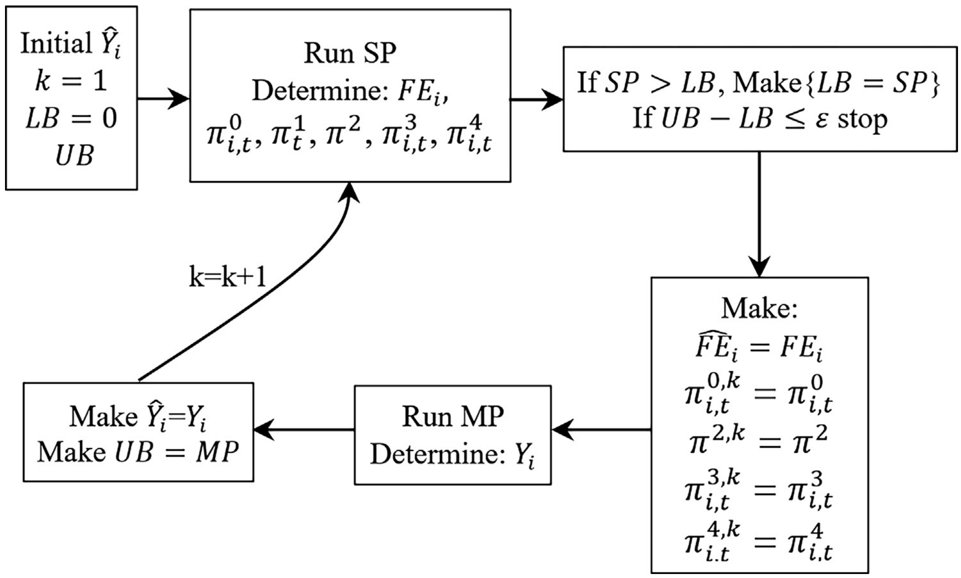

Fig. 2. Flow diagram of the Benders algorithm.

reservoirs, and if it uses the natural flow, parallel representation. The conversion from series to parallel representation is straightforward. Suppose that there is a system with only two hydro plants, let's call plant 1 and plant 2. Given that plant 2 is downstream of plant 1, the hydro balance equations for plant 1 and plant 2 are defined as:

$$V_1^{t+1} - V_1^t + Q_1^t + S_1^t = A_1^t = N_1^t \tag{33}$$

$$V_2^{t+1} - V_2^t + Q_2^t + S_2^t - Q_1^t - S_1^t = A_2^t \tag{34}$$

by adding (33) and (34):

$$V_2^{t+1} - V_2^t + V_1^{t+1} - V_1^t + Q_2^t + S_2^t = A_1^t + A_2^t = N_2^t \tag{35}$$

Constraint (35) can substitute constraint (34) in the original formulation (33), (34), and the same solution will be preserved. The example given here, despite quite simple, can be extended to a larger system, in this case, the hydro balance equation of hydro plant i at stage t for the parallel representation of reservoirs can be obtained by summing all hydro balance equations of hydro plants that belong to the set $\hat{M}_i$ composed by hydro plants upstream of plant i including i. Remembering that in this computation, the hydro balance equations summed must be in the series representation, as in (33) and (34).

The Benders SP and MP using parallel representation are described next. The reformulated sub-problem (SP^R) is equivalent to model (23)–(29), without (24) and with the addition of (36).

$$\sum_{m\in\hat{M}_i} (V_m^{t+1} - V_m^t) + V_i^{t+1} - V_i^t + Q_i^t + S_i^t = N_i^t \hat{Y}_i \quad \forall\, i \in I,\, t \in T \quad (\pi_{i,t}^0) \tag{36}$$

In constraint (36), $\hat{M}_i$ is the set of all plants that are upstream of the plant i, differently of M_i that is the set of plants immediately upstream of the plant i. It is also important to understand that the change of the constraint (24) by (36) does not change the solution of the SP for the same $\hat{Y}_i$.

The reformulated master problem (MP^R) is equivalent to model (30)–(32), without (31) and with the addition of (37).

$$MP \leqslant \sum_{i\in I}\sum_{t\in T} Y_i[\overline{Q}_i \pi_{i,t}^{3,\ell} + (\overline{V}_i - \underline{V}_i)\pi_{i,t}^{4,\ell}] + \sum_{i\in I}\sum_{t\in T} (N_i^t \pi_{i,t}^{0,\ell}) Y_i - \pi^{2,\ell} \sum_{i\in I} (\widehat{FE}_i Y_i) \quad \forall\, \ell = 1, \dots, k \tag{37}$$

3.3. An insight about the use of parallel representation in the cooperative game theory MILP model

After simulating the Benders decomposition model using the parallel representation of the reservoirs, it was possible to notice a significant improvement in the computational time when compared with the traditional series representation. Results about CPU time are described in Appendix A. Therefore, it was carried out the parallel representation in the previous MILP formulation presented in Section II.

The only change needed in the CGM formulation to move from its series representation to a parallel representation is to substitute constraint (15) by constraint (38).

$$\sum_{m\in\hat{M}_i} (V_m^{t+1} - V_m^t) + V_i^{t+1} - V_i^t + Q_i^t + S_i^t = N_i^t Y^i \quad \forall\, i \in I,\, t \in T \tag{38}$$

4. Computational experiments

The proposed approaches were implemented using the commercial software CPLEX [27]. The computational experiments were conducted using a personal computer with 8 Gb RAM and Intel(R) Core(TM) i7-3770 processor.

To perform the analysis the data from the Brazilian Electrical System was used and corresponds to the long-term expansion auction A-5 of 2014 [43] (see Supplementary Table I for more information). In the configuration used, the system has about 110[GW] of hydro power installed capacity, 170 hydro plants are simulated (53% with reservoir storage, and 47% run-of-river). There are 84 years of hydrological data available, and the critical period of the system is of 5 years (1951–1955) defined using model (1)–(10).

4.1. Cooperative game theory applied with LA method

Here, we evaluate which model configuration is more suitable for solving the CGM via the MILP procedure. Table 1 presents the total CPU time to compute the FE_MILP algorithm with a different number of plants. In the column "Number of Constraints", it is listed the number of constraints of the cooperative game needed to reach algorithm convergence. The convergence criteria adopted was $\varepsilon = GFE_I \times 0.1\%$, that is, in order to stop the algorithm, the violation computed by the CGM has to be smaller than 0.1% of the system FE.

We note that the parallel representation overcomes in speed the

Table 1
Performance of the MILP in the series and parallel representation of reservoirs.

Number of plants	Total CPU Time (s)		Number of constraints	*GFE*
	Series	Parallel		
5	0.5	0.3	1	431.5
10	16.3	5.6	3	1754.5
15	31.1	3.7	1	4384.1
20	115.4	9.6	2	4572.7
25	1640.6	32.4	5	5785.6
30	13620.9	99.4	5	6397.0
35	31547.0	100.7	3	9343.3
40	70255.5	423.5	3	9655.7
170	–[a]	264756.9[b]	21	59178.5

[a] The computer runs out of memory before finding the first constraint.
[b] Equivalent to 10 h of wall clock time.

series representation. For the benchmark with 40 hydro plants, that is the larger benchmark that the series representation was capable to run, the parallel representation is at least 150 times faster than the series representation.

Also, the parallel representation was capable to solve an allocation problem of about 170 hydro plants. Finally, it is interesting to notice the small number of constraints needed by the algorithm presented in Fig. 1 to converge.

Fig. 3 shows a histogram for the system composed of 170 hydro plants. The x-axis bins represent percentage differences between the FE allocation using the LA method (allocation of reference) and using the FE_MILP.

Even with a small number of cooperative game constraints (21 constraints), it is possible to notice the significant contribution that the cooperative game theory can provide when applied together with the LA method. There are several hydro plants that changed their FE_i significantly, showing that an isolated application of the LA method could lead to allocations far from the core of the game, and therefore discourage a cooperative behavior among the hydro plants.

Fig. 4 shows the violation $\widehat{GFE}_{I_s^{k+1}} - \sum_{i \in I_s^{k+1}} \widehat{FE}_i^{k}$ computed by the FE_MILP algorithm at each iteration k, during the simulation time, for the 170 hydro plants test case. It is possible to notice a fast decrease in violation during the early stages of the algorithm with a significant slowdown as the number of iterations increases.

4.2. Benders decomposition performance

As shown in Appendix A, the most efficient configuration of the Benders algorithm in computing the core constraints is using the

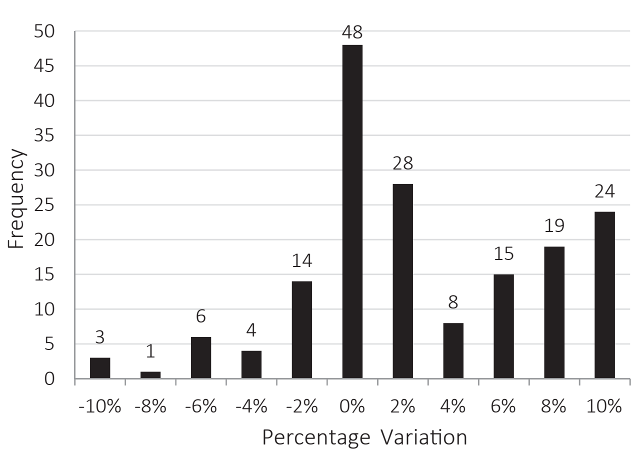

Fig. 3. Percentage difference between last addition allocation and FE_MILP algorithm.

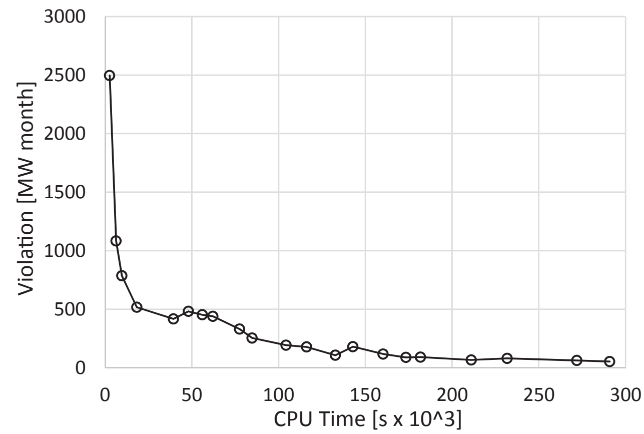

Fig. 4. Evolution of the violation computed by the FE_MILP algorithm.

parallel representation of reservoirs with Papadakos enhancement technique.

This section compares the performance of the CGM in the MILP configuration and the CGM in a Benders configuration (with Papadakos enhancement), both considering the parallel representation of reservoirs. The idea is basically to confront the best models developed so far in order to see which one is more efficient in computing core constraints.

Fig. 5 shows the CPU time required for Benders (vertical axis) and for MILP (horizontal axis), to search for the first core constraint using benchmark systems with 10 to 170 hydro plants. We employ the LA method to define the initial FE allocation. The black line in Fig. 5 represents the limit where Benders and MILP have the same performance in terms of CPU time. In order to illustrate the influence that the number of plants has in the computational time there are three color scales. Marks in blue represent simulations with 10 up to 70 hydro plants, marks in green represent simulations with 75 up to 125 hydro plants, and marks in red represent simulations with 130 up to 170 hydro plants.

From Fig. 5, it is possible to notice that the MILP procedure overcomes the Benders procedure most of the times. For more details about the Benders algorithm performance refer to Appendix A.

5. Conclusion

This work investigates the application of cooperative game theory in the computation of firm energy rights. It proposes a hybrid model that uses the last addition allocation method and cooperative game theory to

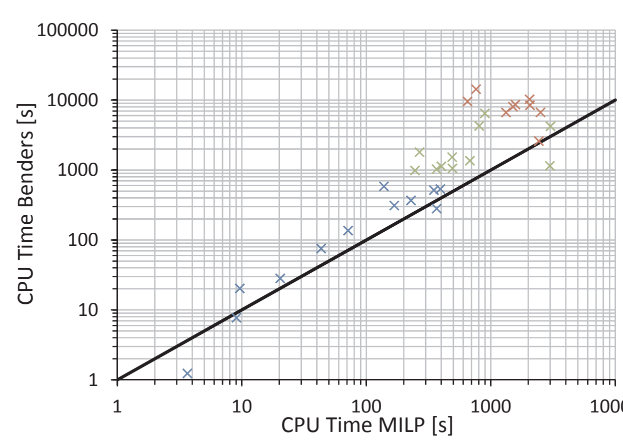

Fig. 5. MILP vs Benders CPU time.

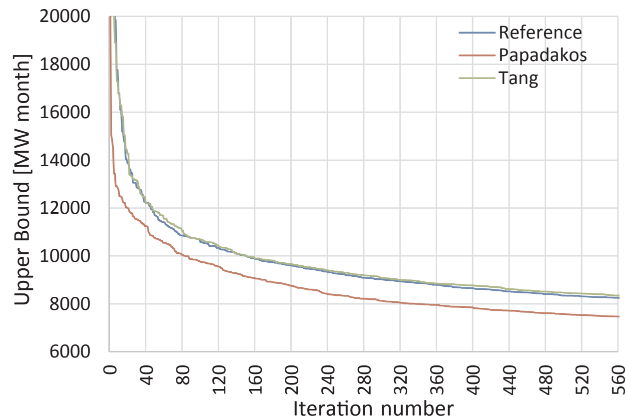

Fig. 6. Benders decomposition MP upper bound-series representation of the reservoirs.

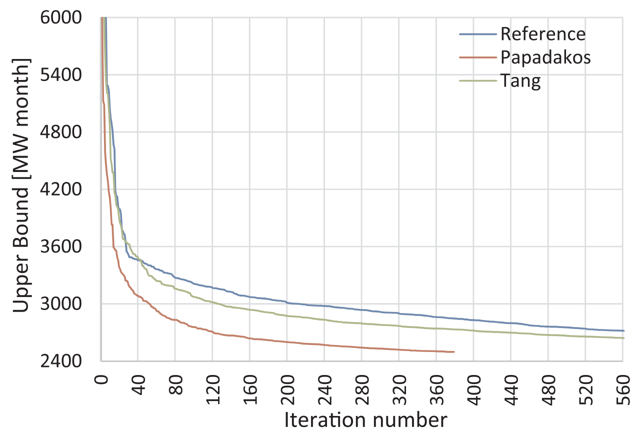

Fig. 7. Benders decomposition MP upper bound - parallel representation of the reservoirs.

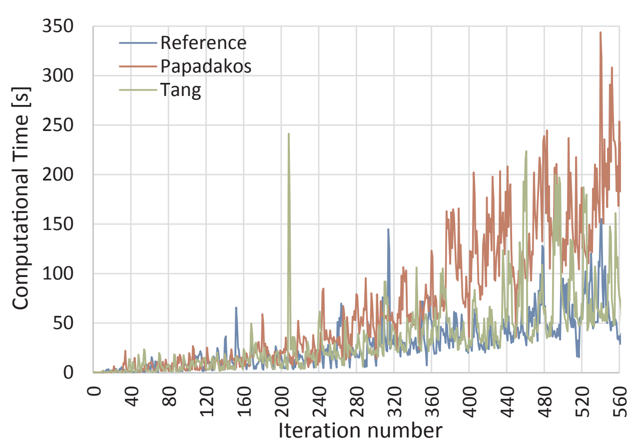

Fig. 8. Benders decomposition MP CPU time per iteration- series representation of the reservoirs.

create an allocation model that encourage the cooperation between the hydro agents while meeting the needs of the regulatory agencies. The limitations of the traditional mixed integer linear programming models in computing the core constraints are eliminated with a new framework that requires less computational time to converge, and also fewer constraints of the cooperative game to be added.

Comparisons between the performance of the Benders algorithm and the mixed integer linear programming when dealing with the firm energy allocation problem are performed. There is no significant advantage in solving the problem via Benders decomposition. Therefore, the approach using the mixed integer linear programming with parallel representation of reservoirs that combines the last addition method with cooperative game theory should be employed to efficiently and fairly determine firm energy rights in hydro power generation systems.

Future work should investigate other allocation methods, different from the last addition method, that promote incentives to the optimal development of the electrical systems. These methods could be used in

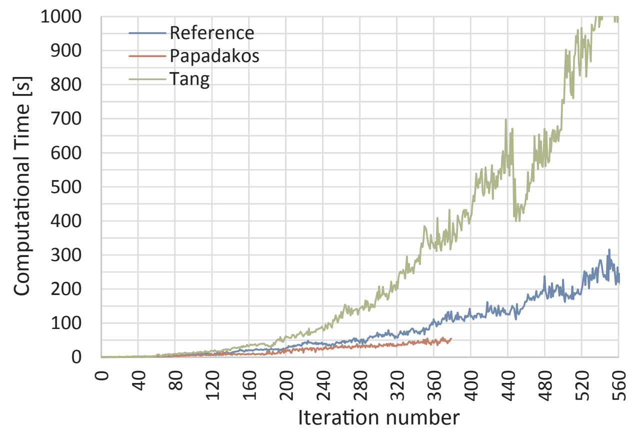

Fig. 9. Benders decomposition MP CPU time per iteration- parallel representation of the reservoirs.

combination with cooperative game theory, as the approach suggested here, to encourage the cooperation of hydro agents while keeping fairness. Also, methods that aim to reduce computational time in computing the cooperative game constraints should be analyzed.

Acknowledgements

The authors would like to express their gratitude to FAPEMIG, CNPQ and CAPES in Brazil for the financial support of this work. The second author would like to also thank the support of the National Science Foundation under Grant CyberSEES-1442909.

Appendix A

This portion of the paper evaluates which model configuration and enhancement technique is more suitable for solving the CGM using Benders scheme. Simulations are performed using the system with 170 hydro plants and the initial allocation adopted is given by the values obtained by the LA allocation. In this case, the most violated coalition presents a difference of 2496 [MW month] between what is allocated by the LA method and the minimum that the coalition should receive (14).

Simulations are performed using series and parallel representation of reservoirs, with Papadakos [38], and Tang [34] enhancement techniques. It is possible to notice that more than 99.5% of the simulation time was spent in solving the MP. Thus, the focus here will be in analyzing the MP results. Fig. 6 shows the evolution of the upper bound (UB) using the Benders model and the series representation of reservoirs.

As it follows in the legend in Fig. 6, the blue curve represents the traditional Benders model without any enhancement procedure, the red curve represents the Benders model with Papadakos [38] enhancement procedure, and the green curve represents the Benders with Tang [34] enhancement procedure. In this case, none of the models achieved convergence in 560 iterations.

Fig. 7 presents similar information using the parallel representation of reservoirs. By comparing Figs. 6 and 7, it is possible to notice that the only configuration that reaches convergence in less than 560 iterations is the parallel representation using Papadakos enhancement technique. It is also interesting to notice the large influence that the model configuration has in the performance of the Benders algorithm. From these results, we notice a considerable efficiency enhancement with the parallel representation.Figs. 8 and 9 show the total CPU time required to solve the MP at each iteration (for both series and parallel representation).

By comparing Figs. 8 and 9, it is possible to notice a trend of significant increase in CPU time, at least for the traditional Benders model (blue) and Tang model (green), when moving from the series to the parallel representation of reservoirs. However, if we analyze carefully the Papadakos CPU time, we notice that during 382 iterations (that is the number of iteration when the model converges for the first time – Fig. 7) the parallel representation only consumes a total of 67% of the time that the series representation takes to achieve the same 382 iterations.

From the results presented in the Appendix, it is possible to say that the model with the best performance in the Benders decomposition scheme is the parallel representation of reservoirs using the Papadakos enhancement technique.

Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apenergy.2018.06.065.

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