Performance Comparison of Equivalent Reservoir and Multireservoir Models in Forecasting Hydropower Potential for Linking Water and Power Systems

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Abstract: To link water and power systems on a regional scale, equivalent reservoir models—an aggregated representation of a multireservoir system—are commonly used because conventional river-basin scale optimization models become computationally expensive with increasing dimensionality. Although equivalent reservoir models are widely applied in power system operation, analyses comparing the performance of equivalent reservoir models with multireservoir cascade models are limited. To this end, this study systematically compares two equivalent reservoir models, an aggregated water balance and an energy balance representation, with a multireservoir cascade representation for a system of three reservoirs in series in Savannah, South Carolina, in terms of the total end-of-period release, hydropower and storage based on simulation, simulation optimization, and analytically over a 30-year period. Findings from the pilot basin are generalized by altering the storage-to-demand ratio (SDR) to understand the effect of different system characteristics on the equivalent reservoir representation under observed and predicted inflows of different skills. Equivalent reservoir models perform similarly to the cascade model for systems with large SDRs, but for systems with smaller SDRs, equivalent reservoir models perform poorly because spill and other losses from individual reservoirs cannot be effectively represented in the aggregated approach. **DOI: 10.1061/(ASCE)WR.1943-5452.0001343.**

Introduction

Multipurpose multireservoir systems allocate water for agricultural and municipal sectors and meet hydroelectric demand within a system/region while ensuring instream ecological demand and flood protection. Increasing withdrawals for human (Sankarasubramanian et al. 2017) and irrigation (Das et al. 2018) needs have challenged water availability for hydropower and ecological needs (Kominoski et al. 2018). Hence, modeling multipurpose multireservoir systems has received considerable attention in the water system (Yeh 1985; Labadie 2004) and power system literature (Arvanitidis and Rosing 1970a; de Queiroz 2011). While studies in the water resources literature have modeled complex reservoir networks using various system techniques—linear programming (Crawley and Dandy 1993; Dahe and Srivastava 2002), dynamic programming (Nandalal and Bogardi 2007), nonlinear programming (Simonovic and Teegavarapu 2000; Labadie 2004), stochastic dynamic programming (Maceira et al. 2008; Dias et al. 2013), simulation-based optimization (Koutsoyiannis and Economou 2003a; Fang et al. 2014) and evolutionary algorithms (Rani and Moreira 2010; Ghimire and Reddy 2014)—by considering various programming techniques, the computational complexity of such methods, and the need for a low-dimensional, aggregated system representations have also been recognized (Arvanitidis and Rosing 1970a; Yeh 1985; Koutsoyiannis and Economou 2003a; de Queiroz 2011) to reduce the so-called curse of dimensionality in linking water and power systems.

The main aim of this study is to compare the performance of a multireservoir model, a cascade representation, with two equivalent reservoir modeling approaches—an aggregated energy balance representation (e.g., Arvanitidis and Rosing 1970a; Brandão 2010; Guo et al. 2013) and an aggregated water balance representation (Koutsoyiannis and Economou 2003a)—for understanding monthly-to-seasonal coordination of water and power systems under observed and forecasted inflows. The basis behind equivalent reservoir formulation is that the conservation of mass or energy must be ensured in the composite representation of a reservoir network system (Koutsoyiannis and Economou 2003a; de Queiroz 2011). The aggregated energy balance model uses the potential energy stored in the system and energy flows into the system estimated based on reservoir storage and inflows respectively to estimate the total hydropower by developing an empirical function relating the cumulative hydropower generation to the simulated end-of-period potential energy stored and hydropower from the system (Arvanitidis and Rosing 1970a). The method proposed by Arvanitidis and Rosing (1970a) has since its first publication been widely adopted to represent hydropower generation from multiple reservoirs in power systems engineering (Turgeon 1980, 1981, 1982; Pereira and Pinto 1985, 1991). The energy balance of the aggregated system is ensured instead of considering the energy balance of individual hydropower plants. Further, in the energy balance modeling approach, release for other uses (e.g., irrigation, municipal supply) that do not contribute to hydropower generation are deducted

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upfront before calculating the available water for hydropower. Recent studies (Maceira et al. 2008; de Matos et al. 2008; de Queiroz 2011; Goor et al. 2011) extended the original formulation of Arvanitidis and Rosing (1970a) to accommodate run-of-the-river hydroelectric plants to support power system planning. Studies have also proposed an aggregation/disaggregation method for water and power system coordination (Turgeon 1981, 1982; Saad et al. 1994), but these studies on equivalent reservoir modeling focus on hydropower availability and the design of rule curves over the long term, not on how the aggregated models estimate monthly hydropower generation under given operational rule curves or policies. To summarize the energy balance modeling approach, the main limitation is not considering individual plants' constraints, such as their intended uses, generation details, and target storage information. Further, working with an aggregated system also could face challenges in providing stochastic representation to incorporate energy inflows and power demand uncertainty.

The second approach in equivalent reservoir formulation is based on an aggregated water balance of multireservoir systems that are parallel to each other (Koutsoyiannis and Economou 2003a). In water management studies, however, use of this approach is limited, mainly because water allocation to individual users from a given reservoir is not available in the equivalent reservoir model representation (Koutsoyiannis and Economou 2003a). Still, this approach can be used to quantify the total amount of water available at any stage and the total downstream release that can be expected at any stage particularly for flood warning/control. Koutsoyiannis and Economou (2003a, b) proposed a theoretical basis for the application of an equivalent reservoir model for the New York City (NYC) system by considering a system of parallel reservoirs, which cater to a single demand downstream of the system. Under this approach, inflows, outflows, and storage for the equivalent reservoir would be a simple sum of the respective quantities for individual reservoirs primarily for parallel reservoir systems. However, for reservoirs in series, a proper theoretical formulation of equivalent reservoir representation is not available in the literature to support water allocation and hydropower generation on a monthly-to-seasonal time scale. *One goal of this study is to develop an equivalent reservoir representation for a cascade system using aggregated water balance and energy balance approaches and to quantify the aggregated hydropower generated for supporting monthly-to-seasonal operation of both systems.*

Despite the rich literature on equivalent reservoir modeling, few studies focused on bridging the energy balance approach and water balance approach to address the limitations of both modeling approaches (e.g., Santos et al. 1989; Maceira et al. 2011). Santos et al. (1989) proposed an equivalent reservoir representation considering both aggregated water and energy balance constraints in the context of hydropower generation. However, Santos et al. (1989) suggested the combined water and energy balance approach for capacity expansion problems, but not for reservoir operations linking water and power systems as shown by Arvanitidis and Rosing (1970a) and Fang et al. (2014). Even though equivalent reservoir formulation provides an aggregated, low-dimensional representation, it still requires a considerable number of parameters and functions to estimate the aggregated system attributes, which could compromise model parsimony, thereby leading to increased computational time (Cain et al. 2012). For instance, Arvanitidis and Rosing (1970a) suggested estimating the total hydropower generated from the basin based on an empirical function that relates the hydropower generated from the entire system with the estimated energy outflow and potential energy. Nandalal and Bogardi (2007) proposed a novel approach where storage of the composite reservoir is assumed to be equal to the sum of individual reservoir storage volumes at each time stage, but suggested only a fraction of each reservoir inflows contributes to the composite system, which results in increasing the parameter space as the number of reservoirs in the network increases. Fang et al. (2014) and Brandão (2010) used long-term mean productivity for converting water storage and water flows into corresponding energy terms, but this simplification can lead to significant discrepancies, especially when the length of observed data is small or the relation between total hydroelectric generation, storage, and water outflow is highly nonlinear. Studies on optimal transmission switching based on IEEE test systems have shown that the need for a simplified optimal power flow model with fast computational capacity is required to support reliable power generation and transmission and to reduce the generation cost (Soroush and Fuller 2013). Simplified representation of alternating current optimal power flow could save approximately $19 billion per year for an improved efficiency of 5% (Cain et al. 2012). Given that simplified models, such as equivalent reservoir model performance, are also largely influenced by inflow characteristics of basins and the operational policy of reservoir networks (Arvanitidis and Rosing 1970a; Brandão, 2010; Koutsoyiannis and Economou 2003a), a critical assessment of their performance under forecasted flows is also needed. *To our knowledge, few if any studies provide effective parameterization or equivalent water and energy balance representation, in terms of aggregated storage, hydropower, and release, of equivalent reservoir models for multiple reservoirs in series. A systematic comparison of the performance of equivalent reservoir representation based on water balance and energy balance approaches with a multireservoir model for a cascade (i.e., reservoirs in series) is currently lacking*. Hence, a systematic comparison of equivalent reservoir models, both aggregated water and energy balance representations, with multireservoir models in a cascade system under observed and forecasted inflow conditions and under different reservoir system configurations would provide useful information for integrating water and power systems on a regional scale.

The primary motivation for the comparison of these two representations—cascade model and equivalent reservoir model—is primarily in integrating hydropower generated from multiple river basins with power systems. Since power systems operate on a regional/subnational scale connecting multiple hydropower units that are in series and parallel across various river basins, it is important to have an aggregate reservoir representation for integrating hydropower units with other power generation units to support power system planning and operation (Arvanitidis and Rosing 1970a; Ahmed and Lansey 2001; Brandi et al. 2015; de Queiroz 2016; Fredo et al. 2019). However, reservoir modeling using cascade models typically limits itself to the river basin scale, strictly adhering to rule curves for releases across multiple uses (Golembesky et al. 2009; Oludhe et al. 2013). For that reason, we present a systematic performance comparison of equivalent reservoir models developed using a water and energy balance representation with a multireservoir cascading system for forecasting hydropower potential and generalize the comparison under various reservoir system configurations by deriving relationships for equivalent energy generated from the cascade system. For this purpose, we develop a three-reservoir multipurpose cascading system in Savannah, South Carolina, and conduct a systematic comparison with equivalent reservoir models. We also generalize the findings on the performance of equivalent reservoir modeling to humid to temperate systems by considering additional reservoir system configurations and changing the storage-to-demand ratios (SDRs) of the Savannah system (Sankarasubramanian et al. 2009). Based on current and potential system configurations, we intend to address the following research questions:

1. What are the potential challenges in utilizing an equivalent reservoir model for coordinating water and power system operation on a monthly to seasonal time scale? How do the equivalent reservoir models perform in estimating actual hydropower generation from a cascade system under wet/dry inflow conditions?
2. How do we effectively relate the hydropower generated from the multireservoir cascade model with equivalent reservoir model configurations? How does the revised parameterization of an equivalent reservoir model compare with the hydropower generated from the multireservoir cascade system?
3. Is there an optimal system configuration, represented by SDR, at which the equivalent reservoir performance is significantly inferior to the multireservoir cascade model performance for humid/temperate basins? How do we generalize the findings on the utility of equivalent reservoir configurations for integrating water and power systems' coordination?

The manuscript is organized as follows. The next section, "Methodology," presents the formulations of multireservoir and equivalent reservoir modeling along with new relationships that relate hydropower generation from equivalent reservoir models to the multireservoir cascade model. Here we also discuss pilot basin details along with the experimental design to evaluate the equivalent reservoir under alternative system configurations. Next, the "Results and Discussion" section compares the performance of the equivalent reservoir model with the multireservoir cascade model under current and potential system configurations as well as under different inflow conditions using a simulation-optimization scheme. Finally, we summarize the findings and conclude with remarks on how an equivalent reservoir formulation can facilitate linking water and power systems over a large region comprising multiple reservoirs over multiple basins.

Methodology

This section describes the multireservoir model using the simulation-optimization procedure, followed by a presentation of two candidate equivalent reservoir models. All models presented here are at a monthly time step. For reservoir systems with significant storage such as the Savannah system, initial storage typically guarantees monthly power demand subject to release and storage constraints. Thus, a power system model may operate at a finer time scale; if the initial storage guarantees power demand, then the power system will utilize hydropower subject to generation constraints (e.g., ramp-up and ramp-down constraints) of nonhydropower plants (de Queiroz et al. 2019). Since the within-the-month variability in streamflow is also dampened by reservoir storage, a monthly model is sufficient (Sankarasubramanian et al. 2009; Oludhe et al. 2013) for comparing the performance of equivalent reservoir models with the performance of the multireservoir cascade model in estimating the monthly hydropower demand. Thus, the presented analysis in comparing equivalent reservoir models with the cascade representation provides insights on power system planning on a monthly to seasonal time scale, as opposed to real-time power system operation.

Multireservoir Cascade Model

The multireservoir model uses mass balance to route water flow from upstream reservoirs to downstream reservoirs in each time step. Since reservoirs in parallel are two independent systems, they can be easily represented as equivalent reservoir systems. Hence, we limit our discussion to a multireservoir model with reservoirs in series based on Eq. (1). The vertical bar in Eq. (1) and in other equations in this article denotes *when* or *conditional*:

$$S_t^j + R_t^j = S_{t-1}^j + (R_t^{j-1}|_{j>1} + I_t^j) - W_t^j|_{S_t^j > S_{\max}^j} + def_t^j|_{S_t^j < S_{\min}^j} \tag{1}$$

Water releases and incremental net inflow [i.e., streamflow + (precipitation − evaporation) over the lake] from and into the jth reservoir at the tth time step (month) are R_t^j [hm^3 (million m^3)/month] and I_t^j (hm^3/month), respectively. The term R_t^{j-1} indicates releases from an immediate upstream reservoir at the tth time step that contributes to the net inflow to the jth reservoir. Storage volume (hm^3/month) at the beginning and end of the time period are denoted by S_{t-1}^j and S_t^j, respectively. Water elevation in the reservoir provides the storage, which is obtained from the storage–elevation relationship for a given reservoir. For run-of-the-river systems lacking significant storage, the head available for power generation could be effectively incorporated by computing the effecting storage volume and the head on the upstream reservoirs (Arvanitidis and Rosing 1970a, b; Brandão 2010; de Queiroz 2011). W_t^j and def_t^j are spills and deficits, respectively, which occur when the end-of-month storage violates the maximum and minimum storage constraints. The actual release, $R_t^j - def_t^j$, is less than the target release, R_t^j, if S_t^j drops below the minimum storage. This approach sets storage limits [Eq. (2)], but release constraints are often controlled by minimum flow requirements for meeting environmental standards and by the maximum allowable downstream flood release:

$$\begin{aligned} S_{\min}^j &\le S_t^j \le S_{\max}^j \\ R_{\min}^j &\le R_t^j \le R_{\max}^j \end{aligned} \tag{2}$$

For a hydropower system, it is a common practice to consider an additional constraint to incorporate power generation [Eq. (3)]:

$$G_{\min}^j \le E_t^j \le PC_{\max}^j \tag{3}$$

where $G_{\min}$ (in megawatts, MW) is the minimum generation controlled by the minimum energy demand for the time stage considered; and PC_j (MW) is the generation capacity of the jth system. The minimum and maximum generation for the Savannah system is obtained based on the USACE reported minimum weekly hydropower demand from three reservoirs (USACE 2013). Reported weekly minimum (maximum) hydropower values for each reservoir typically occur with allowable minimum (maximum) releases and storage for each reservoir. Power generation by the jth system is calculated based on the generation efficiency of the plant (η), release from the reservoir R_t^j, and head in the reservoir h_t^j, which is a function of the storage as given in Eq. (4*a*). For simplicity, releases for all uses (e.g., municipal, irrigation) go through penstocks to generate hydropower. If release for different uses goes through different penstocks, then Eq. (4*a*) could be modified to estimate the different head for the given release for that use. Thus, the generated hydropower from that penstock would simply be an additive term in Eq. (4*a*) for that use:

$$E_t^j = \gamma \eta h_t^j R_t^j \tag{4a}$$

The generated hydropower E_t^j (MW) in Eq. (4*a*) could be written as a nonlinear function of release, R_t^j, by substituting the effective head

$$h_t^j = \phi^j\left(\frac{S_{t-1}^j + S_t^j}{2}\right)$$

where $\phi(S)$ is defined by the storage-to-head relationship of the reservoir. Because S_t^j is linearly dependent on R_t^j [Eq. (1)], hydropower generated, E_t^j, in turn becomes a nonlinear function of release during the time step. Thus, hydropower generated in Eq. (4a) could be written as Eq. (4b):

$$E_t^j = \gamma\eta\alpha^j R_t^{j\beta^j} \tag{4b}$$

where α and β provide a power-law representation for the nonlinear relationship between release and generated hydropower.

Equivalent Reservoir Models

Energy-Balance Equivalent Reservoir Model

The energy-balance equivalent reservoir (EqR-EB) is modeled using an energy balance for the system by expressing the water storage and water flows in terms of potential energy storage (PE_t) and energy flows ($E_{\mathrm{inflow},t}, E_{\mathrm{outflow},t}$), both in MWmonth. Energy outflows are obtained by converting the release with the energy generation factor, calculated using a simulation with observed outflows and power generation for the system. Here, we present the formulation given by Arvanitidis and Rosing (1970a), in Eqs. (5)–(8), for a system of n reservoirs in series. For any jth reservoir, $j = 1, \ldots, n$, potential energy storage is a function of the jth reservoir's storage and all mth downstream reservoirs' ($m = j, \ldots, n$) energy generation factors, p^m, $m = j, \ldots, n$, so that water storage at any upstream reservoir will be available as a potential energy head for downstream reservoirs, depending on the current (jth) and downstream reservoir (m, $m = j, \ldots, n$) energy generation factor [Eq. (5)]:

$$\mathrm{PE}_{t-1} = \sum_{j=1}^{n} S_t^j \left(\sum_{m=j}^{n} p^m \right) \tag{5}$$

$$E_{\mathrm{inflow},t} = \sum_{j=1}^{n} (R_t^{j-1}|_{j>1} + I_t^j) p^j \tag{6}$$

$$E_{\mathrm{outflow},t} = \sum_{j=1}^{n} R_t^j p^j \tag{7}$$

$$\mathrm{PE}_t + E_{\mathrm{outflow},t} = \mathrm{PE}_{t-1} + E_{\mathrm{inflow},t} \tag{8}$$

While conversion of the water inflows and outflows from a plant depends only on its own mean productivity, potential energy stored in an upstream plant is the sum of the potential energy stored in all downstream plants, including itself. Brandão (2010) called p the mean productivity of a hydropower plant. This energy generation factor (p) is referred to as productivity, which is defined as the hydropower generation by unit turbine release. Another way to interpret p is as a first-order (i.e., linear) approximation of Eq. (4b) that represents the hydropower generated from a reservoir j for unit discharge.

Water Balance–Based Equivalent Reservoir (EqR-WB)

For the water balance–based equivalent reservoir formulation, with n reservoirs indexed by j, we can rewrite Eq. (1) as Eq. (9), in the same units as previously:

$$S_{eq,t} + R_{eq,t} = S_{eq,t-1} + I_t - W_t|_{S_{eq,t} > S_{eq,\max}} + def_t|_{S_{eq,t} < S_{eq,\min}} \tag{9}$$

where

$$I_t = \sum_{j=1}^{n} I_t^j \qquad S_{eq,t-1} = \sum_{j=1}^{n} S_{t-1}^j \qquad S_{\max} = \sum_{j=1}^{n} S_{\max}^j$$
$$S_{\min} = \sum_{j=1}^{n} S_{\min}^j \qquad R_{\max} = R_{\max}^n \qquad R_{\min} = R_{\min}^n \tag{10}$$

To estimate hydropower generation from the EqR-WB model, we modify Eq. (4) for equivalent head of water ($h_{eq,t}$) and equivalent water release ($R_{eq,t}$), as in Eq. (11a):

$$E_{eq,t} = p_{eq,t} R_{eq,t} = \gamma\eta_{eq} h_{eq,t} R_{eq,t} \tag{11a}$$

Appendix II [Eq. (24)] provides an expression for calculating equivalent head ($h_{eq,t}$) and productivity ($p_{eq,t}$). Since $R_{eq,t}$ is equal to the downstream release, one can estimate $h_{eq,t}$ based on the release from the reservoir that is furthest downstream. Thus, $h_{eq,t}$ is the effective head of water, which is assumed to be a nonlinear function of the individual reservoir's release from the cascade and of $R_{eq,t}$, which equals the downstream release. Eq. (11a) can also be expressed as a function of the equivalent storage, $S_{eq,t}$ [Eq. (26)]. Thus, hydropower generation in Eq. (11a) could be written as Eq. (11b) subject to the constraints as in Eq. (11c):

$$E_{eq,t} = \gamma\eta_{eq}\phi(S_{eq,t}) R_{eq,t} \tag{11b}$$

$$G_{\min} \le E_{eq,t} \le PC_{\max}$$
$$PC_{\max} = \sum_{j=1}^{n} PC_{\max}^j \quad \text{and} \quad G_{\min} = \sum_{j=1}^{n} G_{\min}^j \tag{11c}$$

Relating EqR-EB and EqR-WB Models to 3R-WB Model for Simulation Optimization

Fig. 1 shows a schematic comparison of the multireservoir cascade model (nR-WB) along with two different equivalent reservoir model approaches (EqR-EB and EqR-WB). The first two sections of this article provide primarily a simulation framework for estimating the hydropower generated from each model. However, to compare the performance of multireservoir cascade model with equivalent reservoir models, there are no direct relationships that can relate the output from the nR-WB cascade model to the equivalent reservoir models.

This section provides a basis for comparing the cascade model with equivalent reservoir models and utilizes it for the simulation-optimization framework based on expressions from Appendixes I and II. A summary of the simulation-optimization analysis for both nR-WB model and EqR models is presented in Table 1. In the simulation-optimization framework for both nR-WB and equivalent reservoir models, we used an interior point algorithm for

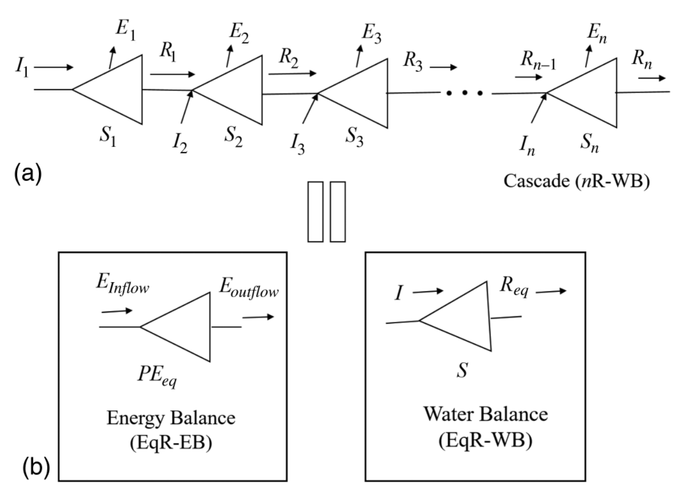

Fig. 1. (a) Schematic representation of multireservoir cascade model; and (b) two equivalent reservoir models considered in this study.

Table 1. Details of simulation-optimization setup for three models

Model	Objective function	Decision variable	Constraints
3R-WB	$\arg\max \sum_{j=1}^{n} E_t^j$	$S_t^j, R_t^j, j = 1,2,3$	Eqs. (1)–(3)
EqR-EB	$\arg\max \sum_{j=1}^{n} \{p^j - p^{j+1}\} R_t^j$ [Eq. (12)]	PE_t, $E_{\text{outflow},t}$	Eqs. (5)–(8)
EqR-WB	$E_{Eq,t}$ [Eqs. (11*c*) and (27)]	$S_{Eq,t}$, $R_{Eq,t}$	Eqs. (9)–(11*c*)

optimization with the reservoir simulation model (Barros et al. 2003; Rani and Moreira 2010). This method solves the constrained optimization by Lagrangian multiplier with a linear approximation technique. All three models were run on MATLAB version 9.6 (R2019a). For simulation-optimization we used the *fmincon* nonlinear solver that uses an interior point algorithm for optimization with the reservoir simulation model. To run the monthly simulation-optimization models for the Savannah system, we considered both observed inflows and realizations of forecasted inflows. Although conventionally maximizing hydropower is not the objective in power system studies, the power system model utilizes the available hydropower demand that meets the transmission and generation constraints since the marginal cost of hydropower production is the smallest among all available technologies (de Queiroz et al. 2019). Thus, maximizing hydropower based on release constraints justifies the objective of this study.

The multireservoir model (nR-WB) described in the section "Multireservoir Cascade Model" is used as a simulation-optimization model, with the objective being to maximize monthly hydropower generation:

$$\sum_{j=1}^{n} E_t^j$$

subject to the release and storage constraints given in Eqs. (1)–(3), with E_t^j estimated using Eq. (4*a*).

For the EqR-EB model, given PE_{t-1} and $E_{\text{inflow},t}$ from Eqs. (5) and (6), we can write $E_{\text{outflow},t}$ as in Eq. (12) (Appendix I):

$$PE_t + \sum_{j=1}^{n} \{p^j - p^{j+1}\} R_t^j = PE_{t-1} + E_{\text{inflow},t} \tag{12}$$

The objective function for the optimization of the EqR-EB model is the second term on the left-hand side (LHS) of Eq. (12):

$$\text{argmax} \sum_{j=1}^{n} \{p^j - p^{j+1}\} R_t^j$$

subject to the release constraints given in Eq. (2). Energy outflow from the nR-WB model can be simply a sum of the hydropower generated [Eq. (4*a*)] from all the reservoirs, but it could also be expressed using the productivity factor, p, of the reservoirs (Appendix I):

$$E_{\text{outflow},t} = \Delta PE + \sum_{j=1}^{n} p^j \sum_{m=1}^{j} I_t^j \tag{13}$$

For the EqR-WB model, given $S_{eq,t-1}$ and I_t, the objective is to maximize the equivalent hydropower generation, $E_{eq,t}$ [Eq. (11*a*)], subject to downstream release and storage constraints, R_t^n. But there is no relationship readily available to relate the equivalent downstream release to the total hydropower generated from the system. Appendix II relates the nR-WB information to estimate the equivalent hydropower, $p_{eq,t}$, generated from the EqR-WB model:

$$p_{eq,t} R_{eq,t} = \sum_{j=1}^{n} I_t^j \sum_{m=j}^{n} p^m + \sum_{j=1}^{n} (S_{t-1}^j - S_t^j)^j \sum_{m=j}^{n} p^m \tag{14}$$

Since $R_{eq,t} = R_{3,t}$, maximizing Eq. (14) will be equal to maximizing the total hydropower generated from the EqR-WB model. However, the LHS of Eq. (14) uses the full cascade form to estimate the total hydropower from the entire system. Hence, we develop a regression relationship [Eq. (27)] relating the EqR-WB terms [Eq. (9)], equivalent storage, $S_{eq,t}$, and downstream release, $R_{eq,t}$, for estimating the LHS Eq. (14). For the EqR-WB model, we maximize the total hydropower estimated from the regression relation based on the equivalent release and storage constraints in Eq. (9).

Study Area and System Details

We selected a case study area in the upper Savannah Reservoir system that consists of three reservoirs on the border of South Carolina (SC) and Georgia (GA). The basin experiences the June–September as the wettest and warmest period, with a fairly constant amount of rainfall over the rest of the year. From upstream to downstream, these three reservoirs—Hartwell Lake (HWL), Richard B. Russell (RBR) Lake, and J. Strom Thurmond (JST) Lake (Fig. 2)—are operated by the USACE to meet the water supply, flood control, and power generation needs of the Savannah River basin. RBR Lake, a pump-back hydropower facility, has a small storage capacity and is used primarily for peak power generation. HWL and JST Lakes have similar storage capacities (Table 2), and both are also used for peak power generation. The current operating policy involves meeting water and power demands and maintaining seasonal storage targets for drought management. Hydropower generated from these three USACE plants is marketed by the Southeastern Power Administration (SEPA). Located in the Upper Savannah watershed, HWL and JST facilities produced 452,911 and 677,202 MW·h of hydropower in 2014. The RBR facility produced 661,032 MW·h of

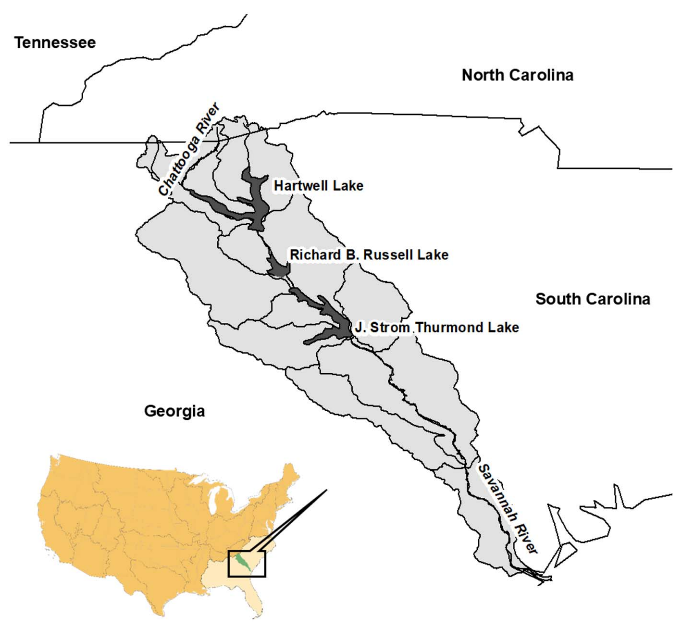

Fig. 2. Savannah River basin with three reservoirs in series, HWL, RBR, and JST lakes from upstream to downstream.

Table 2. Storage, outflow, and hydropower constraints for Savannah system (water elevation levels appear in parentheses)

Reservoir characteristic	HWL	RBR	JST	Total
$S_{\max}$ (hm^3) [elevation (m AMSL)]	3,506 (202)	1,275 (145)	3,577 (102)	8,359
$S_{\min}$ (hm^3) [elevation (m AMSL)]	1,389 (190)	1,109 (143)	1,807 (95)	4,312
$R_{\max}$ (hm^3/month)	2,090.6	4,401.2	2,200.6	8,692.4
$R_{\min}$ (hm^3/month)	278.7	278.7	278.7	836.2
Storage capacity/mean annual demand	1.12	0.29	0.59	1.39
Hydropower generation capacity (MW)	426	320	364	1,110

Note: Water elevation levels appear in parentheses.

hydropower in 2014. The monthly net inflow, outflow, and lake level data for the period 1985–2014 were obtained from USACE Savannah District Water Management's historic project data archives (USACE 2013). Spill elevation indicates the elevation corresponding to the top of flood storage. Historical data suggest that only RBR Lake has experienced elevations above its spill elevation of 145 m (475.3 ft) above mean sea level (AMSL). Hydropower is calculated using Eq. (4). For additional details on the operational policy of the system, please visit USACE's publications on Savannah River Basin Water Management. All the three plants are operated by USACE. Additional hydropower plants operated by Duke Energy or Georgia Power are on the Savannah basin downstream of the JST reservoir. These were not considered for this study because of limited data availability. The objective is to maximize the hydropower generation from these three systems by ensuring the streamflow constraints to be met at Augusta, downstream of JST. Power system coordination—purchasing and trading of electricity across different entities, or other drivers of power markets—is a topic that goes beyond the scope of this study. Because reservoirs situated in a cascade and operated by different entities are required to follow upstream and downstream flow regulation guidelines, our study provides a baseline evaluation on how equivalent reservoir representation performs in estimating hydropower generation in comparison to the cascade representation.

Generalization of Equivalent Reservoir Performance under Different Reservoir Configurations

To generalize the findings for basins with a similar hydroclimatology, we alter the reservoir system configurations and evaluate the three models using the monthly reservoir inflows recorded for the Savannah system as well as inflow forecasts generated with specified predictive skill (Appendix III). The proposed generalization will provide a regional perspective for linking water and power systems and for analyzing the nexus under various system configurations with a similar hydroclimatology. Findings from smaller system configurations, particularly aggregating run-of-the-river systems, would be very useful because they have comparatively less of an environmental impact.

To create different reservoir system configurations, we change the total storage capacity to the mean annual demand ratio (SDR) of the Savannah system. Studies have used SDRs and storage to mean annual runoff ratios for quantifying the utility of inflow forecasts on different system configurations (Vogel and Stedinger 1987; Maurer and Lettenmaier 2004; Sankarasubramanian et al. 2008; Li et al. 2014). Inflow forecasts for each month are generated from a multivariate lognormal distribution that preserves spatiotemporal dependence among reservoir inflows at Savannah (Sankarasubramanian et al. 2008; Li et al. 2014), with two forecast skills $\rho = 0.75$, 0.5 (Appendix III). Our aim here is to quantify the departures of the equivalent reservoir models from the cascade model for a given SDR and inflow forecast skill. The ratio of storage to mean annual runoff is 0.92 for reservoir systems in the South Atlantic–Gulf region (Graf 1999). For the Savannah system, this ratio is 1.39. The rationale behind choosing different SDRs are, first, that the average SDR in the generalization experiments is 0.94, which is close to 0.92, as estimated by Graf (1999) for the region; second, the proposed SDR range covers essentially run-of-the river systems as well as facilities with storage capacity double that of Savannah; and, third, systems with relatively smaller storage capacities (SDRs) are often operated together. We consider 10 different sets of hypothetical reservoir systems that maximize hydropower generation using the observed inflows into the Savannah system.

To develop a hypothetical cascade configuration for the 3R-WB model, we linearly modify the existing Savannah system by preserving the fractional storage contribution of each reservoir and the mean annual downstream release (i.e., demand) from the system. Based on this assumption, we obtain different SDRs of the system by changing the total storage capacity of the cascade system. Hence, to develop the cascade system configuration, any change (increase/decrease) in the total storage capacity of the Savannah system will be proportionately distributed based on the current fractional storage contributed by the individual reservoir. Thus, a total of 10 different SDRs (including the current system configuration) and 3 different inflow forecast [including observed inflows as perfect forecast (PF)] skills were considered, resulting in a total of 30 different comparisons between equivalent reservoir models with the 3R-WB model. Steps for evaluating these three candidate models under a given SDR and inflow forecast skill are as follows:

1. Using the observed monthly inflows and reported monthly releases for the Savannah system, we first run the 3R-WB model for the given SDR to simulate the storage at the end of the month. This simulated storage is then used as an initial storage for running both the cascade model and equivalent reservoir models for optimizing the releases under a given inflow forecast with a specified skill. This approach is reasonable because the current system configuration (in the preceding section) was set using the observed initial storage obtained for comparing the equivalent reservoir models with the cascade model to maximize hydropower releases.
2. Using the simulated initial storage obtained based on observed inflows from the 3R-WB model in the previous step, equivalent reservoirs are run with inflow forecasts with a specified skill for optimizing hydropower release. Initial storage and forecasted inflows for each ensemble member are aggregated to account for equivalent water and energy storage and inflows for running the EqR-WB and EqR-EB models. It is important to note that PF has only one member, which is the observed inflow. Thus, both equivalent reservoir models and the 3R-WB model are run separately with each inflow forecast ensemble member to obtain the releases for three reservoirs that maximize the hydropower generation under the given system configuration. These optimized releases obtained for each candidate model are averaged across the 50 ensemble members.
3. Using the averaged optimized release for EqR and 3R-WB models obtained for the inflow forecast, we evaluate the forecast-based

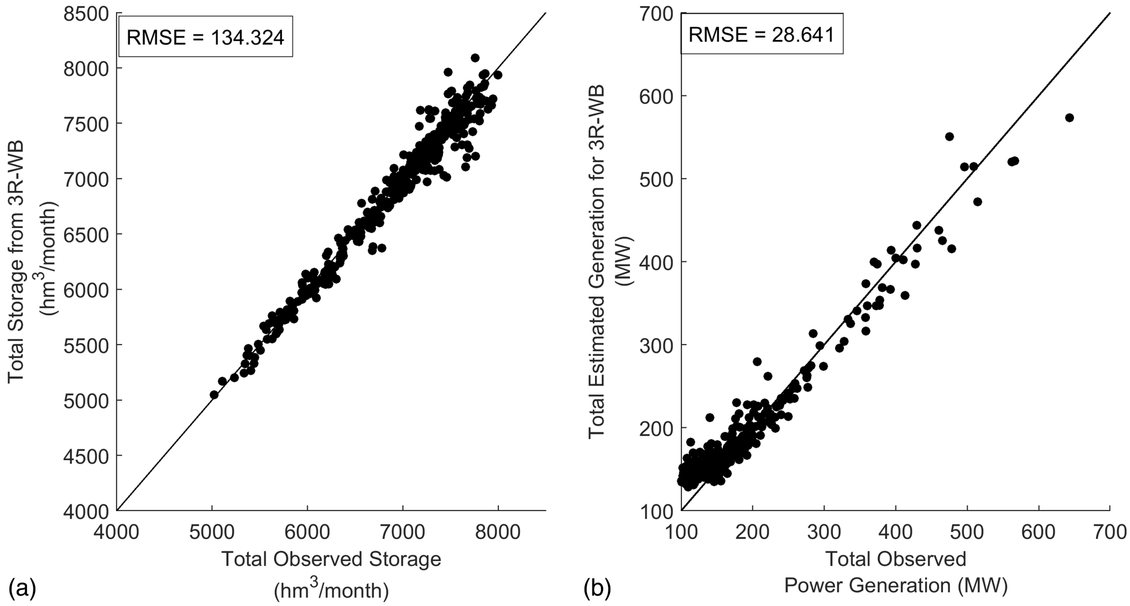

Fig. 3. Performance of cascade model (3R-WB) in estimating (a) total storage; and (b) total hydropower generation under simulation.

decisions (i.e., optimized releases) from each candidate model under observed inflows in estimating the end-of-month storage and total hydropower. For this purpose, we run EqR-EB, EqR-WB, and 3R-WB models with observed inflows and the simulated initial storage (Step 1) in a simulation mode with the respective forecast-suggested releases for each model. This gives the validation of the candidate models' performance—end-of-month storage and generated hydropower—under inflow uncertainty. Then the error in estimating the end-of-month storage and generated hydropower by the EqR-EB and EqR-WB models with respect to the corresponding estimates obtained by the 3R-WB model is computed for each month.

On an eight-core 2.6 GHz Intel Core i7 processor machine, each simulation run of the 3R-WB, EqR-WB, and EqR-EB models took about 40, 15, and 20 s, respectively.

Results and Discussion

We discuss the performance of both equivalent reservoir models for two scenarios of operation. First, we study the equivalent reservoir models' behavior for the Savannah basin under current USACE operating policy (i.e., simulation) and under an objective (i.e., simulation optimization) to maximize hydropower generation. Then we expand our understanding of the relative model performances under potential/alternative SDRs for generalization. To analyze the models' behavior under current policy, we use 30 years of historical monthly inflows and initial storage from 1985 to 2014 and compare the estimated end-of-month storage volumes (S_t^j) and hydropower generation (E_t^j) using the 3R-WB multireservoir cascade model with the corresponding observed values (Fig. 3). Two competing equivalent reservoir models' (EqR-EB and EqR-WB) estimates of hydropower generated ($E_{\text{out flow}}$ for EqR-EB, E_{eq} for EqR-WB), and potential energy/storage (PE_{eq} for EqR-EB, S_{eq} for EqR-WB) are then compared with the corresponding estimates of the multireservoir cascade model (3R-WB) of the Savannah system under both simulation and simulation optimization. Comparing the results in simulation mode allows for a baseline comparison with current operational policy. Analyses under simulation optimization through maximization of the total release gives information on two models' performance under potential operating policies. We compare monthly optimized/simulated storage, release, and hydropower from the multireservoir model (3R-WB) and the equivalent reservoir models (EqR-WB, EqR-EB) over the considered periods of analysis. A summary of these experiments is shown in Table 3.

Performance Comparison under Existing Operating Policy Using Observed Inflows (Simulation)

To begin with, we first compare the multireservoir cascade model, 3R-WB, in simulation mode in terms of its ability to estimate the observed end-of-month storage [Fig. 3(a)] based on the net inflows, release, and previous month's storage as inputs, using Eqs. (1)–(4*a*). Using the elevation–storage relationship provided by USACE for individual reservoirs, we estimate the total hydropower generated from the system [Fig. 3(b)]. The releases from each reservoir are set to meet downstream, drought management, and hydroelectric demands. Based on Fig. 3, the 3R-WB model for the Savannah

Table 3. Summary of model comparisons for different inflow scenarios

Scenario	Models compared	Inputs	Figures
Performance comparison under existing operating policy (simulation)	3R-WB with observations	Historical data for Savannah system along with historical monthly inflows	Fig. 3
Performance comparison under potential operating policy (simulation-optimization)	EqR-EB, EqR-WB, and 3R-WB		Figs. 4–7
Equivalent reservoir performance under different reservoir configurations	EqR-EB, EqR-WB, and 3R-WB	Different reservoir systems (as in Table 4) with synthetic inflows (Appendix III)	Fig. 8

system captures the observed storage well, except for very high storage volumes; but in the case of generated hydropower, the cascade model slightly overestimates the hydropower for drought conditions. However, such error is not seen in low storage values [Fig. 3(a)]. This is because the same turbine efficiency (0.88 for HWL Lake, 0.88 for RBR Lake, and 0.86 for JST Lake) was used for all the storage (or elevation) levels, and tailwater head information was unavailable. In reality, turbine efficiency varies with respect to available head or reservoir storage volume. This simplification is the reason behind the error in the overestimation of hydropower under drought conditions (Fig. 3). Overall the 3R-WB model underestimates total historical storage by 0.35% and overestimates total historical hydropower generation by 14.92%. Errors in estimating power generation mainly occur under drought conditions at the HWL facility, with a root-mean-square error (RMSE) of 15.164 MW. This mainly happens by not including the tailwater head that varies considerably due to the operation of the downstream RBR Lake. RMSE is defined as

$$\sqrt{\frac{\sum_{t=1}^{T}(\hat{\chi}_t - \chi_t)^2}{T}}$$

where $\hat{\chi}_t$ and χ_t represent estimated and observed quantities, respectively.

The RMSE for hydropower generation at the RBR and JST facilities are 9.7 and 9.8, respectively (Fig. S1). Given this benchmarking of 3R-WB in estimating the observed storages and hydropower, henceforth only the 3R-WB model simulated/optimized hydropower, storage, and releases are considered in evaluations of the performance of the EqR-WB and EqR-EB models.

For all the comparisons under simulation (Figs. 4 and 5), all the models are run with the releases observed for each reservoir, including the downstream release and the observed energy outflow calculated from Eqs. (11) and (7) with observed initial conditions [S_{t-1}^{j}, $S_{eq},_{t-1}$ from Eq. (9) and PE_{t-1} from Eq. (8)]. The performance of EqR-EB and EqR-WB are compared with the 3R-WB model in estimating end-of-month cumulative storage [Fig. 4(a)] using Eq. (9), potential storage [Fig. 4(b)] using Eq. (8), total energy outflow [Fig. 4(c)] using Eqs. (7) and (11), and downstream release [Fig. 4(d)] using Eq. (9). Figs. 4 and 5 show a comparison of the normalized root-mean-square error (NRMSE) [Eq. (15)] to shed light on the relative performance of EqR-WB and EqR-EB models:

$$\text{NRMSE} = \frac{1}{\bar{\chi}}\sqrt{\frac{\sum_{t=1}^{T}(\hat{\chi}_t - \chi_t)^2}{T}} \times 100 \tag{15}$$

In Eq. (15), $\hat{\chi}_t$ represents storage, release, or hydropower generation using two candidate equivalent reservoir models (EqR-EB, EqR-WB), and χ_t represents corresponding terms using the multireservoir cascade model (3R-WB). Since ranges of the estimated quantities are different, we have used a normalizing factor, $\bar{\chi}$, which is the mean of the measured quantities (estimates of 3R-WB model). We compare coefficient of correlation (R^2) for each of

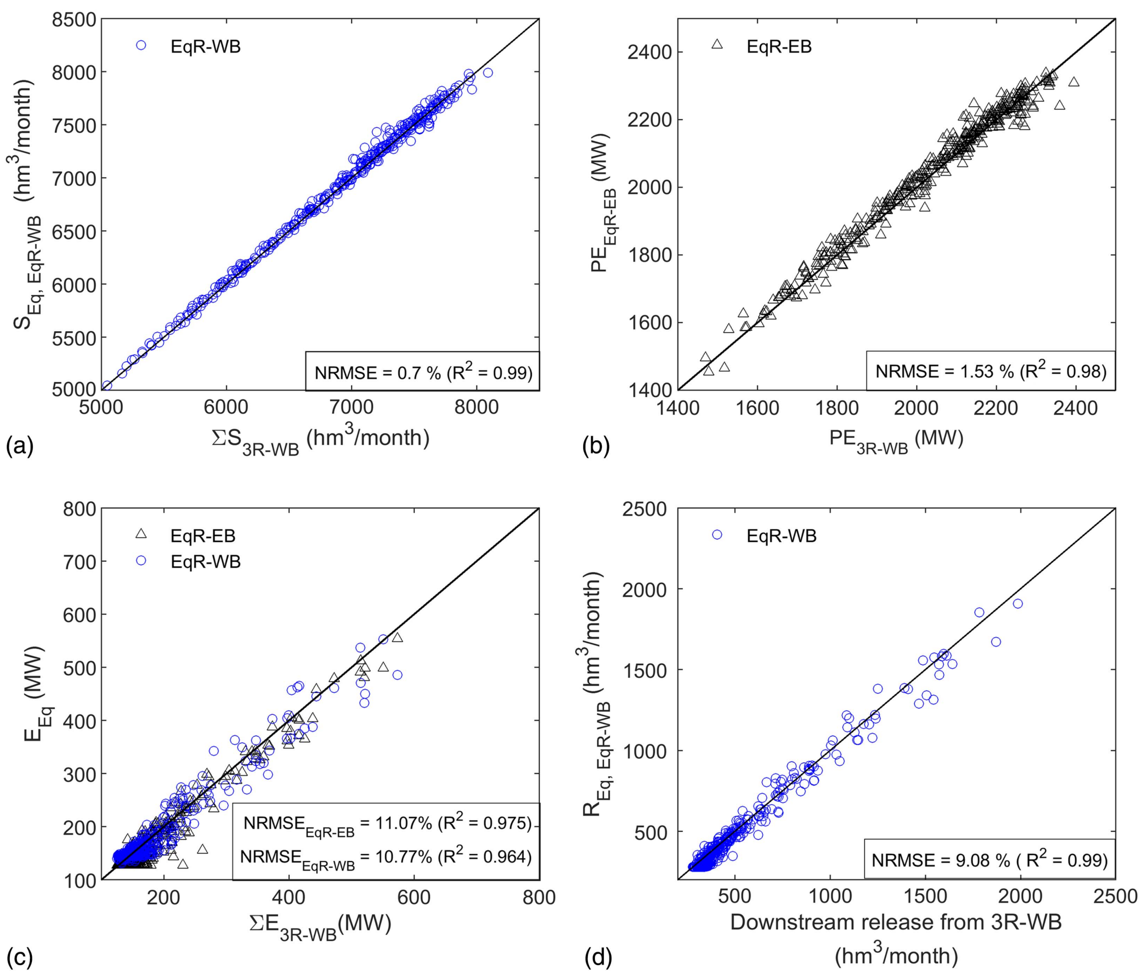

Fig. 4. Performance comparison of equivalent reservoir models with cascade model (3R-WB) in estimating (a) total storage; (b) potential energy storage; (c) total hydropower generation; and (d) downstream release under simulation.

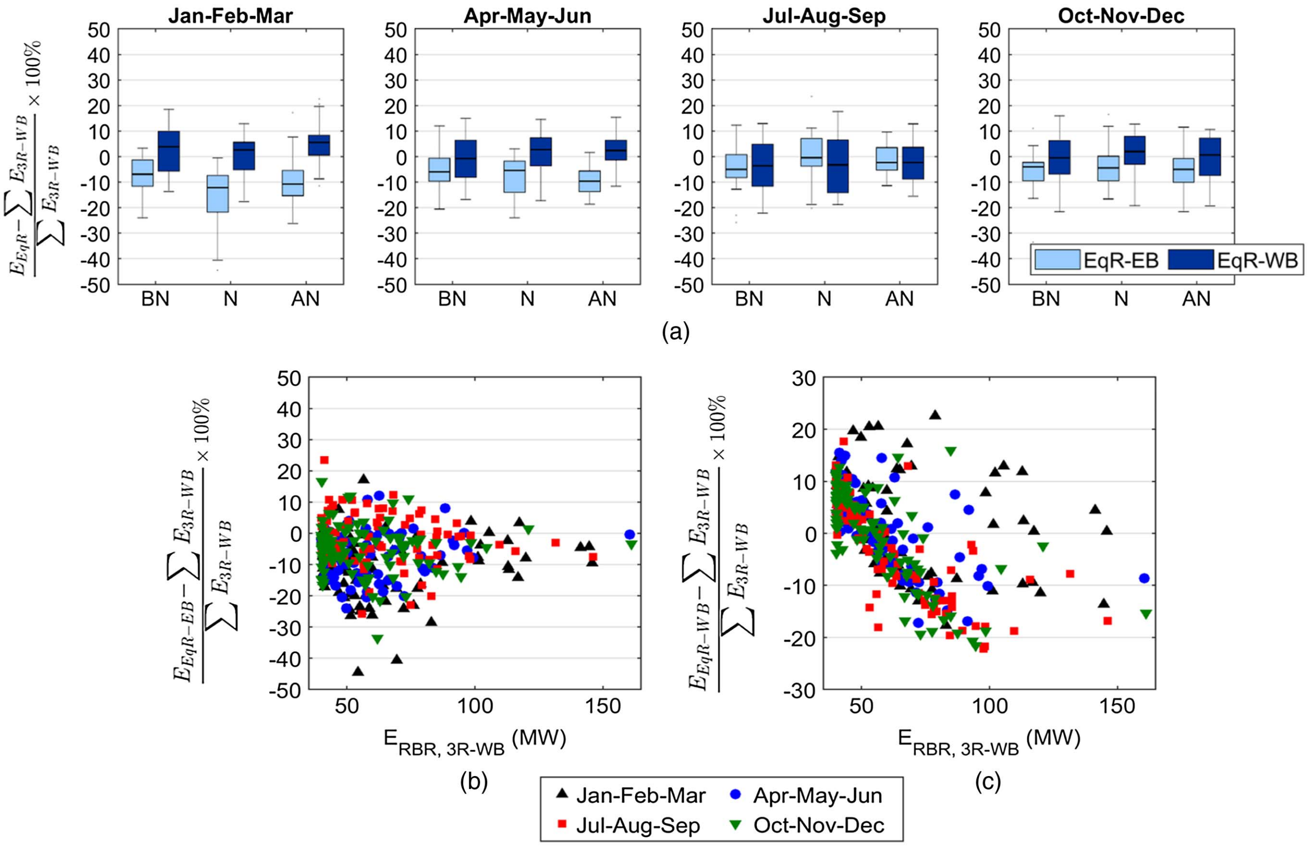

Fig. 5. Relative error in estimating the generated hydropower of 3R-WB model under simulation by (a) both equivalent reservoir models over four seasons for inflow categories Below Normal (BN) (<33rd percentile), Normal (N) (33rd–67th percentile), and Above Normal (AN) (>67th percentile); (b) EqR-EB model with hydropower generated by Russel Lake; and (c) EqR-WB model with hydropower generated by Russel Lake.

the estimated quantities ($\hat{\chi}_t$) by equivalent reservoir models and respective 3R-WB estimates (χ_t) by calculating R^2 as $\mathrm{cov}(\chi, \hat{\chi})/(\sigma_\chi \sigma_{\hat{\chi}})$. From Figs. 4(a and d), equivalent reservoir representation using water balance of reservoirs in series estimates very well the cumulative storage ($R^2 = 0.99$) and the downstream release ($R^2 = 0.99$) from the cascade model. We present cumulative water storage and potential energy storage separately as they represent the end-of-month storage available for next month. From Figs. 4(b and c), EqR-EB model estimates the end-of-month potential energy stored ($R^2 = 0.98$) and the energy outflow from the 3R-WB estimates. Further, the total energy outflow for the EqR-WB model derived from this study using Eq. (11) estimates the total energy outflow from the 3R-WB model. EqR-EB model ($R^2 = 0.975$) performs slightly better than EqR-WB model ($R^2 = 0.964$) in estimating hydropower generation from 3R-WB model [Fig. 4(c)]. However, most models estimate the energy outflow during drier conditions with error as inferred from the increased scatter for lower values. This is partially due to the inaccurate representation of (p) for low flows as EqR-EB model considers a single value of (p) for all the flow levels. To understand this issue better, we compare the equivalent reservoir models under three different inflow categories over four seasons (Fig. 5) by considering the relative error [Eq. (16)] in comparison to the 3R-WB model:

$$\text{Relative error} = \frac{\chi_{Eq} - \chi_{\text{3R-WB}}}{\chi_{\text{3R-WB}}} \times 100 \tag{16}$$

In Eq. (16), χ_{Eq} represents storage, release, or hydropower generation using two candidate equivalent reservoir models (EqR-EB, EqR-WB) and $\chi_{3R\text{-}WB}$ represents corresponding terms using the multireservoir cascade model (3R-WB). Inflow categories—Below Normal (BN) (<33rd percentile), Normal (N) (33rd–67th percentile), and Above Normal (AN) (>67th percentile)—are based on the long-term climatological naturalized inflows into the system.

Fig. 5(a) extends the analysis present in Fig. 4(c) under the different flow categories—BN, N, and AN. Relative error in estimating the cumulative storage is lesser than the relative error with respect to hydropower and downstream releases. For calculating relative error for EqR-EB estimates using Eq. (12), we have used the energy equivalent storages and releases from 3R-WB model as used before in Figs. 4(b and c). It should be noted that, energy outflow from EqR-EB model does not correspond to the actual water release available at the downstream of the cascade. This is expected because the energy balance-based model is essentially used for developing hydropower scheduling problems (de Queiroz 2016; de Matos and Finardi 2012; Dias et al. 2013). Seasonal pattern of release estimates (total release, and downstream release) are reflected in seasonality in hydropower generation, as expected. Although relative error in hydropower estimates is small, there is much variation across model estimates for the months of July, August, and September than other months in the year. With increased water availability, variation in estimates is also increased. When more water is available, the EqR-WB tends to store more water and reduce its downstream release. The differences in the estimates of 3R-WB and EqR-WB are caused by the fact that equivalent reservoir cannot represent the allocation of different storage limits across reservoirs, and, for EqR-WB only downstream release is compared and not individual reservoir releases and storages.

Table 4. Correlation and p-values (in parentheses) between seasonal residuals in hydropower estimates by equivalent reservoir models and individual reservoir's hydropower generation

Season	HWL EqR-EB	HWL EqR-WB	RBR EqR-EB	RBR EqR-WB	JST EqR-EB	JST EqR-WB
Simulation						
JFM	0.26 (0.011)	−0.20 (0.064)	0.07 (0.522)	**−0.33** (0.001)	0.13 (0.225)	0.02 (0.856)
AMJ	**0.29** (0.006)	**−0.40** (0*)	0.18 (0.091)	**−0.66** (0*)	0.18 (0.098)	**−0.30** (0.004)
JAS	0.02 (0.886)	−0.25 (0.015)	−0.20 (0.059)	**−0.77** (0*)	0.10 (0.357)	−0.16 (0.135)
OND	0.15 (0.156)	**−0.29** (0.005)	−0.05 (0.644)	**−0.68** (0*)	0.08 (0.476)	−0.14 (0.181)
Optimization						
JFM	**0.60** (0*)	−0.05 (0.667)	**0.59** (0*)	−0.08 (0.445)	**0.60** (0*)	0.15 (0.167)
AMJ	**0.36** (0*)	**−0.40** (0*)	**0.30** (0.004)	**−0.54** (0*)	**0.49** (0*)	**−0.21** (0.046)
JAS	**0.56** (0*)	−0.19 (0.076)	**0.42** (0*)	**−0.48** (0*)	**0.63** (0*)	−0.04 (0.707)
OND	**0.75** (0*)	−0.11 (0.322)	**0.65** (0*)	**−0.32** (0.001)	**0.67** (0*)	−0.01 (0.950)

Note: 0* = p-value $\ll 0.05$. Bold values indicate statistically significant correlation.

Except for the July–August–September (JAS) period, EqR-EB underestimates hydropower generation more during above-normal conditions and less during below-normal conditions, compared to the EqR-WB model when compared with the hydropower estimated from the 3R-WB model [Fig. 5(a)]. EqR-EB error in estimating the hydropower estimating during above/below normal conditions primarily results due to the approximation of the productivity function, p, which ignores the head variation due to storage by considering the average hydropower generated for unit release. Here, we note that 3R-WB model overestimates the reported generation (Fig. 3), such that EqR-EB actually estimates hydropower closer to the reported values (Table 4) for all four seasons. The mean productivity, p, for individual reservoirs, are computed based on the observed release and generation for the Savannah system, as no guide curve relationship exists for the energy balance approach. Therefore, EqR-EB estimated hydropower [Eq. (7)] is closer to the reported generation when the model is run under simulation. Though the reservoir simulation model in EqR-WB is able to capture the end-of-month storage and downstream releases precisely compared to the 3R-WB model (Table 4), estimation of hydropower generation is more error prone during low-flow conditions. This is because, to develop a hydropower generation function [Eq. (11)], elevation–storage–generation relationships, provided by USACE, are used. These relationships reflect guide curve operations but cannot capture properly the drought contingency management plans adopted by water managers of both Duke Energy and USACE during different drought events.

To understand the sources of error in estimations of hydropower generation from aggregated representations, we compared the seasonal residuals between equivalent reservoir models and the 3R-WB model's total hydropower generation with an individual reservoir's hydropower generation from the 3R-WB model (Fig. 5) and storage and downstream release (Fig. S2). The primary reason for the errors in the EqR-EB model is due to the error in the productivity function, p, which considers the average hydropower generated for unit release across various storage levels for different releases. Hence, such an average productivity function, p, is expected to underestimate hydropower during low-flow conditions and overestimate during high-flow conditions. In the case of the EqR-WB model, performance is more uniform because the total hydropower is estimated using a nonlinear equation that relates the equivalent storage and release [in Eq. (9)] to the total hydropower estimated by the 3R-WB model.

In Figs. 5(b and c), we have plotted the seasonal residuals from the EqR-EB and EqR-WB models, with energy generated from RBR Lake obtained from the 3R-WB model, $E_{\mathrm{RBR},3R\text{-}WB}$, respectively. From Fig. 5(b), we can see that there is no systematic bias in the seasonal residuals of the EqR-EB model with RBR Lake's generation ($E_{\mathrm{RBR},3R\text{-}WB}$); however, a prominent systematic bias is present in the seasonal residuals of the EqR-WB model with $E_{\mathrm{RBR},3R\text{-}WB}$. The correlation between the relative errors of the EqR-WB model with the hydropower generated from RBR Lake from the 3R-WB model is negative [−0.33 for January–February–March (JFM), −0.66 for April–May–June (AMJ), −0.77 for JAS, and −0.68 for October–November–December (OND)] and statistically significant (Table 4) for all seasons. This indicates that the EqR-WB model overestimates (underestimates) when RBR Lake's hydropower is lower (higher). Compared to the storage capacities of HWL and JST Lake (Table 4), RBR Lake acts practically as a run-of-the-river system meeting the peak power demand of the upper Savannah basin. Hence, we found a dependence between the relative errors in the total generation for the EqR-WB model and RBR hydropower generation, but we found no such dependence for HWL or JST. The bias in hydropower generation in the EqR-WB model is due to poor representation of pump-back flows in the model, besides other sources of error described for Fig. 4. Because reservoir simulation models are run on a monthly time scale and RBR Lake, as a pump-back facility, operates for peak power demand on daily to hourly time scales, it is intuitive that RBR Lake would impart a significant bias, particularly in the EqR-WB model in simulation. This is because, from the formulations of the EqR-EB and EqR-WB models, it is evident that EqR-EB directly estimates potential energy and energy outflow using total release from the cascade system, whereas in the case of the EqR-WB model, the equivalent release is limited by the channel capacity downstream of JST Lake. Thus, under existing operating policy, hydropower generation during low-flow conditions has more error from both equivalent reservoir models.

Performance Comparison under Potential Operating Policy (Optimization-Simulation)

Under this scenario, we assumed the Savannah system was operated to maximize hydropower generation given the initial conditions (storage and inflows for the EqR-WB model or potential energy and energy inflows for the EqR-EB model), while ensuring storage, release, and hydropower generation constraints were met. Given the optimal release, the end-of-month storage, potential energy, and hydropower generated are calculated. Thus, Fig. 6 presents a model comparisons as in Fig. 4, but with calculations based on the optimal release estimated by each model.

To evaluate the reservoir models under alternative operating policies, we need to use an updated formulation of the EqR-EB model, separating upstream releases from a reservoir's net inflows [Eq. (6)].

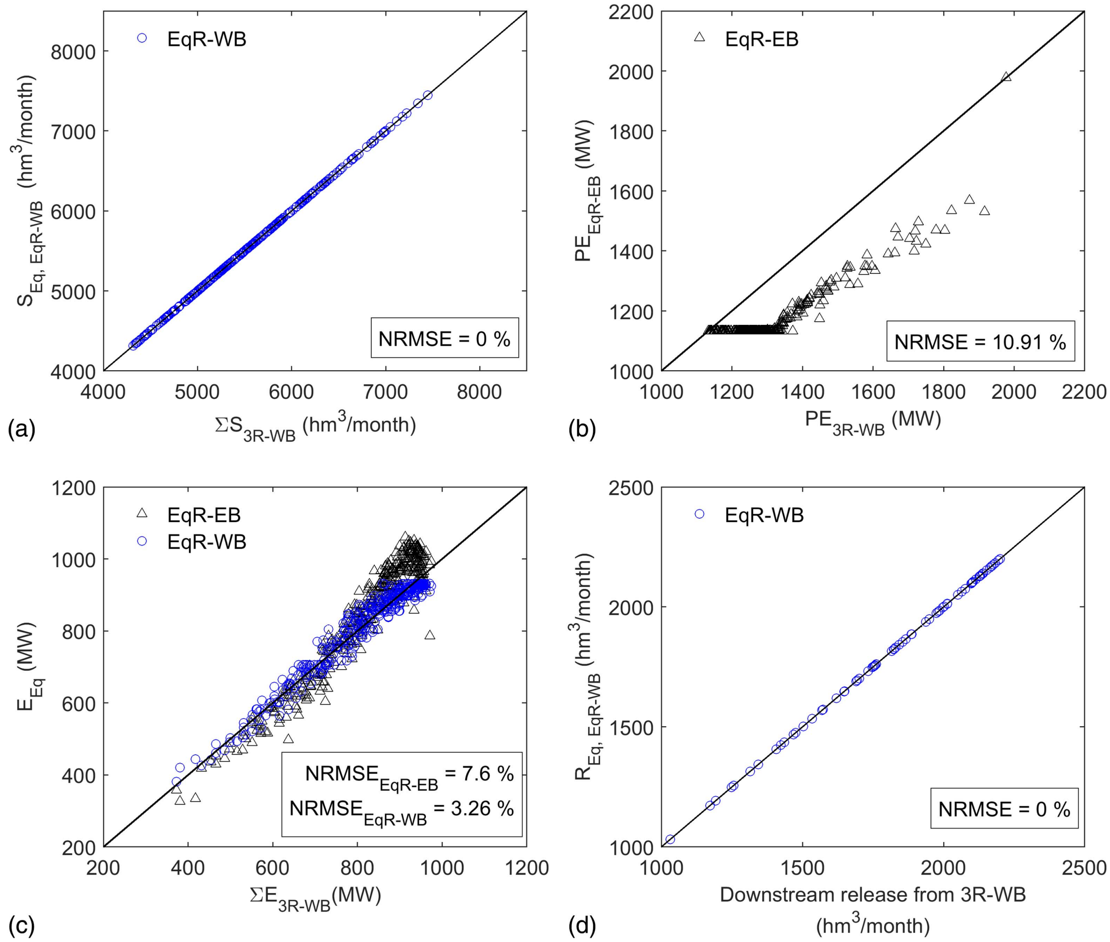

Fig. 6. Performance comparison of equivalent reservoir models with cascade model (3R-WB) in estimating (a) total storage; (b) potential energy storage; (c) total hydropower generation; and (d) downstream release under maximizing total hydropower generation (i.e., simulation optimization).

Thus, in this case, we need to rewrite Eq. (8) as Eq. (17). On the LHS of Eq. (17), only a fraction of the actual energy outflow is considered. Total energy outflow [Eq. (8)] in this case will represent the sum of energy inflows and the fraction on the LHS of Eq. (17) (see Appendix I for details):

$$PE_t + \sum_{j=1}^{n} R_t^j (p^j - p^{j+1} | j < n) = PE_{t-1} + \sum_{j=1}^{n} I_t^j p^j \tag{17}$$

In Fig. 6 we see that the performance of the EqR-WB model is precise in estimating the cumulative storage and downstream release of the 3R-WB model. However, the EqR-EB model shows more deviation in estimating hydropower generation from the estimates of the 3R-WB model. The EqR-EB model performs poorly during high-flow conditions. This is primarily because the EqR-EB model maximizes hydropower generation without considering the constraints of individual reservoirs. This results in overestimation of energy outflow by the EqR-EB model within the prescribed limits.

To understand this, we evaluate the performance under different flow conditions (Fig. 7). Fig. 7 extends the analysis presented in Fig. 5, but based on optimized releases from each reservoir model under different inflow categories: BN, N, and AN. Under potential alternative operating policies (optimization simulation), relative errors in estimating hydropower are less for the EqR-WB model compared to the EqR-EB model. The correlation between the relative errors of the EqR-WB model with the hydropower generated from Russel Lake from the 3R-WB model is negative (−0.54 for AMJ, −0.48 for JAS, and −0.32 for OND) and statistically significant for the three seasons [Fig. 7(b); Table 4]. This indicates that the Eqr-WB model overestimates (underestimates) when RBR Lake hydropower is lower (higher). The performance of the EqR-WB model is also consistent across the seasons, with the spread of relative errors being small. The EqR-EB model overestimates during the spring and summer seasons compared to the cascade model, which indicates the aggregated model informs the power system with increased hydropower availability than what could be obtained from the total release from the system. The correlation between the relative errors of the EqR-EB model with the hydropower generated from Russel Lake based on the 3R-WB model is positive (0.59 for JFM, 0.30 for AMJ, 0.42 for JAS, and 0.65 for OND) and statistically significant for all seasons. This indicates that the EqR-EB model overestimates when RBR Lake generation is higher, and vice versa. This arises partly as a result of the overestimation of hydropower by the EqR-EB model under optimization. In accordance with our findings presented in the previous section, under an alternative operating policy, systematic bias introduced by RBR Lake into the system now is only prominent in the seasonal residuals of hydropower generation from the EqR-EB model and from the EqR-WB model. It is important to note that both equivalent reservoir models show increased deviation from the total hydropower estimated by the 3R-WB model for RBR Lake under

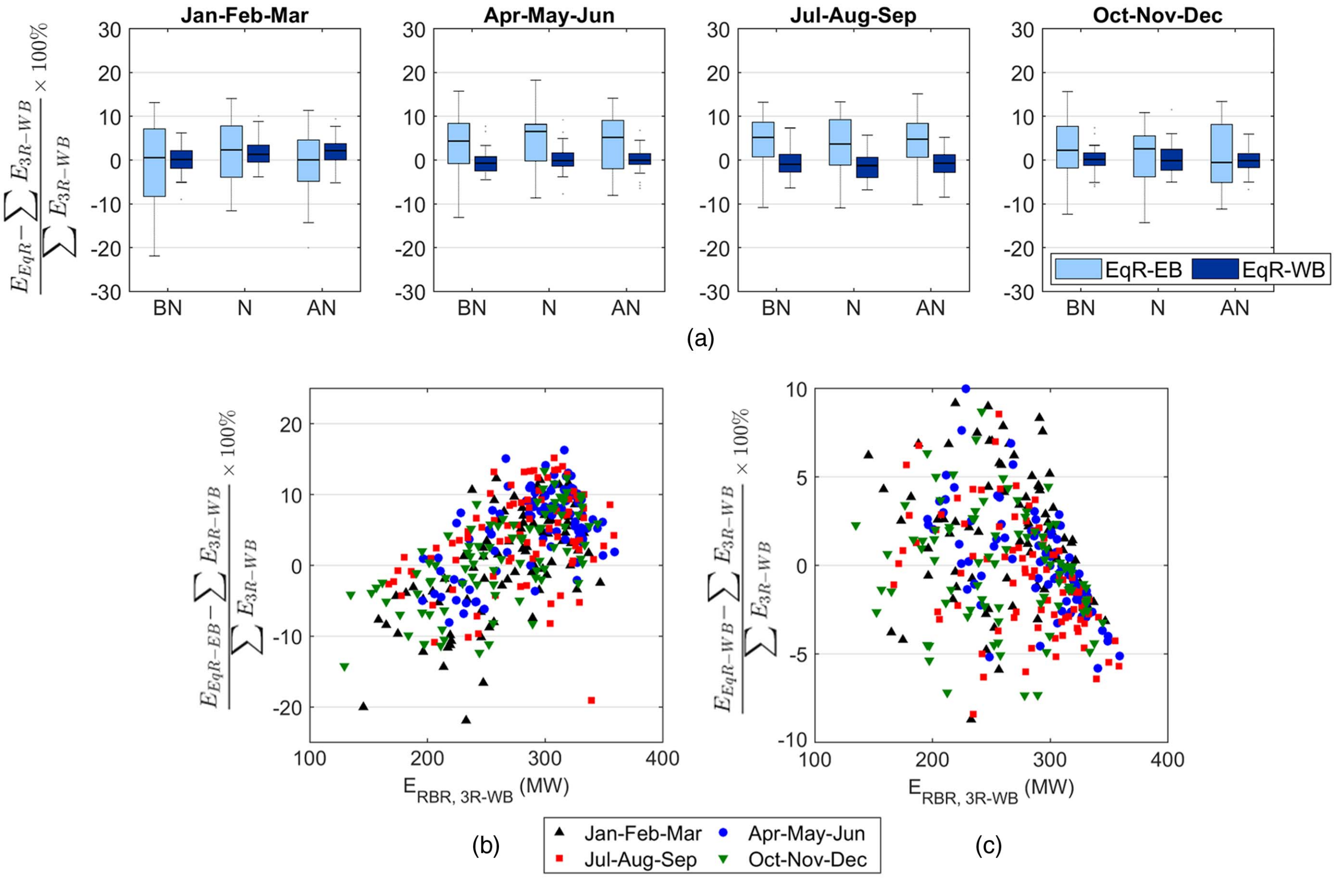

Fig. 7. Relative error in estimating generated hydropower of 3R-WB model under maximization of total hydropower (i.e., simulation optimization) by (a) both equivalent reservoir models over four seasons for inflow categories BN (<33rd percentile), N (33rd–67th percentile), and AN (>67th percentile); (b) EqR-EB model with hydropower generated by Russel Lake; and (c) EqR-WB model with hydropower generated by Russel Lake.

optimization. Thus, the performance of the equivalent reservoir models in representing the total hydropower generated varies substantially under optimization.

Equivalent Reservoir Performance under Different Reservoir Configurations

The analysis in earlier sections showed that the EqR-WB model performs as well as 3R-WB for the Savannah system under current and potential operating policies. Such performance is expected provided the equivalent reservoir model is able to estimate the hydropower generated from the multireservoir cascade model based on the developed relationships in Appendixes I and II. With proper parameterization, the EqR-WB model can estimate the total storage, downstream release, and total hydropower generation as precisely as the 3R-WB model for the Savannah system.

To understand how the equivalent reservoir models perform in comparison to the 3R-WB model under different system configurations and under different synthetic inflow forecast skills (see details in the "Methodology" section), we estimate the end-of-month storage and total hydropower generation by three models for each combination of system configuration and forecast skill, resulting in a total of 30 different comparisons between equivalent reservoir models with the 3R-WB model (Fig. 8). The mean relative error in estimating the end-of-month storage and generated hydropower by the EqR-EB model and EqR-WB model with respect to the corresponding estimates obtained by the 3R-WB model is plotted in Fig. 8. For instance, in Fig. 8, the results under the current system configuration (vertical line) with the inflow forecast being observed inflow (i.e., PF—filled markers), correspond to the errors in estimating the end-of-month storage and hydropower by the EqR-EB and EqR-WB models shown in Figs. 6 and 7. Thus, we compare the equivalent reservoir models' performance with the 3R-WB model under both inflow uncertainty and varying system configurations.

The mean relative error in estimating storage, hydropower (numbers in italics in Fig. 8), and downstream release (Table 5) from the EqR-WB model is negligible under PF. The mean relative error of the equivalent reservoir models is greater in the case of systems with a SDR less than 0.3 (Fig. 8). For small storage systems, the potential for spillage from individual small reservoirs is higher (Fig. S4), so the error in estimating the 3R-WB storage is higher because the EqR-WB model does not accurately reflect spillage owing to the aggregated nature of the system. For SDRs of 0.07 and 0.14, EqR-WB spills only when natural inflow reaches the maximum reported value for the Savannah system (2,177 $\mathrm{hm}^3/$month). In this situation, EqR-WB estimates spillage of 107.3 and 50.05 $\mathrm{hm}^3/$month, at SDR $= 0.07$ and 0.14, respectively. This can be inferred from the increased relative error in the downstream release for small SDR systems for the EqR-WB model (Table 5). The mean relative error in the estimation of potential energy storage (EqR-EB) [Fig. 8(a)] and energy outflow [Fig. 8(b)] indicates that the performance of EqR-EB varies considerably with the cascade SDR. Because we consider the optimized solution of each 3R-WB model for each SDR under a given inflow forecast as the *truth*, we estimate the mean productivity p and storage–elevation–generation relationships for EqR-EB and EqR-WB based on 3R-WB model estimates, respectively. From Fig. 8, it is evident that the relative

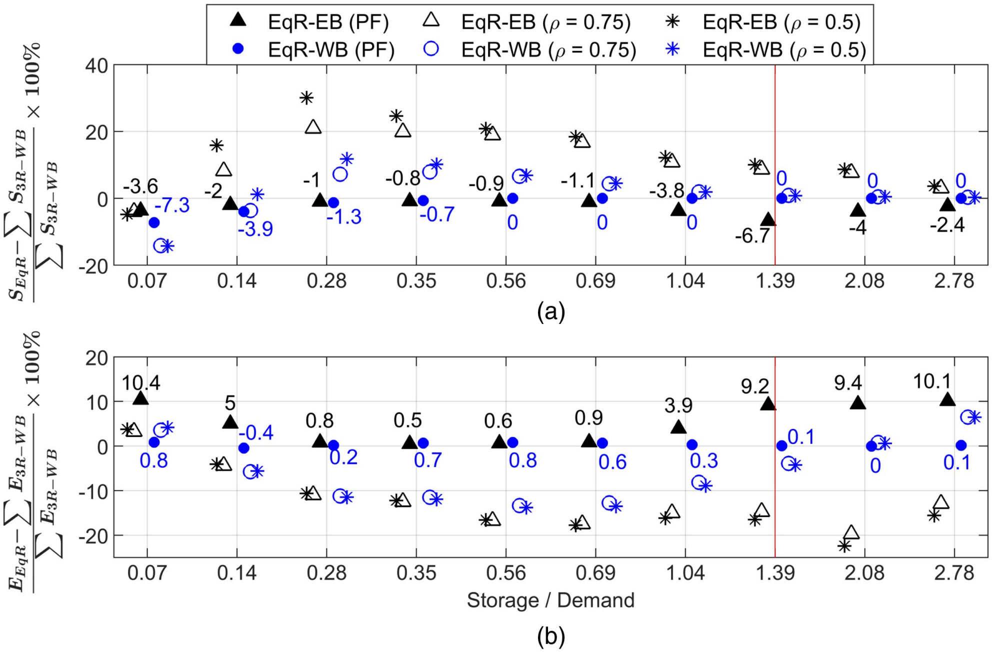

Fig. 8. Relative error in estimating (a) water/energy storage; and (b) hydropower by equivalent reservoir models in estimating respective attributes estimated by cascade model (3R-WB) under different storage capacity to mean annual demand ratios with objective of maximizing total hydropower generation. The Savannah system's SDR is 1.38 (vertical line). Numbers indicate mean relative error (%) for PF for EqR-EB (triangles) and EqR-EB (circles) models compared to 3R-WB.

Table 5. Storage-to-demand ratios considered for analyzing equivalent reservoir model performance; mean annual demand is 6,023 hm^3 for all configurations. A SDR of 1.39 indicates Savannah system; p-values are for simulation optimization for different system configurations

Storage capacity of multireservoir cascade (hm^3)	SDR	Mean productivity (p) of reservoirs in cascade			Relative error (%) in estimation of release by EqR-WB w.r.t. 3R-WB (95% CI) (PF)
		Upstream	Middle	Downstream	
418	0.07	0.10	0.09	0.07	−1.46 (−15.7, 10.8)
836	0.14	0.11	0.10	0.07	−2.1 (−13.2, 7.9)
1,672	0.28	0.12	0.10	0.07	−0.84 (−10.54, 3.78)
2,090	0.35	0.12	0.11	0.07	−0.32 (−7.9, 2.4)
3,344	0.56	0.18	0.12	0.05	0.06 (−1.38, 0)
4,180	0.69	0.21	0.13	0.04	−0.02 (−0.44, 0)
6,269	1.04	0.22	0.13	0.06	—[a]
8,359	1.39	0.20	0.14	0.09	—[a]
12,539	2.08	0.18	0.13	0.16	—[a]
16,718	2.78	0.15	0.01	0.29	—[a]

Note: CI = confidence interval; and w.r.t. = with respect to.
[a]Relative mean error.

performance of different equivalent reservoir models depends on the SDR.

Under PF scenario, for hydropower generation, the EqR-WB model performs consistently, with wider range of errors for smaller SDR systems and accurately estimating the hydropower generation for larger SDR systems. The primary reason for the poor performance for smaller SDR systems is due to the difference in the downstream release (Table 5) and to spillage (Fig. S4) that occurs from individual systems, which is not accounted for in the equivalent reservoir. For larger SDR systems, since the initial storage itself guarantees the total monthly demand for both the 3R-WB and EqR-WB models, the amount of hydropower generated varies minimally from the cascade model, as shown in Table 5, which reveals that the difference in downstream release is smaller under PF. But the EqR-EB model overestimates the actual hydropower generation of the 3R-WB model across all configurations since the physical limits of individual reservoirs' storage capacities are not well captured in the EqR-EB model formulation, which conforms to our previous findings for the Savannah system (Fig. 6).

Under the PF scenario, the overestimation of hydropower by EqR-EB compared to 3R-WB is more pronounced for smaller SDR configurations and for larger SDR systems. The overestimation of hydropower [Fig. 8(b)] and underestimation of potential energy storage [Fig. 8(a)] by the EqR-EB model are minimal for systems with SDR ranging between 0.28 and 0.69. By defining $\upsilon = PE_{eq,\max}/PC_{\max}$, the ratio between potential energy storage capacity ($PE_{eq,\max}$) and total hydropower generation capacity ($PC_{\max}$) of a cascade [Eq. (11c)], we infer that for system configurations where SDR ranges between 0.28 and 0.69, υ is almost equal to SDRs. Another way to interpret υ is that it is an energy-equivalent system characteristic of the SDR because it denotes potential energy storage and demand denoted by maximum generation capacity. When SDR and υ are similar, hydropower and potential energy estimates by EqR-EB are precise, compared to both the 3R-WB and

EqR-WB models. This observation has significant implications in terms of linking water and power systems, especially for within-year storage systems. For smaller systems ($\mathrm{SDR} < 0.28$), poor performance of the EqR-EB model arises from its inability to capture spillage from the system, particularly under wet conditions. For larger systems ($\mathrm{SDR} > 1$), error in the estimation of hydropower and potential energy by EqR-EB can be attributed to two main reasons. First, the fractional storage contribution of each reservoir does not translate into a fractional potential energy contribution because the latter depends on the productivity of each hydropower facility. Second, because the SDR is quite different from υ, the downstream release limit seen by the cascade model is not captured by the EqR-EB model, so the EqR-EB model estimates higher hydropower generation for both dry and wet conditions owing to the availability of large potential energy storage.

To assess a model's relative performance under different inflow forecast skills (hollow markers in Fig. 8), we address the questions of whether the system was operated using the forecasted inflows and how much deviation in the estimation of end-of-month storage and hydropower generation would result between EqR models and the 3R-WB model once the actual inflows for the month occurred. To this end, we compare the EqR-WB and EqE-EB models' performance with the corresponding 3R-WB model with the same inflow forecast skill (not with the PF case) under a given system configuration. From Fig. 8 we can infer that as the inflow forecast skill decreases, the error of the EqR model increases for a given system configuration. Comparing across different system configurations, we infer that, except for $\mathrm{SDR} = 0.07$, the errors of the EqR models in estimating storage in general decrease as the SDR increases. This is consistent with earlier findings (Maurer and Lettenmaier 2003; Sankarasubramanian et al. 2009) that systems with large SDRs guarantee monthly demand with high reliability, thereby reducing differences across models in estimating monthly storage. Comparing the performances of the EqR-WB and EqR-EB models, we infer that the EqR-WB model consistently performs better with smaller errors in estimating the monthly storage obtained from the 3R-WB model. However, under a given forecast skill, differences in the errors between the EqR-WB and EqR-EB models in estimating the hydropower versus the 3R-WB model are not significant until $\mathrm{SDR} = 1.39$. For SDRs of 2.08 and 2.78, the EqR-EB model performs poorly owing to an error in the estimation of mean productivity, p. The utility of the inflow forecast is more pronounced in the case of the EqR-WB models for systems with $\mathrm{SDR} > 1$ because the error in estimating the storage and hydropower approaches that obtained under PF (i.e., observed inflows). The poor performance of the EqR-EB models in these cases arises as a result of their inferior approximation of mean productivity p. Estimation of p can be improved by accounting for tailwater information and stage-dependent turbine efficiency.

When the SDR is varied and a model is run with different inflow forecast skills equivalent reservoir models for large SDRs tend to behave similarly to a cascade system, suggesting the possibility of integrating water and power systems over large regions. Another implication of the presented analyses is that to aggregate a multireservoir model, one could group reservoirs in such a way that the SDR and υ are similar for the proposed aggregated system, which would ensure similar performances of the EqR-EB and EqR-WB models. In other words, the SDR and υ act as a characteristic scale/metric for the aggregation of a multireservoir system. However, the performance of this integration could result in significant bias, as shown in Figs. 5, 7, S2, and S3, during below-normal and above-normal conditions and depending on the seasonal characteristics of the basin.

Concluding Remarks

Power systems depend on water systems, particularly for hydropower to meet peak demand as they are easy to start/stop and have low marginal operating costs. Because the power system grid is highly interconnected with generation units over a larger area/region, hydropower availability is typically quantified by equivalent reservoir representation using an energy balance (Arvanitidis and Rosing 1970a; de Queiroz 2016). Aggregated water balance representation is available only for parallel reservoir systems (Koutsoyiannis and Economou 2003a), but cascade systems are critical in multireservoir models at the river basin scale (Archibald et al. 2006; Koutsoyiannis and Economou 2003a). While most studies (Turgeon 1981, 1982; Pereira and Pinto 1985, 1991; Brandão 2010) focus on long-term power system planning, our approach is to emphasize monthly to seasonal coordination of water and power systems because hydropower plays a significant role (de Queiroz et al. 2019). In this study, we systematically evaluate the performance of two equivalent reservoir models—water balance and energy balance approaches—with a cascade reservoir representation and generalize the findings for systems with different SDRs. To this end, we considered three reservoirs from the Savannah system in South Carolina and compared two equivalent reservoir models with the cascade model under current operating policy, a modified operating policy intended to maximize hydropower generation, and under different reservoir system configurations having different inflow forecast potentials. Considering the cascade model, 3R-WB, the so-called true model, we compared the total end-of-month storage, downstream release (total release in case of EqR-EB), and total hydropower generation estimated from each of the equivalent reservoir models with corresponding estimates from a multireservoir model.

In simulations with known releases, both the EqR-WB and EqR-EB models perform equally well based on RMSE and NRMSE in estimating hydropower generation [Fig. 4(c)], but the EqR-EB model slightly underestimates total hydropower generation across most seasons [Fig. 5(a)] and flow conditions, which happens as a result of its inability to estimate hydropower generation from the smallest reservoir, RBR Lake, in the system [Fig. 5(b)]. Under the policy to maximize hydropower optimization, EqR-WB performs better overall than the EqR-EB model in estimating hydropower generation [Fig. 6(c)], resulting in overestimation of hydropower generation across all flow categories and seasons [Fig. 7(a)]. This overestimation happens because the energy balance model takes into consideration only the aggregated energy outflow without making allowances for releases in individual reservoirs, particularly for run-of-the-river systems [Fig. 7(b)]. However, it would be natural to extend the revised formulation [Eq. (17) and Appendix I], with decision variables in the EqR-EB model being the individual reservoirs' releases. In such a scenario, it is imperative to add equality constraints to the storage of individual reservoirs. But such an addition of storage constraints would essentially turn the EqR-EB model into a cascade model, with the primary difference in hydropower generation arising as a result of mean productivity (p) representation. Hence, we did not pursue such a formulation for the purpose of comparing equivalent reservoir and cascade models.

Comparing the EqR models and cascade model under different reservoir configurations reveals that for systems with small SDRs, the error in the performance of equivalent reservoirs in estimating storage and hydropower generation is high owing to the difference in the downstream releases and to spills occurring in individual systems, which is not well represented in the equivalent reservoir representation (Fig. S4). For larger SDR systems, since the initial storage itself guarantees the total monthly demand for both 3R-WB model and the equivalent reservoir models (EqR-WB and EqR-EB), the amount of hydropower generated varies minimally from the cascade model. For the EqR-EB model, the aggregated model overestimates [Fig. 8(b)] the actual hydropower generation of the 3R-WB model for all SDRs since maximization of total hydropower from the three systems does not take into account individual reservoir constraints, resulting in the potential storage of the EqR-EB model being less than that of the cascade model [Fig. 8(a)]. However, this overestimation of hydropower and underestimation of potential energy storage by the EqR-EB model is at its minimum for SDRs between 0.28 and 0.69 since the total energy generated by the equivalent reservoir model and cascade model is very close. The EqR-EB model's performance is closest to the cascade model when SDR is similar to υ. This observation will be crucial for future studies on linking power and water systems because it indicates a characteristic scale for the two aggregated system representations.

On the other hand, the EqR-WB model consistently performs closer to the cascade model as the SDR increases because most often the total demand is met based on initial storage. Thus, generalization based on different system characteristics shows that aggregation of systems with different SDR—run-of-the-river storage system (SDR < 0.28), a within-year storage system (0.28 < SDR < 1.0), and an over-year storage system (SDR > 1)—characteristics could result in bias in hydropower generation estimated by the equivalent reservoir models for integration with power systems. In situations where allocations for irrigation and public water supply uses are significant, releases should be adjusted accordingly in developing equivalent reservoir models depending on whether the release for the specified use goes through a turbine or not. Further, the generalization developed here is more relevant for humid basins with similar inflow characteristics dominated by a rainfall-runoff regime over the Southeastern US (Petersen et al. 2012, 2018). But the findings have potential value for larger SDRs even if the hydroclimatology differs from one basin to the next (e.g., snowmelt regime in the Northwest) because the initial storage guarantees demand and dampens seasonal inflow variability (Maurer and Lettenmaier 2004; Sankarasubramanian et al. 2009). It is also important to note that arid basins in the Western US typically have larger SDRs (Graf 1999), which implies that initial storage will ensure the required releases, so the findings presented here could provide insights on the performance of EqR models in other regional settings as well.

This study does not aim at substituting multireservoir network models with equivalent reservoir representations for water allocation and management; rather it focuses on the large-scale integration of water and power systems. Most research on equivalent reservoir models for power systems have focused on developing a planning model (de Queiroz 2016) or on evaluating their importance in reducing dimensionality (Koutsoyiannis and Economou 2003a) in reservoir model formulation. Evaluation of equivalent reservoir models for monthly operation has been carried out primarily in simulation settings (Arvanitidis and Rosing 1970a). Findings from this study offer potential for developing equivalent reservoir models to maximize hydropower generation for supporting power system operation under inflow uncertainty. Both equivalent reservoir models and cascade models considered reservoir inflows as an exogenous input. However, some studies have considered power system generation in hydrologic models to shed light on water and energy issues under climate change (Ehsani et al. 2016; Tarrajo et al. 2016; Turner et al. 2019). Findings from this study emphasize that the aggregation of reservoir systems with large SDRs should be considered when representing power systems in hydrologic models. Further, a traditional cascade modeling framework, such as RiverWare and Hec-ResSim, could also consider the suggested SDRs for aggregating various run-of-the-river generation systems or systems with very small storage capacities as part of the river basin. Further, large-scale water and power system coordination has also garnered considerable attention recently because operating reservoirs for maximization of hydropower generation substantially alters flow alteration (Wang et al. 2015; Kominoski et al. 2018). These are critical issues for large-scale interconnections across North American Electric Reliability Corporation (NERC) regions for improving power system reliability and for promoting renewable energy generation. For such large-scale interconnections, it is imperative to develop low-dimensional, aggregated models that represent both systems using critical system attributes belonging to water and power systems to support monthly to seasonal coordination. Even for systems with significant hydropower (e.g., Columbia and Missouri River basins), evaluation of the hydropower potential based on streamflow forecasts have considered aggregated system representation of the cascade reservoir system (Hamlet and Lettenmaier 1999; Maurer and Lettenmaier 2003). Our comparison of EqR-WB and EqR-EB models under different inflow forecast potential shows that the EqR-WB model performs better than the cascade model. Thus, our study provides initial results in this complex integration problem using a case study and also generalizes the findings considering different reservoir system characteristics.

Appendix I. Parameterization of Equivalent Reservoir Model EqR-EB

In Eq. (8), expanding $E_{\text{inflow},t}$ and $E_{\text{outflow},t}$ using Eqs. (6) and (7), respectively, yields the following energy balance relationship [Eq. (18)] that is used in optimization:

$$\begin{aligned} PE_t + \sum_{j=1}^{n} R_t^j p^j &= PE_{t-1} + \sum_{j=1}^{n} (R_t^{j-1}|_{j>1} + I_t^j) p^j \\ \Rightarrow PE_t + \sum_{j=1}^{n} R_t^j p^j &= PE_{t-1} + \sum_{j=1}^{n} R_t^{j-1}|_{j>1} p^j + \sum_{j=1}^{n} I_t^j p^j \\ \Rightarrow PE_t + \sum_{j=1}^{n} R_t^j p^j - \sum_{j=2}^{n} R_t^{j-1} p^j &= PE_{t-1} + \sum_{j=1}^{n} I_t^j p^j \\ \Rightarrow PE_t + \sum_{j=1}^{n} (p^j - p^{j+1}|_{j<n}) R_t^j &= PE_{t-1} + \sum_{j=1}^{n} I_t^j p^j \end{aligned} \tag{18}$$

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It is interesting to note that, in Eq. (18), the second term on the LHS is not energy outflow ($E_{\text{outflow},t}$), but only a fraction of it. For simulation optimization, when the objective is to maximize energy outflow ($E_{\text{outflow},t}$), we can only effectively optimize that fraction of energy outflow that is contributed by an individual reservoir's release and not the portion contributed by local inflow to the reservoirs. In other words, our decision variable is the second term on the LHS of Eq. (18). If we denote this by $E^*_{\text{outflow},t}$ [Eq. (19)], then $E_{\text{outflow},t}$ can be expressed as a nonlinear function of $E^*_{\text{outflow},t}$ [Eq. (20)]:

$$E^*_{\text{outflow},t} = \sum_{j=1}^{n} (p^j - p^{j+1}|_{j<n}) R_t^j \tag{19}$$

$$\begin{aligned} E_{\text{outflow},t} &= \Delta PE_t + \sum_{j=1}^{n} I_t^j \left(\sum_{m=j}^{n} p^m \right) \\ &= E^*_{\text{outflow},t} + \sum_{j=1}^{n-1} I_t^j \left(\sum_{m=j+1}^{n} p^m \right) \end{aligned} \tag{20}$$

The validity of Eq. (20) can be seen in Fig. S5(a). For the EqR-EB model, $E_{\text{outflow},t}$ can be expressed as in Eq. (21):

$$E_{\text{outflow},t} = \sum_{j=1}^{n} R_t^j p^j = \sum_{j=1}^{n} (S_{t-1}^j + R_t^{j-1}|_{j>1} + I_t^j - S_t^j) p^j \tag{21}$$

For any n, simplifying Eq. (21), we obtain

$$\begin{aligned} E_{\text{outflow},t} &= \sum_{j=1}^{n} R_t^j p^j = \sum_{j=1}^{n} S_{t-1}^j \left(\sum_{m=j}^{n} p^m \right) - \sum_{j=1}^{n} S_t^j \left(\sum_{m=j}^{n} p^m \right) \\ &\quad + \sum_{j=1}^{n} I_t^j \left(\sum_{m=j}^{n} p^m \right) = \Delta PE_t + \sum_{j=1}^{n} I_t^j \left(\sum_{m=j}^{n} p^m \right) \end{aligned} \tag{22}$$

It should be noted that the second term on the right-hand side of Eq. (22) is not energy inflow ($E_{\text{inflow},t}$) but the potential energy equivalent of the local inflows into the cascade.

For $n = 3$, for example, Eq. (21) can be expanded as follows:

$$\begin{aligned} &\sum_{j=1}^{3} (S_{t-1}^j + R_t^{j-1}|_{j>1} + I_t^j - S_t^j) p^j \\ &= (S_{t-1}^1 + I_t^1 - S_t^1) p^1 + (S_{t-1}^2 + R_t^1 + I_t^2 - S_t^2) p^2 \\ &\quad + (S_{t-1}^3 + R_t^2 + I_t^3 - S_t^3) p^3 = (S_{t-1}^1 + I_t^1 - S_t^1) p^1 \\ &\quad + (S_{t-1}^2 + S_{t-1}^1 + I_t^1 - S_t^1 + I_t^2 - S_t^2) p^2 \\ &\quad + (S_{t-1}^3 + S_{t-1}^2 + S_{t-1}^1 + I_t^1 - S_t^1 + I_t^2 - S_t^2 + I_t^3 - S_t^3) p^3 \end{aligned} \tag{23}$$

Appendix II. Parameterization of Equivalent Reservoir Model EqR-WB

Hydropower generated using the EqR-WB model is assumed to be equal to total hydropower generation by the 3R-WB model, such that

$$\begin{aligned} E_{eq,t} = \sum_{j=1}^{n} E_t^j &\Rightarrow \gamma \eta_{eq} h_{eq,t} R_{eq,t} = \sum_{j=1}^{n} \gamma \eta^j h_t^j R_t^j \\ &\Rightarrow p_{eq,t} R_{eq,t} = \sum_{j=1}^{n} R_t^j p^j \end{aligned} \tag{24}$$

For $n = 3$, for example, Eq. (24) can be expanded as in Eq. (25):

$$\begin{aligned} E_{eq,t} &= E_t^1 + E_t^2 + E_t^3 \\ &\Rightarrow \gamma \eta_{eq} h_{eq,t} R_{eq,t} = \gamma \eta^1 h_t^1 R_t^1 + \gamma \eta^2 h_t^2 R_{eq,t}^2 + \gamma \eta^3 h_t^3 R_{eq,t}^3 \\ &\Rightarrow p_{eq,t} R_{eq,t} = p^1 R_t^1 + p^2 R_t^2 + p^3 R_t^3 \end{aligned} \tag{25}$$

The validity of Eq. (25) can be seen in Fig. S5(b). Moreover, expressing each of the release terms under summation in terms of the summation of inflows and change in storages, we can rewrite Eq. (24) as Eq. (26). This is similar to Eq. (22):

$$p_{eq,t} R_{eq,t} = \sum_{j=1}^{n} (S_{t-1}^j - S_t^j) \left(\sum_{m=j}^{n} p^m \right) + \sum_{j=1}^{n} I_t^j \left(\sum_{m=j}^{n} p^m \right) \tag{26}$$

Eqs. (24) and (26) establish the relationship between the EqR-WB, 3R-WB, and EqR-EB models for generating hydropower. The hydropower generation function [Eq. (11*b*)] for the EqR-WB model is derived as given in Eq. (27) and Fig. S6, subject to constraints given in Eq. (11*c*):

$$\begin{aligned} E_{eq,t} &= \gamma \eta_{eq} \phi(S_{eq,t}) R_{eq,t} \\ &= \begin{cases} \beta_{00} + \beta_{10} R_{eq,t}, & R_{eq,t} < R_{eq}^{\max} \\ \beta_{01} + \beta_{11} S_{eq,t} + \beta_{12} S_{eq,t}^2, & R_{eq,t} = R_{eq}^{\max} \end{cases} \end{aligned} \tag{27}$$

Appendix III. Generation Scheme: Synthetic Inflow Forecasts of Specified Skill

The steps for generating synthetic inflow forecast (Sankarasubramanian et al. 2009; Li et al. 2014) of a specified skill, ρ, are as follows:

1. From 30 years of monthly inflow series, inflow series of each calendar month, $t = 1, 2, \ldots, 12$, are modeled separately and then combined to run the reservoir models.
2. We assumed a trivariate lognormal distribution ($Y_{k,t} = \log X_{k,t}$) for modeling the inflow at each reservoir, k, where $k = 1, 2, 3$ for Hartwell, Russell, and Thurmond inflow series, respectively.
3. Calculate each of the transformed inflow series, mean $\mu_{k,t}$, and standard deviation $\sigma_{k,t}$.
4. To preserve the spatial correlation $\gamma_t = [\gamma_{12}, \gamma_{13}, \gamma_{23}]$ between natural inflows across the reservoir for each month, t, the covariance matrix, Σ_Y, for each month, t, is specified to generate lognormal variates. The subscript t on the variance and cross-correlation terms are dropped for ease of notation:
$$\Sigma_{Y,t} = \begin{bmatrix} \sigma_1^2 & \gamma_{12}\sigma_1\sigma_2 & \gamma_{13}\sigma_1\sigma_3 \\ \gamma_{12}\sigma_1\sigma_2 & \sigma_2^2 & \gamma_{23}\sigma_2\sigma_3 \\ \gamma_{13}\sigma_1\sigma_3 & \gamma_{32}\sigma_3\sigma_2 & \sigma_3^2 \end{bmatrix}$$
5. Inflow forecasts with different predictive skill (ρ) are obtained using Eq. (28):
$$\hat{I}_{t,k} = \rho I_{t,k} + \epsilon_{t,k} \tag{28}$$
where Gaussian error $\epsilon_{t,k} \sim N(\mu_{k,t}(1-\rho), \sigma_{k,t}\sqrt{1-\rho^2})$ and $I_{t,k}$ and $\hat{I}_{t,k}$ are the log-transformed observed and generated inflow into a reservoir, k, in month t.
6. We generated 50 traces of lognormal flows using Eq. (28) and transformed them back to the data space to obtain the monthly flows with a specified skill, ρ, for analysis in the section "Generalization of Equivalent Reservoir Performance under Different Reservoir Configurations."

The month–month temporal correlation is preserved since the noise is added to the observed inflows in each month. We assumed two forecast skills, $\rho = 0.75$, 0.5, and generated 1,000 realizations of monthly flows for each site for each specified skill. The ability of the generation scheme to preserve the skill and monthly inflow characteristics are shown in Tables S1–S3 (all values are in $hm^3/month$).

Data Availability Statement

All data used in this analysis are available from the authors upon request.

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Supplemental Materials

Figs. S1–S7 and Tables S1–S3 are available online in the ASCE Library (www.ascelibrary.org).

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