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RESEARCH ARTICLE

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Transmission towers spotting in power systems considering engineering and environmental aspects: A dynamic programming approach

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Summary

In recent years, electricity transmission systems' planning has become a subject of significant discussions worldwide due to increasing investments in renewable power and the need to optimize resources. Planning results directly affect the price of electricity for the final consumers; therefore, it is necessary to determine precise, robust, and relevant plans for the system expansion. Optimization techniques have been successfully employed in several problems associated with transmission line expansion planning, with emphasis on electricity interconnections, routing studies, and tower spotting, among others. The use of these techniques is intended to support planning processes with information that will assist the analyst in the pursuit of defined goals. The present work proposes a methodology based on dynamic programming that seeks to obtain the optimal spotting of transmission towers considering environmental (type of land use, slope, and geotechnical class of the terrain) and engineering characteristics (minimum distance between the electric conductor and ground and tensions supported of each tower type) associated with the problem. The methodology is tested in two different case studies including a real transmission line project with 39 km of extension. The results obtained show, approximately, 3.8% of cost savings obtained using the proposed approach when comparing with the real transmission line project. We also note that it is possible to verify a great similarity between the tower arrangements defined in the real project and the optimal decisions generated by the proposed approach, demonstrating its usefulness as a tool to support decision-making in early stages of investment planning and long-term auctions.

List of Symbols and Abbreviations: ANEEL, Brazilian Electricity Regulatory Agency; AS, average span; AWR, allocation without recursion; CEER, Council of European Energy Regulators; DP, dynamic programming; FC, foundation cost; FERC, Federal Electricity Regulatory Commission; MD, maximum supported deflection; RA, recursive allocation; SD, slope deflection; TL, transmission lines; WS, weight span.

1 | INTRODUCTION

The electric transmission system plays an important role in the power systems' planning and operation, since the purpose of the grid is the continuous supply of electricity to the final consumer. In recent years, due to the rising of renewable energy sources, there has been a significant increase in investments in assets related to the transmission system to integrate such resources. In electric power systems, the transmission capacity must operate in parallel with the generating capacity, avoiding an underutilization of energy resources, thus contributing to the system optimization and the supply of energy to the final consumers at lower prices.[1] In the broad energy systems planning area,[2] high-voltage/capacity transmission lines (TLs) help with the overall utilization of resources available at different regions to satisfy end-user demands, which represent a higher degree of flexibility to the system.

In the global context, new renewable resources are being introduced in many countries in the search for diversification of the energy matrix. Reasons such as global warming and the possibility of depletion of fossil fuels give indications about the need to exploit new energy sources and shift systems away from thermal-based generation. Some countries such as Norway, Colombia, Brazil, and Canada already have most of their generation supplies from hydropower plants, which is a traditional form of renewable power.[3] However, hydropower plants depend on rainfall regimes, and more importantly, local topography, making the potential of this source saturated or economically unattractive in many places. With the increasing demand for electricity, and new incentives targeted to promote other renewables,[4] wind power, solar photovoltaics, and hybrid generation systems have gained momentum,[5,6] as well as natural gas as the streamline contributors to improve generation capacity.[7] The work of Ref. 8 states that over the past 10 years, cumulative global wind power capacity has increased by 6.5 times, and solar photovoltaic capacity has increased by 43 times. Nonetheless, the incorporation of renewable sources must be carried out after a detailed planning of the generation and the transmission system to integrate the supply to the market; the works of Refs. 2,8-10 are examples of studies that address aspects related to the increase in the share of renewables in markets and the role of transmission systems to interconnect these sources to the load.[11]

The incorporation of TLs to integrate new-generation resources assists in electricity trading, as it acts as a facilitator in the process of buying and selling energy from different sources according to their availability. TLs projects implementation costs millions of dollars, and this value is passed on to final consumers through electricity tariffs, which highlight the importance of the planning process of new TL to find the solutions of lower costs.[12] The hiring and implementation of TLs projects are usually controlled by regulatory agencies, such as the Federal Electricity Regulatory Commission in the US,[13] the Brazilian Electricity Regulatory Agency (ANEEL) in Brazil, and the Council of European Energy Regulators, one of the most important regulatory entities in the European Union.[14] In some countries, such as Brazil, new TLs are contracted by auctions through long-term contracts, in which the winner is the competitive agent that requires the lowest annual revenue to construct and operating the TL for 30 years.[15]

The use of mathematical optimization to support decision-making regarding investments in the electric power sector has been a constant practice over the years. Examples of recent work can be found in Refs. 8-10,16 and others. More specifically in transmission systems, mathematical optimization models are constantly applied in the search for the balance between maximum efficiency and the lowest cost in the economic planning of a TL. Generally, the focus is to seek reductions in implementation and maintenance costs, while ensuring a certain level of reliability and other minimum operational requirements.[17] In addition, during the design of optimization models, one must take into account normative, legal, physical, economic, and environmental aspects,[18] considering that new TLs will be integrated to the existing system without compromising or limiting its operation.

The work of Ref. 13 highlights the growth of studies in transmission planning in the last decade and assesses the impact of the use of a mixed-integer linear programming models in the planning of transmission in the Western US. The results show that with the possibility of optimizing the use of the available resources the investments in TLs increase due to higher levels of trading between regions and, consequently, the electricity prices decrease. The work of Ref. 19 proposes a stochastic optimization model that is solved by an approach based on Benders' decomposition with progressive incorporation of contingencies in the transmission expansion planning. The case study presented a high participation of renewable sources covering the Southwestern European region (Portugal, Spain, and France). The authors showed the applicability of their methodology in large transmission systems, finding an investment cost of 1279 M € / year associated with the construction of 27 200 km of TLs. Other examples using mathematical optimization models can be found in Ref. 20, which presents a two-stage stochastic optimization model for the planning of interregional transmission in Great Britain, in Ref. 21 that makes use of risk aversion techniques in transmission and generation planning in the Western US, and in Ref. 22, where a genetic algorithm is presented for cost evaluation and optimization design of TL projects. The work of Ref. 23 proposes a framework based on Geographic Information Systems (GIS) systems, together with multicriteria and shortest path evaluation methods, to provide cost savings and computational time when routing TLs.

The work of Ref. 1 presents a general review of the transmission sector, seeking to discuss concepts that should be used in expansion studies. The authors present a synthesis about the main aspects of the optimization models and economic analysis used in the design, implementation, and evaluation of TL projects. Different optimization techniques are used to model TLs problems ranging from electrical studies, through the definition of structures and components, to the optimal routing definition and optimal spotting (or allocation) of towers in a TL. The present work deals specifically with the optimal allocation of TL structures using a recursive methodology based on the Bellman principle.[24] The transmission tower spotting problem is addressed here using dynamic programming (DP) to find optimal decisions among the various allocation possibilities. The recursive problem characteristic is directly related to the definition of a site to locate a tower as well as the height, type, and location of the towers that precede it and proceed it.

References 25,26 present a discussion about the allocations and heights that should be considered in the tower location, in which two towers' types (suspension and tension towers) are presented for composition of the minimum cost in TLs design. In Ref. 26, when the use of suspension tower is impossible, the methodology detects and monitors the situation returning to the point of the course that a tension tower must be leased to reach an optimal solution. In Refs. 27,28, three variables are optimized, site, height, and span between structures, in the allocation of towers. Due to computational limitations as well as necessary simplifications, Refs. 25-27 focus only on extremely simplified case simulations. For instance, in Ref. 27, the author compares the results for the simplified and detailed models (where the step size for deciding tower allocation vary, resulting in an explosion of computational time) for small stretches of TLs. However, Ref. 27 highlights that (at that time) the computational time required for the more detailed problem representation was very high and that the gain in response accuracy was not relevant for the reduced TL portion under analysis. Due to the computational advances that have occurred in the last decades, it is possible to develop more detailed and accurate models to be applied in more realistic and large case studies. In the work of Ref. 28, the results obtained by a tower allocation methodology are presented, but the authors currently lack a detailed explanation and analysis of the mathematical model and the optimization approach used to carry out the work. The work of Ref. 29 presents a graph theory approach for the tower allocation process, in which the proposed model eliminates paths by preference relation between nodes not yet used, in order to improve the efficiency in determining the minimum cost of the TLs. In order to apply the preference relation, the algorithm generates several alternative paths, implying the need for intensive information storage (often generating memory allocation problems) that makes it difficult to apply the model to obtain optimal solutions for the problem itself, especially in realistic case studies

The literature shows that new tools and methodologies have been developed over the years to improve TLs planning and are mostly focused on GIS.[1,23,30-32] Such methodologies aim to establish the optimal routing of the TL corridor, and the tower spotting aspect of the problem is generally neglected from the initial analysis. Generally, tower spotting is considered only in advanced stages of the planning process (eg, after the hiring of the transmission project in long-term auctions) using traditional engineering software such as Power Line Systems–Computer-Aided Design and Draft (PLS-CADD). These commercial software and tools applied for the final design require detailed information that is not available in the preliminary steps to support decision-making with respect to investment planning and auctions strategies. Therefore, more accurate knowledge of TL projects, including the number, position, type, and tower's height, can improve the cost estimates in early stages of the planning process, helping companies to devise bid strategies in auctions and government agencies in transmission system planning.

Considering the Brazilian system, the regulatory agency provides previous studies before the TL auctions, containing socio-environmental characteristics of the study area, preliminary routes alternatives, and electrical studies that include economical conductor suggestion.[33] Despite the information provided, companies competing in auctions need to plan their bid through cost estimates and risk analysis; this emphasizes the need for methodologies that can accurately provide cost estimates. Therefore, the main goal of this article is to propose a reliable tower spotting methodology able to provide accurate results to support the decision-making process in real-world TL investment planning and long-term auctions strategies. This novel methodology incorporates environmental criteria (type of land use, slope, and geotechnical class of the terrain) and TL project engineering criteria (minimum distance between the electric conductor and ground, and tensions supported of each tower type) to define optimal solutions using DP. The approach is applied to a real TL project that has 39 km of extension (TL Machadinho–Campos Novos), located in the southern region of Brazil, and the results obtained with the approach are compared with the real TL project costs.

The remainder of the article is segmented as follows: Section 2 presents an overview of the methodology focused on mathematical optimization applied to transmission system decision-making. Section 3 presents an example case and a real case study based on the TL 525 kV Machadinho–Campos Novos project along with the results obtained. Section 4 presents the conclusions and suggestions for future studies.

2 | METHODOLOGY

Generally, the process of transmission towers allocation can be divided into three main steps: (a) Route Guideline Definition; (b) Vertex Siting, and (c) Tower Spotting. In the first step, the focus is to define the best area for the LT, also known as the optimal TL corridor. The second step defines where the vertex siting will be located (strong towers sites). And the third step seeks to define the best possibility to allocate each tower in the TL for a predetermined conductor. This work is focused on the third step of TL spotting, and we propose an approach based on DP to find the optimal distribution of towers along a specific topographical profile of the route obtained after the other steps.

The tower spotting problem considers different tower types and heights, terrain slope, geotechnical class, minimum distance between the electric conductor and ground (d_{min}), average span (AS), and minimum and maximum weight span (WS), in order to achieve the minimum cost to implement the project. In this article, the tower allocation process will be divided into two stages (discussed in Section 2), the allocation with recursion and the allocation without recursion (AWR). Variable distances between towers are considered, and in addition to testing the most appropriate structures for the project, the optimal tower layout along the topographic profile is defined. Figure 1 presents an overview of the analysis process carried out here, as well as the representation of the input information, the tower allocation methodology, and the output information.

Optimization techniques applied to transmission towers allocation are often aimed to minimize deployment costs by evaluating many possible towers combinations. Such allocation depends on the type of land use, site deflection in the route, foundation type, tower types, and heights, in addition to the costs linked to these characteristics and to the topographic route profile. The cost and type of tower are directly related to its own height and location, and to the height and location of the back and ahead towers, in which limits for traction in the conductor and efforts on the structures must be respected. Therefore, the allocation of the supports must be optimized economically while respecting

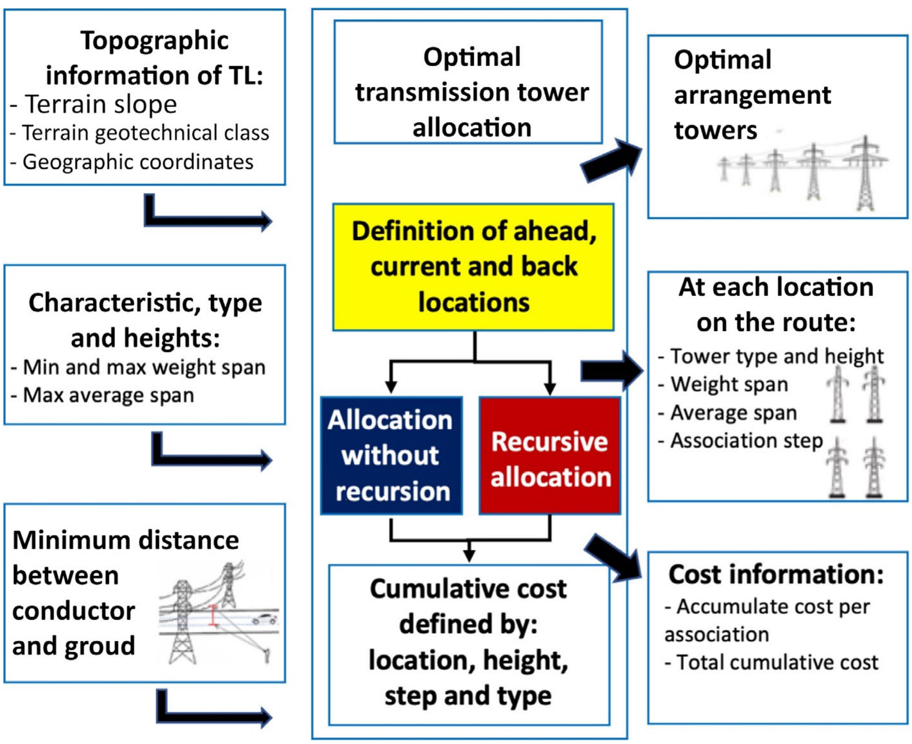

FIGURE 1 Overview of the proposed methodology for optimal transmission tower allocation

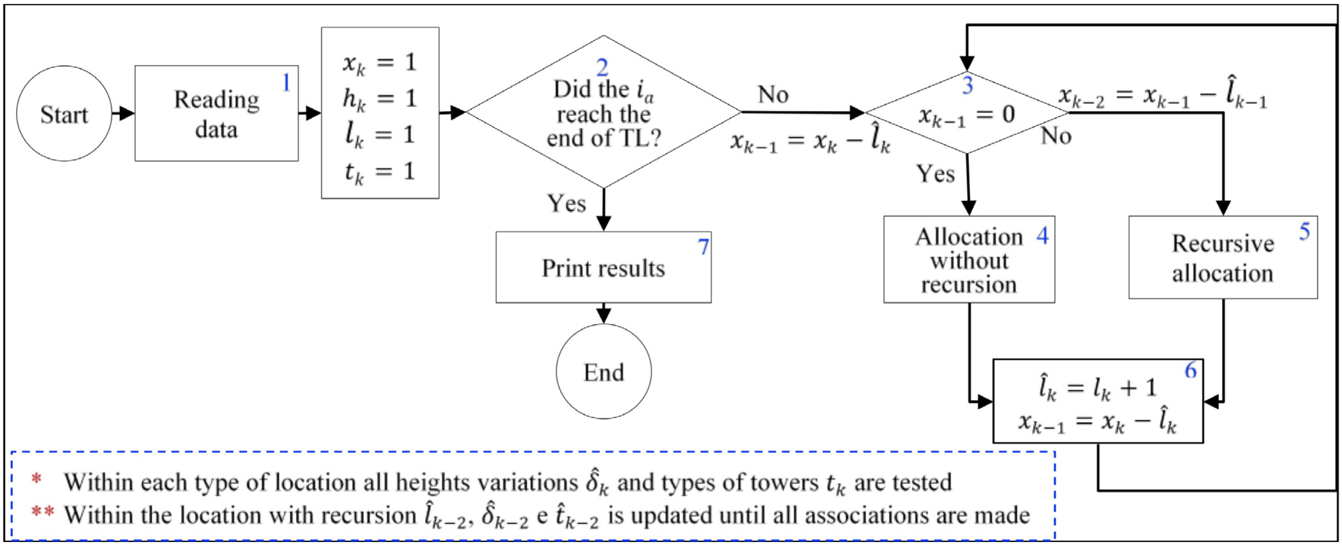

FIGURE 2 Simplified flowchart of the optimized tower allocation process based in DP

technical, geographical, and safety restrictions (which define the obstacles of the ground, the prohibited and compulsory allocation points, and the safety heights of the conductors in each section of the route).

The use of the DP technique in this article has the goal to find a set of towers to compose a TL route of minimum cost that respects the defined technical and environmental restrictions. The employment of DP technique allows one to identify the true optimal solution for the model instead of quasi-optimal solutions derived from heuristic methods of expert judgment approaches. The main input for the optimization model is a list of points referring to the topographic profile of the TL with the following information: dimension, slope deflection (SD), land type, and safety height. In addition, for each type of tower, the following parameters are defined: maximum supported deflection (MD), AS, WS, and foundation cost (FC) by ground type and structure costs by height.

The flowchart of Figure 2 presents a simplified form of the proposed tower allocation procedure. The first step is to read the input data (block 1), the second step verifies if the TL has already been completely covered (block 2), and then a decision must be made for the allocation process to stop or continue. If the decision for block 2 is to continue, meaning that the end of TL has not been reached, the decision for block 3 defines which process of tower allocation will be performed at the site under analysis, if the site is directly associated with the first site of the TL the type of allocation is without recursion (block 4). Otherwise, allocation with recursion is performed (block 5). At the end of the allocation, data must be updated (block 6), and the program returns to the decision block 2 until TL ends, then results are printed (block 7).

2.1 | Representation of the tower allocation problem

The mathematical representation of the problem relies in the definition of three test sites, identified by two-dimensional coordinates (x,y). The x-coordinate represents the location of the site on the route from the location of the initial tension tower to the location of the terminal tension tower of the TL. The y-coordinate represents the information about the total height (quota of the site + tower height) for each test site. These sites are then represented by the coordinates $(x_k,y_k),(x_{k-1},y_{k-1}),(x_{k-2},y_{k-2})$, which correspond to the specifications of the ahead site, middle site, and back site, respectively, as presented in Figure 3. Therefore, $x_k,x_{k-1},x_{k-2} \in X$, where X is the set of possible sites to place transmission towers and y_k,y_{k-1},y_{k-2} are the respective total heights.

The use of three references to represent sites is a way to obtain the global optimal solution for the problem using DP, once it is necessary to test all the associations with the back site (x_{k-2}) and with the ahead site (x_k) to obtain the best possibility for the middle site (x_{k-1}). The determination of the x-coordinate of the middle site is obtained using the coordinate of the ahead site minus a step l_k, that is, $x_{k-1} = x_k - \hat{l}_k$. The x-coordinate of the back site is obtained with respect to the coordinate of the middle site minus a step $\hat{l}_{k-1}$, that is, $x_{k-2} = x_{k-1} - \hat{l}_{k-1}$. Knowing that $l_k \in L$ represents the set of steps that define the possible distances for a given tower type, it is possible to conclude that by establishing a

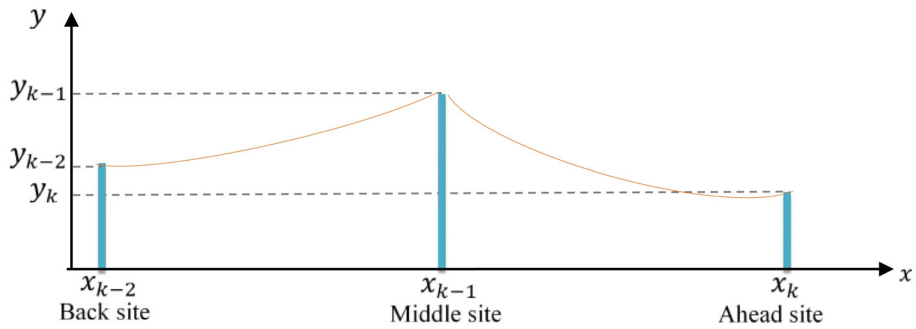

FIGURE 3 Representation of test sites

value for x_k and testing $\hat{l}_k$ and $\hat{l}_{k-1}$, the x-coordinates for x_{k-1} and x_{k-2} are obtained. Thus, the entire optimization problem can be structured with respect to the tower to be placed in the ahead site.

A DP model formulation for the problem at hand can be represented by Equation (1). The function $F_k(s_k)$ represents the goal to minimize the total cost, where $F_{k-1}(s_{k-1})$ is the previous cost established up to location x_{k-1} added to the actual location x_k costs represented by $C_k(s_k, u_k)$. Here, the term $s_k = (x_k, x_{k-1}, y_k, y_{k-1})$ represents the state vector at x_k, the term u_k is associated with the decision vector in the same spot, the term N represents the TL total length, and the problem stages and locations are represented by $k = 0, 1, 2, ..., N-1, N$.

$$F_k(s_k) = \min \{C_k(s_k, u_k) + F_{k-1}(s_{k-1})\}. \quad (1)$$

$$\text{s.t. } s_k = m(s_{k-1}, u_{k-1}). \quad (1a)$$

$$g(s_k, u_k) \geq 0. \quad (1b)$$

Equation (1.a) can be generally represented as defined here in Equation (2). The state vector s_k is in this case defined using conditions carried forward from the previous stage, while current decisions are made to ensure that the Equation (2) are met. Also, u_k represents a vector of the decisions performed at stage k, and s_{k-1} is the state vector values defined as parameters previously defined at stage $k-1$.

$$m(s_{k-1}, u_{k-1}) = \begin{cases} x_k = x_{k-1} + \hat{l}_k, \\ w_k = x_{k-1}, \\ y_k = y_{k-1} + \hat{\delta}_k, \\ z_k = y_{k-1}. \end{cases} \quad (2)$$

Other structural constraints represented initially presented on Equation (1.b) are expanded here in Equations (3.a)-(3.c). Where $u_k = \left(\hat{\delta}_k, \hat{l}_k, \hat{t}_k\right)$ represents the decision vector at stage k, the difference in height from previous stage is defined by $\hat{\delta}_k$, the route step length is defined here as $\hat{l}_k$, and the tower type selected is represented by $\hat{t}_k$.

Here, physical efforts due to the weight of the wires are represented in Equations (3.a) and (3.b), while Equation (3. c) denotes the safety heights to be enforced in order to diminish the likelihood of potential faults such as short circuits. Physical and safety criteria have to be satisfied by the model when attempting to make location decisions and the distance among towers. In this case, such distance is denoted considering average WS limits of each tower type $t_k \in T$. The term d_{min} enforces the minimum height for all the TL extension due to the wire height being directly connected to the tower total height. The terms c_1 and c_2 represent constants in Equations (3.b) and (3.c), calculated as the horizontal effort of the wire in kgf, given the temperature considered, divided by the wire's unitary weight in kg/m. c_1 is calculated for the minimum wire temperature and c_2 for the maximum wire temperature. Equation (3.a) represents a constraint that checks if $\hat{l}_k$ and $\hat{l}_{k-1}$ create an AS that is smaller than MAS_{tk} (upper bound for AS). We note that Equation (3.a) relates to the horizontal efforts created by the cables in the towers. Equation (3.b) represents a constraint that guarantees that the difference in height for towers placed at (x_k,x_{k-1} and x_{k-2}) and the step lengths sizes ($\hat{l}_k$ and $\hat{l}_{k-1}$) form a

WS that ranges between $\underline{\mathrm{WS}}_{t_k}$ (lower bound for WS) and $\overline{\mathrm{WS}}_{tk}$ (upper bound for WS). We note that Equation (3.b) relates to the vertical force caused by the wires on the towers. Therefore, with the verification of the horizontal efforts as well as the vertical efforts, the approach directly considers the span. Because many tower types are considered, different MAS_{tk}, $\underline{\mathrm{WS}}_{t_k}$, and $\overline{\mathrm{WS}}_{tk}$ have to be analyzed. Equation (3.c) represents a constraint that checks if the cables are positioned at a safe distance from the soil. In this case, for all the points defined, the wire height should be higher than $d_{\min}$. In this model, x' represents each existent point among x_k and x_{k+1}, where $d_{\min}$ must be enforced. To simplify the notation in the reminder of the article, we consider the right-hand side of Equation (3.c) as τ_k, which represents the height requirement evaluation.

$$g(s_k, u_k) = \frac{\hat{l}_k + \hat{l}_{k-1}}{2} \leq \mathrm{MAS}_{t_k}. \tag{3a}$$

$$g(s_k, u_k) = \underline{\mathrm{WS}}_{t_k} \leq \frac{\hat{l}_k + \hat{l}_{k-1}}{2} - c_1 \left[\frac{\hat{\delta}_k}{\hat{l}_k} - \frac{\hat{\delta}_{k-1}}{\hat{l}_{k-1}} \right], \overline{\mathrm{WS}}_{tk} \geq \frac{\hat{l}_k + \hat{l}_{k-1}}{2} - c_2 \left[\frac{\hat{\delta}_k}{\hat{l}_k} - \frac{\hat{\delta}_{k-1}}{\hat{l}_{k-1}} \right]. \tag{3b}$$

$$g(s_k, u_k) = d_{\min} \leq y_k + \hat{\delta}_k + \frac{\left(x_k + \hat{l}_k - x'\right)^2}{2c_2} - \left(x_k + \hat{l}_k - x'\right) \cdot \left(\frac{\hat{l}_k}{2c_2} - \frac{\hat{\delta}_k}{\hat{l}_k}\right), \forall (x_k \leq x' \leq x_{k+1}). \tag{3c}$$

2.2 | Tower allocation process

From the set of allocation possibilities, the DP method chooses the least cost configuration of towers that satisfies the constraints imposed on the model. At the beginning and at the end of the TL, the types of towers are fixed, since these places present restrictions and peculiarities linked to interconnections with electrical substations. The trajectory vertices (deflection points) are also the required items for the allocation, but different from the starting and ending sites, the tower types may vary.

For other sites, the allocation is carried out in two different ways: the AWR and the recursive allocation (RA). The AWR is the one performed between the first test location of TL and the initial site (x_0). In this allocation, the only possible association step is the distance between the test site x_k and x_0. On the other hand, the RA deals with the definition of test sites, considering the characteristics of the configurations already tested. Thus, in order to perform an RA, it is necessary that at least one AWR has been evaluated.

In Figure 4, the possible allocation tests for AWR and RA are represented using a single height and span determined by $l_k \in L$, where the values assigned to the minimum and maximum allowed spans between towers are 8 and 10. In the AWR, the second tower of the segment is tested, in which the association with the initial site is mandatory. In such cases, as this is the first span of the route, the information of the back association does not exist. Therefore, they are not part of the analysis. In the RA, the allocation of the third tower of the TL is tested. In this case, there is already information of back allocations to be considered. Therefore, this information must be considered in such tests.

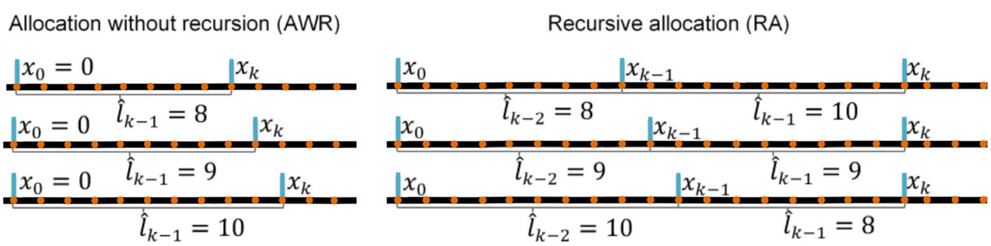

FIGURE 4 Tower allocation considering AWR and RA processes

2.3 | Allocation without recursion

In the combination of towers for each test site, it is checked whether the $d_{\min}$ is being met. The verification of $d_{\min}$ is performed using the parabola equation, in which each soil type has a different safety height. If this distance is respected for all points along the span, a cost $f_k(s_k)$ is defined as a function of the height of the terminal tension tower (t_0) in Equation (4). If $d_{\min}$ is not respected for a height variation $\hat{\delta}_k = y_k - y_{k-1}$, the value of $f_k(s_k) = \infty$.

$$F_k(s_k) = \min f_k(s_k) = \min\left\{\left[\vartheta(\mathrm{GC}_k)\cdot \mathrm{SC}_k + \mathrm{CT}\left(\hat{t}_k, \hat{\delta}_k\right)\right] + \left[\vartheta(\mathrm{GC}_0)\cdot \mathrm{SC}_0 + \mathrm{CT}\left(t_0, \hat{\delta}_0\right)\right]\right\}. \tag{4}$$

Where $\vartheta(\mathrm{GC}_k)$ is the FC due to the geotechnical class GC_k of the site x_k; SC_k represents a factor multiplier of the FC related to the slope of the site x_k; and $\mathrm{CT}\left(\hat{t}_k, \hat{\delta}_k\right)$ represents the cost associated with the tension tower type defined in k. Therefore, C_0 is only allowed for the AWR, tested for site $x_0 = 0$ and at height variation $\hat{\delta}_0$. After defining the first $f_k(s_k)$, a second height variation $\hat{\delta}_k$ is tested and another $f_k(s_k)$ is defined, until all the heights available are tested and their respective $f_k(s_k)$ is determined. The same process must be performed for each tower type t_k.

Taking all the costs of associations from a height variation $\hat{\delta}_k$ (between k and $k-1$) into x_k, it is not necessary to store all the information; for each y_k, only the smallest value of $f_k(s_k)$ should be saved because the purpose of the model is to search for the minimum cost, and this is obtained by the lowest $f_k(s_k)$ value. The height variation $\hat{\delta}_k$ of the stage k used to obtain the minimum cost must also be stored in $F_k(s_k)$, since it is necessary to store the total height for the optimal route defined at the end of the TL. Where $F_k(s_k)$ is the cumulative cost of tower allocation up to site x_k considering a tower with height variation $\hat{\delta}_k$, type $\hat{t}_k$ and with a step $\hat{l}_k$. In the AWR, the checking of the AS and WS is not performed, once the analysis only takes into account two towers and the calculation of AS and WS for one tower considers three sites (ahead, middle, and back). Based on this, only one value of maximum span for the test is fixed MAS_{tk}.

2.4 | Recursive allocation

The RA process is an association between three towers, represented by three coordinates (x_k, y_k), (x_{k-1}, y_{k-1}), and (x_{k-2}, y_{k-2}) (Figure 3) and the steps $l_k \in L$ that interconnect them. All the possibilities of association between the sites x_k and x_{k-1} are tested, for which the $d_{\min}$, MAS_{tk}, $\left(\underline{\mathrm{WS}_{t_k}}\right)$, and $\left(\overline{\mathrm{WS}_{tk}}\right)$ are checked. The AS requirement aims to check if an allocation in site x_k is allowed according to mechanical efforts linked to the wind speed load that the tower is able to support in its electric conductors and to the lightning arresters when exposed to the SD angle at the site x_{k-1} in face of their adjacent sites x_{k-2} and x_k. Knowing that the highest value of MAS_{tk} occurs when the SD angle at the site is zero and its support decreases linearly, the lowest MAS_{tk} value that the tower can support is associated with the highest SD allowed for each tower type.

The WS indicates the vertical stress on the chain of electric insulators, and it is calculated by the distance between the vertices of the parabolas formed by the spans adjacent to site x_{k-1}. For each tower type, a minimum and a maximum limit for WS are established, which must be verified. The value of $\overline{\mathrm{WS}_{tk}}$ is verified for the conductor curve at the maximum temperature when the conductor has a longer length (thermal expansion), and the WS values are consequently higher. On the other hand, the $\underline{\mathrm{WS}_{t_k}}$ must be checked for the minimum temperature condition, when the electric conductor has shorter length, higher traction, and consequently lower values for WS. In extreme cases for minimum temperature, a situation called uplift can occur, when vertical pulls are negative, pulling the tower upwards, which highlights the importance of the check for $\underline{\mathrm{WS}_{t_k}}$. It is important to highlight that the effects of the environment (wind, temperature, and lightning) are considered when the terms c_1 and c_2 of Equations (3.b) and (3.c) are calculated, what is a previous step for the application of the methodology proposed here. We note that horizontal efforts and temperatures (used to calculate c_1 and c_2) can be obtained from a database of similar TL projects with the same conductor type and preliminary studies that the regulatory agency provides before the auctions.

In order to test all possibilities of tower association, it is necessary to vary the heights of the towers at site x_k and x_{k-1}. However, the available heights for site x_k are linked to their possible associations with the site $x_{k-2} = x_{k-1} - \hat{l}_{k-1}$ and the partial costs of these associations. Therefore, during the verification of the RA requirement, it is possible to collect cumulative costs not only for the different steps $\hat{l}_k$ (that associate the ahead site x_k with the middle site x_{k-1}) but also for the different steps $\hat{l}_{k-1}$ (that associate the middle site x_{k-1} with the back site x_{k-2}). The procedure is repeated until all sites have all requirements verified. If all requirements are satisfied for the points between x_k and x_{k-1}, the associated $f(s_k)$ is obtained by using Equation (5), otherwise $f(s_k) = \infty$.

$$F_k(s_k) = min f_k(s_k) \tag{5}$$

2.5 | Total final cost computation

After defining the minimum costs for all tower associations, the minimum TL cost must be determined and, consequently, the optimal tower arrangement related to that cost. To complete the process, a final procedure must be performed to define the optimal tower arrangement, where the sum of the cost of the tension tower allocation at the terminal site N is added to $F_{N-1}(s_{N-1})$, obtaining a minimum cost for each height tested. The minimum of these costs is the optimal cost associated with the tower spotting for the TL. The total cost $F_N(s_N)$ and the total height y_N of the terminal tension tower are represented, respectively, by Equations (6) and (7).

$$F_N(s_N) = \min\{C_N(s_N, u_N) + F_{N-1}(s_{N-1})\} = \min\left[\vartheta(\mathrm{GC}_N)\cdot \mathrm{SC}_N + \mathrm{CT}\left(\hat{t}_N, \hat{\delta}_N\right) + F_{N-1}(s_{N-1})\right]. \tag{6}$$

$$A_N(s_N) = \mathrm{argmin}(F_N(s_N), y_N). \tag{7}$$

It is important to store the height and the total cost of the allocation because this information allows one to find the path with the least cost configuration at the end of the allocation process. The process for determining each location is processed backward because the lowest accumulated cost of a site directs to the next site that previously had the lowest cost. Each site in the path is directly associated with the height of the tower, tower type, and step that lead to the site that precede it on that path. This search process is repeated until the first site of the route is reached.

2.6 | Dynamic programming algorithm for transmission tower spotting

The algorithm developed for the transmission tower spotting is divided into four basic steps described next. The first step deals with the main logic of the algorithm, where the data are processed and the type of allocation is selected. The second step deals with the AWR process; the third step of the algorithm carries on the RA process. Step 4 deals with the calculation of the total cost associated with the TL project and the height of the tension tower at the end of the route. The proposed algorithm was initially implemented in Matlab and later translated to C++ to improve computational performance.

Algorithm to identify the optimal allocation of TL towers based on DP

Input: Positions (X) and total height (y) of each test location, safety height by type of land use ($d_{\min}$), sites which are not feasible to allocate towers, site deflection, geotechnical class (GC_k), type (t) and height variations (δ) for towers available, tower type specifications (MAS_{tk}, ($\underline{\mathrm{WS}}_{t_k}$, ($\overline{\mathrm{WS}}_{tk}$), conductor specifications and costs associated with towers ($C(t,h)$), foundation ($\vartheta(\mathrm{GC}_k)$), and slope multiplier (SC_k)

Output: All allocation sites (x_k) with their respective quota site and towers types (t_k), towers height, WS, and AS, and interconnections with existent infrastructure and cumulative cost

Step 1. **set** $x_k = 1$; $l_k = 1$; $t_k = 1$;
while $x_k < N$ **do:**
  for $l_k \le L$:
    for $t_k \le T$:
      if $x_k - l_k == 0$:

go to Step 2
else:
go to Step 3
set $x_k = x_k + 1$;
go to Step 4

Step 2. **set** $x_{k-1} = x_k - \hat{l}_k$;
if $l_k < \mathrm{MAS}_{tk}(0°)$:
set $\hat{\delta}_k = y_k - y_{k-1}$;
for $h_k \leq H$:
set $\hat{\delta}_{k-1} = y_{k-1} - y_{k-2}$;
for $h_{k-1} \leq H$:
if $\tau_k \geq d_{\min}$:
compute $f_k(s_k)$), i.e.,
$f_k(s_k) = C_k(s_k, u_k) + C_0$;
else:
set $f_k(s_k) = \infty$;
compute $F_k(s_k)$ as Equation (4) i.e.,
$F_k(s_k) = \min f_k(s_k)$;
go back and continue Step 1

Step 3. **set** $x_{k-1} = x_k - \hat{l}_k$; $\hat{\delta}_k = y_k - y_{k-1}$;
for $h_k \leq H$:
set $\hat{\delta}_{k-1} = y_{k-1} - y_{k-2}$;
for $h_{k-1} \leq H, t_{k-1} \leq T, l_{k-1} \leq L$:
if $\mathrm{AS} \leq \mathrm{MAS}_{tk}$ and $\mathrm{WS} \geq \underline{\mathrm{WS}_{t_k}}$ and $\mathrm{WS} \leq \overline{\mathrm{WS}_{tk}}$:
if $\tau_k \geq d_{\min}$:
compute $f_k(s_k)$, i.e.,
$f_k(s_k) = C_k(s_k, u_k) + F_{k-1}(s_{k-1})$;
else:
set $f_k(s_k) = \infty$;
compute $F_k(s_k)$ as Eq. (5) i.e.,
$F_k(s_k) = \min f_k(s_k)$;
go back and continue Step 1

Step 4. compute the total cost $F_N(s_N)$ using Equation (6) and the height associated with the terminal tension tower $A_N(s_N)$ using Equation (7), i.e.,
$F_N(s_N) = \min C_N(s_N, u_N) + F_{N-1}(s_{N-1})$;
$A_N(s_N) = \mathrm{argmin}(F_N(s_N), y_N)$;

3 | CASE STUDY AND ANALYSIS

3.1 | Example case

The example case refers to a portion of a TL with 780 m, divided into 27 sites with 30 m. Table 1 presents the input data for the test of the proposed methodology in which are presented plot data, land use, deflection, and slope for each site. Each site identifies the tower type as 0 or 1, where 1 means that the tower type is allowed on site, and 0 means that tower type is not allowed. Six types of towers are considered: simple suspension tower (SS), reinforced suspension tower (SR), angle suspension tower, middle tension tower, terminal tension tower (AT), and special tower for rivers and reservoirs (S1).

The d_{min} is determined according to the site's land use class (Table 2). In order to define the conductor's curve (parabola) and thus check $d_{\min}$, it is still necessary to specify the characteristics of the conductor, where, for example,

TABLE 1 Input data for the test route

Site	S	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	K
Distance (m)	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780
Quota (m)	448	448	446	442	437	430	422	424	418	413	409	406	404	403	404	406	409	413	418	427	437	450	467	484	498	507	511
Land use	2	2	6	6	6	5	5	6	6	1	1	1	1	1	1	1	1	1	2	2	1	2	2	2	2	2	2
Deflection (SD) (°)	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16
Slope (°)	48	48	43	37	39	40	42	40	42	40	38	36	34	34	33	32	30	27	30	41	46	57	63	61	44	30	19
Slope class	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	3	3	3	2	2	1
Tower type																											
SS	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0
SR	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0
SA	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0
AM	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0
AT	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	1
S1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0

TABLE 2 Values of d_{min} according with to land use type

No	Land Use	Safety Height (d_{min}) in (m)
1	Water	9.53
2	Forest	12.53
3	Pasture	10.53
4	Agriculture	9.03
5	Ground exposed	8.53
6	Urban areas	10.53

1601 [kg/m] was used for the conductor weight and 2100 [kgf/m] for traction. The technical parameters and costs of each type of tower are presented in Table 3. In relation to the MAS, it is worth mentioning that its value is calculated linearly with respect to MD of each tower type. That is, it calculates the variation of the MAS between the location without deflection (0°) and with MD. Afterward, it is necessary to divide this value by the angle that corresponds to the allowed MD for the tower type, and finally for each increment of deflection of the route, the value of allowed MAS is calculated. The FC of each tower type is related to the location's geotechnical class. For example, all locations for allocation are in the same geotechnical class, so only one FC value for each type of tower was used. The slope multiplier is used as a foundation overcapacity factor in relation to slope, in which places with slopes greater than 50° were considered as prohibitive in allocation of structure. The CE is the cost of structure according to tower type and height specification.

The application of the methodology for the example case resulted in a total cost of US $ 270 288.27 and use of three towers. The first tower is located at site 0 and is a 15-m (AT) terminal tension tower, the second one is located at site 5 and is a 40-m SR-reinforced suspension type, forming a span of 150 m between these site and site 0. The third tower is at site *T* and is an AT tower with 20 m, forming a span of 630 m with site 5. The arrangement of the towers in the profile can be seen in Figure 5.

3.2 | Real case study: TL Machadinho–Campos Novos

It is important to notice that, when considering real TL projects (with large extension), the computational performance of the proposed approach is affected due to the well-known DP "curse of dimensionality." In these situations, for the sake of computational tractability, it is possible to segment a TL and apply the proposed approach to each TL segment separately. In this case, vertex sites (for the terminal towers) are predetermined for each TL segment, and then the proposed approach can be applied to identify the optimal tower type, height, and location for the other towers between the vertex sites. After performing this process for all the TL segments, the results are then combined to obtain the overall TL allocation.

A high-voltage TL (525 kV) composed of a simple circuit in the southern region of Brazil was selected as a real case study to evaluate the proposed methodology. This TL is in operation, has 83 freestanding towers, and connects the Machadinho hydro power plant (located in the state of Rio Grande do Sul) to the Campos Novos substation (located in the state of Santa Catarina). In Brazil, ANEEL provides a pilot project for the acquisition of the project by the companies involved, but competing agents can perform studies based on the data provided for construction of alternative projects. Since the calculations in this type of projects are complex, the risk in which the competitors are inserted is high, and the use of optimization models in this case can be applied in order to support and facilitate the decision-making process.

The TL Machadinho–Campos Novos crosses lands of basaltic rocks and clay soils, with different relief configurations, and with levels of smooth topography plateaus and edges with dissected valleys with high slopes. The application of the methodology considered the same class of towers used in the TL executive project[34] and tower heights varying in increments of 5 m within the established limits. For the topographic profile, a mean resolution of 30 m was used, that is, the TL tracing was divided into equally spaced points of 30 m. For the modeling of the conductor's curve, reference mechanical efforts of 1944 and 2199 [kgf] were used for maximum temperature (60°C) and minimum temperature (−6°C), respectively. Land use in the area was determined by Landsat 8 satellite image classification and photointerpretation of high-resolution satellite images.[35] Safety height values were determined using the standards from NBR5422[36]

TABLE 3 Technical parameters, FC, and slope for each tower type and TC

Tower Type	MAS_{ta} **(0°) (m)**	**MD (°)**	MAS_{ta} **(MD) [m]**	**WS (m)**		**FC**	**SM**			**TC (US$)**					
				Min	**Max**	**(US$)**	**(<30°)**	**(30°–50°)**	**C (>50°)**	**15 m**	**20 m**	**25 m**	**30 m**	**35 m**	**40 m**
SS	450	3	370	300	600	11 746	1	1.1	Infeasible	19 285	22 618	25 950	29 283	32 616	–
SR	560	3	478	300	700	15 661	1	1.1	Infeasible	19 406	23 230	27 054	30 878	34 702	38 526
SA	700	10	429	350	850	20 009	1	1.1	Infeasible	21 243	25 546	29 850	34 153	38 457	42 760
AM	1000	25	340	−300	1120	25 095	1	1.1	Infeasible	63 898	71 561	79 225	86 888	94 551	102 214
AT	1400	56	300	−300	1300	30 899	1	1.1	Infeasible	70 556	79 090	87 625	96 159	104 693	113 227
S1	1000	25	340	−300	1120	95 337	1	1.1	Infeasible	63 898	71 561	79 225	86 888	94 551	102 214

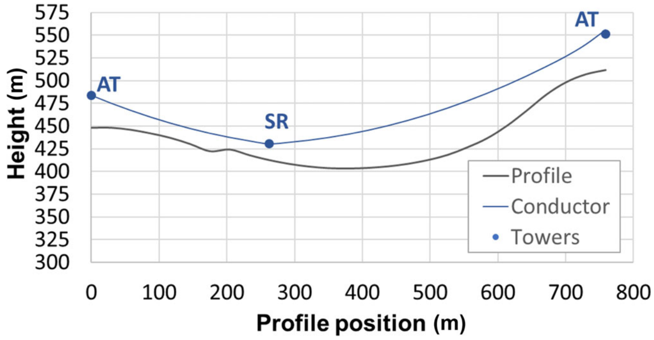

FIGURE 5 Projection in the example case model profile

TABLE 4 Towers characteristics of the 525 kV Machadinho–Campos Novos

	Height (m)						TC (US$)		FC (US$)	
Tower Type	Min	Max	MD (°)	MAS_{tk} (m)	$\underline{WS_{t_k}}$ (m)	$\overline{WS_{tk}}$ (m)	A	B	Soil I	Soil II
SS	15	35	3	450	300	600	734	9670	12 970	24 116
SR	15	40	3	560	300	700	842	8180	17 176	31 947
SA	15	40	10	700	350	850	948	8665	21 982	40 887
AM	15	55	25	1000	−300	1120	1688	44 118	27 705	51 531
AT	20	45	56	1400	−300	1300	1879	48 573	34 587	64 332

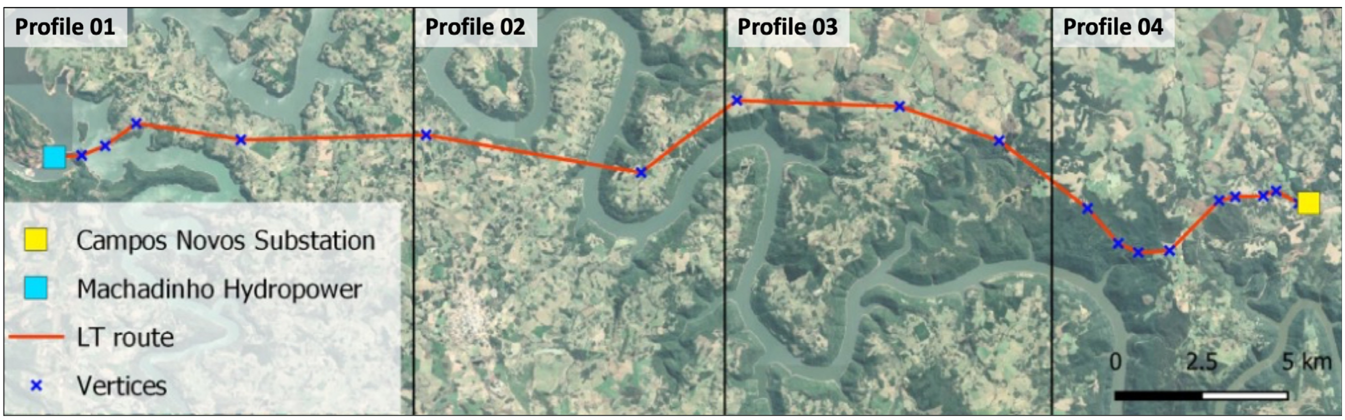

FIGURE 6 Existing transmission line Machadinho–Campos Novos

according to the soil use (8.5 m for water, forest, pasture, 9 m for agriculture, and 10.5 m for streets and avenues). The quote and slope data were obtained from the WorldDEM digital elevation model[37] with 12 m resolution and absolute vertical precision of less than 10 m.

Table 4 presents the technical parameters and costs of each type of tower. The cost of each type of tower is determined as a function of the height adopted in the structure cost equation ($TC = A \times h + B$). The FC are associated with two types of soil: Soil I, which corresponds to soils of good geotechnical quality and does not require special foundations, and Soil II, which corresponds to low-resistance soils that require special foundations such as stakes and shallow foundations. Note that TC and FC were calculated from the ANEEL TL project and budget information sheet.[38] It is important to emphasize that the cost values adopted in this study were converted from the Brazilian currency R$ to US $ using an exchange rate of 1 US$ = 3.75 R$.

Figures 6 and 7 show details of the overall transmission project that connects Machadinho to Campos Novos. Portion of the results associated with the allocation process (highlighted with a square in Figure 6) by the proposed methodology in comparison with the existing TL tracing is shown in Figure 7. It is possible to verify a large similarity

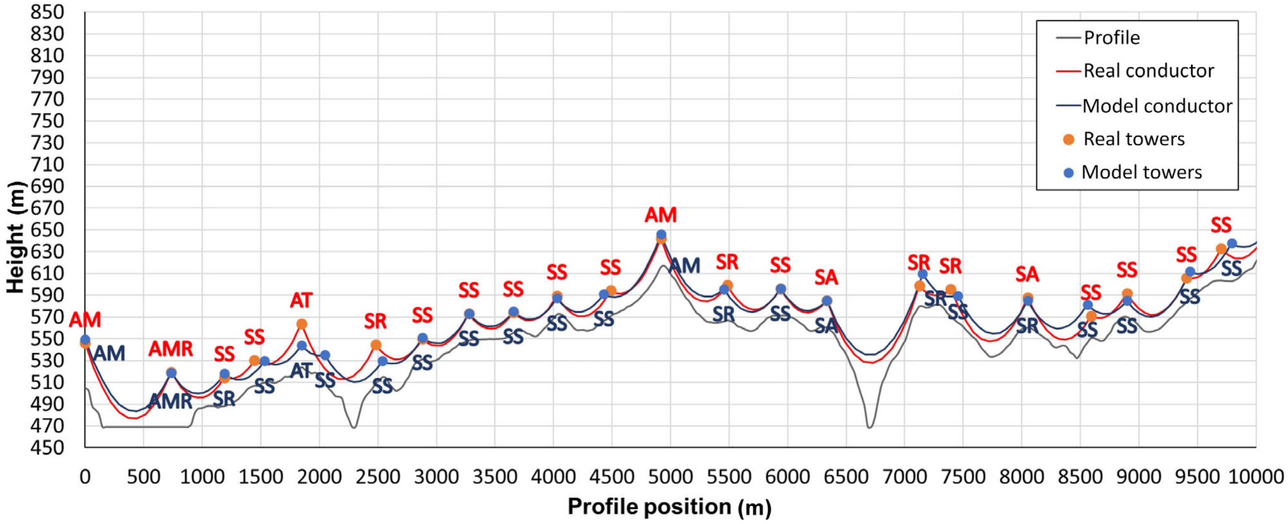

FIGURE 7 Comparison of the structure arrangement in Profile 01—model result vs real TL

TABLE 5 Results of the applied methodology by tower type

Tower Type	Tower Number		Average Height (m)		Average Span (m)		Total Cost (US$)	
	Model	Real	Model	Real	Model	Real	Model	Real
SS	36	27	24	24	410	404	1 457 314	1 083 627
SR	22	23	29	32	509	505	1 096 865	1 197 601
SA	8	11	34	30	622	631	505 832	644 693
AM	9	11	28	34	455	521	1 163 496	1 500 393
AT	10	9	22	30	400	435	1 235 714	1 250 291
TL (total)	85	81	27	29	459	483	5 459 221	5 676 605

between the real tower arrangements and the one generated by the model. The structure arrangement for other segments of the TL Machadinho–Campos Novos can be found in the Appendix. The results also show that decisions made by the DP algorithm approach provide accurate information with respect to cost estimation.

Table 5 shows the heights, AS, and costs (towers + foundation) discretized by tower types. The model solution resulted in an arrangement with four additional towers than the information reported in the real TL design. The towers selected by the proposed approach have slightly lower heights and spans in average, as can it be observed by the results presented in Table 5. The model used more SS-type towers, and because of that, it chose to allocate a larger number of towers than those used in the real TL project. This larger number of SS towers allowed a reduction in the amount of robust towers and, consequently, a reduction in the total cost. The size of the mean span also decreased, due to the inclusion of these four extra towers in the final towers arrangement. This reduction associated with the mean span also enabled the placement of cheaper and lighter towers. With respect to the total cost, the arrangement obtained by the model presented a value 3.83% lower than the real TL estimated cost. Therefore, we note that the current approach can potentially serve as an important tool to support decision-making and to define strategies aimed to reduce the risk associated with the TL project in long-term auctions.

4 | CONCLUSION

This work presented a detailed methodology for the transmission towers allocation based on DP. The approach considers geographical and engineering criteria in the formulation of technical constraints and costs definition.

The results obtained by model for the TL 525 kV Machadinho–Campos Novos with 39 km of extension show similarities with the TL real project, with respect to the approximated number of towers, average height, and AS. The results also show that the model has a good capacity to represent the technical constraints associated with the tower allocation problem. In addition, the towers arrangement total cost obtained by the model was 3.83% lower than the estimated real TL project cost, demonstrating that the methodology was efficient in optimizing the towers distribution in the profile. These results confirm the potential application of the model to support early-stage decision-making with respect to investment planning and long-term auctions strategies, both for consistency in modeling the problem and for the ability to generate optimized results with respect to costs. In addition, such application of the model is highlighted as a tool to support the preparation of basic and executive projects, given the technical consistency in the representation of constraints.

We note that, as the analysis performed by the tower allocation model is two-dimensional, that is, it does not evaluate the quality of the terrain around the observed location, errors as to the quality of the ground may exist and cheaper towers may be misplaced. In this respect, an evaluation by the company must be carried out based on the experience of analysts in this type of undertaking, since the precision is limited. Also, a more detailed computer simulation associated line sagging and the strengths on dead-end or strain structures could be performed after an initial evaluation with the proposed approach.

The outstanding uncertainties open up space for future works that deal with the influence of profile resolution, topography, and height increase in the process of optimal allocation of towers in TLs. Future works may explore the application of the methodology to other TL traces, as well as the joint consideration of the methodology in transmission planning processes to provide more accurate information related to the costs of the projects for the electric planning that determines the buses of the system to be interconnected. Other aspects of the problem could be represented such as maintenance costs of the transmission towers, co-optimization of tower spotting and optimal corridor definition, and the exploration of different mathematical programming and heuristic techniques to solve the model.

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PEER REVIEW

The peer review history for this article is available at https://publons.com/publon/10.1002/2050-7038.13000. [Correction added on 16 September 2021, after first online publication: Peer review history statement has been added.]

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

INDICES AND SETS

$x_k \in X$	Set of possible sites to place transmission towers
$l_k \in L$	Set of steps that define the possible distances for each transmission tower
$k = 1, 2, ..., N-1, N$	Optimization models' stages as well as the locations
$t_k, t_0 \in T$	Set of transmission tower types
$h_k \in H$	Set of transmission tower heights available at k

PARAMETERS

(x_k, y_k)	Two-dimensional coordinates (x,y) of the site ahead
(x_{k-1}, y_{k-1})	Two-dimensional coordinates (x,y) of the middle site
(x_{k-2}, y_{k-2})	Two-dimensional coordinates (x,y) of the back site
N	Transmission line length
$d_{\min}$	Minimum distance between the electric conductor and ground
MAS_{tk}	Upper bound for average span
$\underline{\underline{WS_{t_k}}}$:	Lower bound for weight span
$\overline{\overline{WS_{tk}}}$:	Upper bound for weight span
x'	Correspond to all sites between $x_k e x_{k+1}$

t_0	Terminal tension tower
GC_k	Geotechnical class in k
SC_k	Multiplier factor of the foundation costs related to k slope
a_k	Quota of x_k
y_k	Total height (quota of the site + tower height) in k
c	Electric conductors that will be used in TL

DECISION VARIABLES

$\hat{l}_k$	Step that associates k and $(k-1)$
$\hat{\delta}_k$	Height difference between k and $(k-1)$
$\hat{t}_k$	Tower type of the site ahead in test in k

FUNCTIONS

$F_k(s_k)$	Cumulative costs of allocating towers in k
$F_{k-1}(s_{k-1})$	Cumulative costs of allocating towers in $(k-1)$
$C_k(s_k,u_k)$	Costs associated with allocating a tower in k
$s_k=m(s_{k-1},u_{k-1})=(x_k,x_{k-1},y_k,y_{k-1})=(x_k,w_k,y_k,z_k)$	State vector in k
$u_k=\left(\hat{\delta}_k,\hat{l}_k,\hat{t}_k\right)$	Decision vector in k
τ_k	Function that evaluates the height requirement
$f_k(s_k)$	Partial cumulative costs of the allocation in k
$g(s_k,u_k)$	Set of model restrictions that depend on decision variables u_k and state variables s_k
$q(u_k)$	Set of model restrictions that depend on decision variables u_k
$\vartheta(GC_k)$	Foundation costs due to the geotechnical class GC_k in k
$A_k(s_k)$	Total height in k for the$F_k(s_k)$

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APPENDIX A.

Figures A.1-A.3.

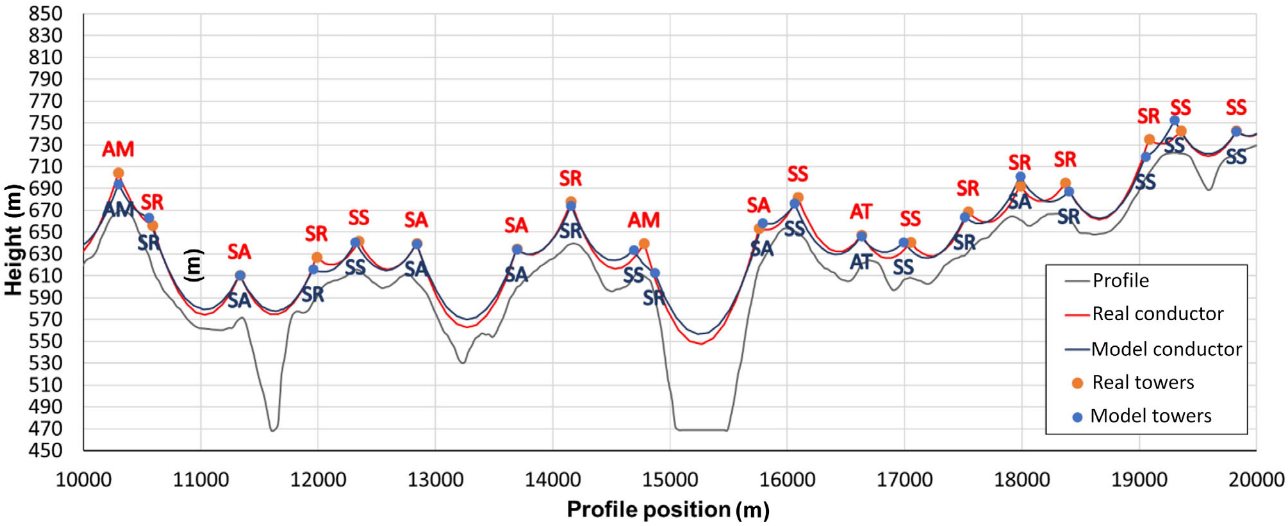

FIGURE A.1 Comparison of the structure arrangement in profile 02—model result vs real TL

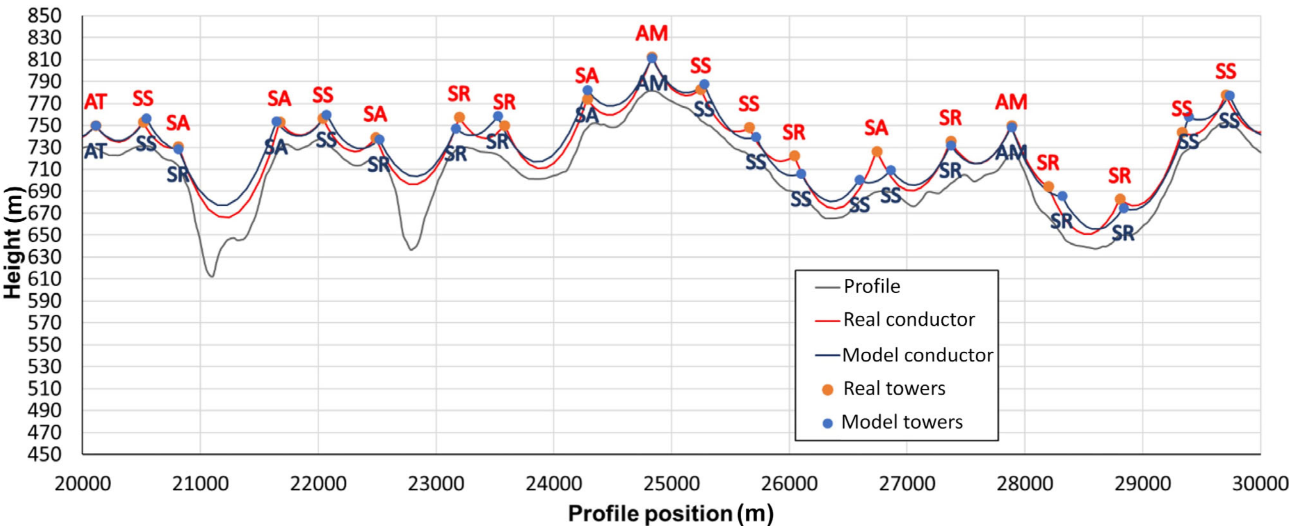

FIGURE A.2 Comparison of the structure arrangement in profile 03—model result vs real TL

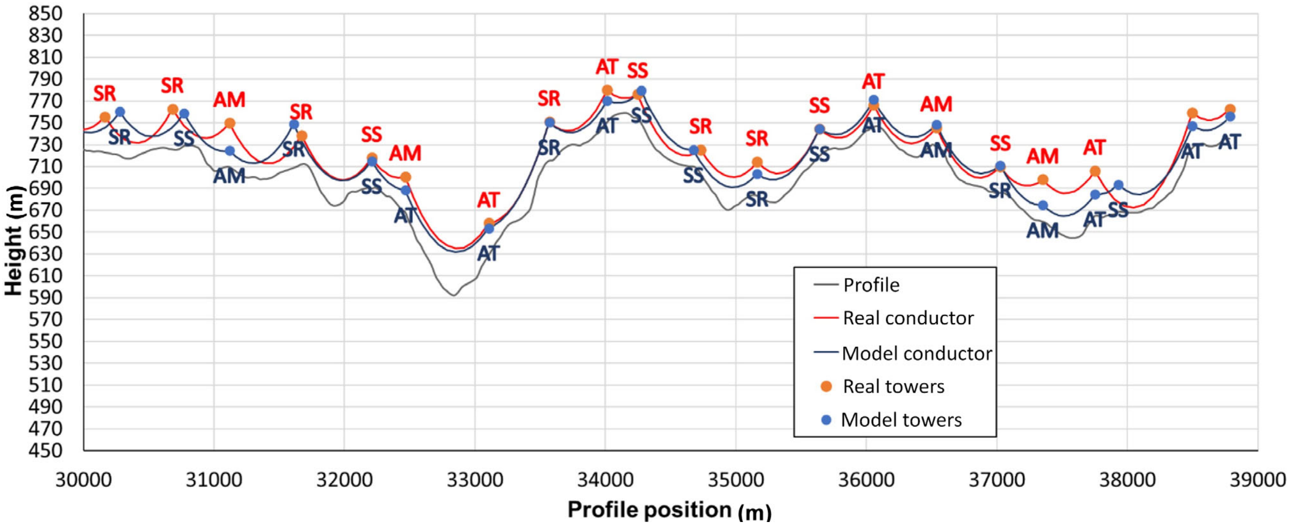

FIGURE A.3 Comparison of the structure arrangement in profile 04—model result vs real TL