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Efficiency analysis for performance evaluation of electric distribution companies

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ABSTRACT

In power systems, the electricity distribution sector requires proper regulation to guarantee power supply security, electricity tariffs modicity and universal service to customers. Generally, electric utilities operate differently and are distinguished in terms of costs, quality of supply, market and network size, and other aspects, that affect their efficiency. In this context, the Data Envelopment Analysis has been used in electricity distribution regulation to define efficiency scores and compare practices. The Data Envelopment Analysis application sometimes comes with weight restrictions and negative variables that modify the original methodology which affects the efficiency scores. The main goal of this paper is to evaluate weights restrictions influence on efficiencies results and to perform a sensitivity analysis of efficiency scores using additional benchmarking techniques. We apply the Cross-Efficiency Analysis and the Ratio-based Efficiency Analysis benchmarking methods, in order to provide relevant quantitative information to compute relative efficiency scores and perform peer evaluations among utilities even if they are outside of the efficient frontier. The Brazilian electricity distribution system is selected as study case. Brazil has strong diversity in terms of economic development, climate and geography, and the current procedure adopted by the regulator determine efficiency metrics for all distribution companies based on their operation cost. Results from our analysis show that the diversity of concession areas significantly influence the stability of efficiency scores. Moreover, considering the approach proposed here it is possible to identify an efficiency relationship among all the distribution companies and not only using the ones that are in the efficiency frontier.

1. Introduction

The electricity transmission and distribution sectors in power systems have characteristics of natural monopolies. In this environment, economic regulation is the key to ensure cost efficiency, quality of supply and an efficient distribution network pricing. The distribution network pricing scheme used to compose electricity rates can be separated into two steps: establishing the required revenue and allocating this revenue among network users. The regulatory revenue incorporates operation expenditure (OPEX), remuneration of regulated asset base and depreciation [1]. The efficient operational cost is usually based on a benchmarking model that ranks electricity distribution companies (DISCOs) using a set of variables. Among commonly used benchmark techniques are the Data Envelopment Analysis (DEA) [2], the stochastic frontier analysis (SFA) [1], and the Corrected Ordinary Least Squares (COLS) [3].

The DEA technique is widely used to define efficiency scores of decision-making units (DMUs), or in the case of electricity distribution sector, DISCOs. The model has inputs such as operational and capital expenditures, while considering outputs such as network length, electricity consumption and number of consumers. The efficiency is measured by weighting these inputs and outputs such that the ratio of weighted outputs divided by weighted inputs is maximized. In this

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Nomenclature

Abbreviations

CAPEX	Capital Expenditure
CEA	Cross Efficiency Analysis
COLS	Corrected Ordinary Least Squares
CRS	Constant Returns to Scale
DEA	Data Envelopment Analysis
DISCO	Electricity Distribution Company a.k.a. electric utility
DMU	Decision-Making Unit
NDRS	Non-decreasing Returns to scale
NIRS	Non-increasing Returns to scale
NTL	Non-Technical Losses
OPEX	Operational Expenditure
PTR	Period Tariff Review
REA	Ratio-based Efficiency Analysis
SFA	Stochastic Frontier Analysis
StoNED	Stochastic Non-smooth Envelopment of Data
VRS	Variable Returns to Scale
nunder	underground network length
nover	overhead distribution network length
nHV	high voltage network length
ncons	number of consumers
enavg	delivered energy weighted by voltage level
CHI	adjusted customer's hours of interruption
NTL	adjusted non-technical losses

Indices and Sets

$k \in K$	Set of DMUs indexed by k
$i \in I$	Set of inputs indexed by i
$j \in J$	Set of outputs indexed by j
$l \in L$	Set of benchmarking DMUs
$r \in R$	Set of linked weight restrictions

Parameters

$\boldsymbol{p}$	DMU in set $\boldsymbol{K}$under analysis
$\boldsymbol{x_i^k}$	Input $\boldsymbol{i}$ value relative to DMU $\boldsymbol{k}$
$\boldsymbol{y_j^k}$	Output $\boldsymbol{j}$ value relative to DMU $\boldsymbol{k}$
α_r	Lower limit for weight restriction $\boldsymbol{r}$
β_r	Upper limit for weight restriction $\boldsymbol{r}$

Decision Variables

$\boldsymbol{u_i}$	Weight of input $\boldsymbol{i}$
$\boldsymbol{v_j}$	Weight of output $\boldsymbol{j}$
$\boldsymbol{\varphi}$	Decision variable regarding returns to scale
$\boldsymbol{z^k}$	Binary decision variable that will be 1 when DMU $\boldsymbol{k}$ has an efficiency ratio higher than DMU $\boldsymbol{p}$

Functions

$\boldsymbol{w_p}$	Efficiency of DMU $\boldsymbol{p}$
$\boldsymbol{w_{pk}}$	Cross-efficiency of DMU $\boldsymbol{k}$ with the multipliers that maximize efficiency of DMU $\boldsymbol{p}$
$\boldsymbol{FPI_p}$	False positive index for DMU $\boldsymbol{p}$, i.e., distance from average efficiency given by CEA and DEA efficiency
$\boldsymbol{r_{min}^p}$	Best position in the ranking interval for DMU $\boldsymbol{p}$
$\boldsymbol{r_{max}^p}$	Worst position in the ranking interval DMU $\boldsymbol{p}$
$\underline{\boldsymbol{D}}_{pk}$	Minimum efficiency of DMU $\boldsymbol{p}$ when DMU k efficiency is set to 1
$\overline{\boldsymbol{D}}_{pk}$	Maximum efficiency of DMU $\boldsymbol{p}$ when DMU k efficiency is set to 1
$\boldsymbol{D}_{p\bar{k}}$	Relative efficiency of DMU p with respect to the most efficient DMU in set $\boldsymbol{K}$.
$\underline{\boldsymbol{D}}_{p\bar{k}}$	Max of $\boldsymbol{D}_{p\bar{k}}$, i.e., maximum relative efficiency of DMU $\boldsymbol{k}$ with respect to the most efficient DMU in set $\boldsymbol{K}$
$\overline{\boldsymbol{D}}_{p\bar{k}}$	Min of $\boldsymbol{D}_{p\bar{k}}$ i.e., minimum relative efficiency of DMU $\boldsymbol{k}$ with respect to the most efficient DMU in set $\boldsymbol{K}$

process, an efficient DMU will have score equal to 1 or 100%. The original model proposed by [2] considers constant returns to scale (CRS), i.e., an increase in inputs cause the same proportional increase in outputs. Alternatives to the DEA CRS to accommodate variable, non-increasing and non-decreasing returns to scale can be found in [4]. For a review of DEA applications to electricity distribution systems see [5] and for international experiences with distribution and transmission benchmarking regulation [6].

Many applications of benchmarking analysis have assessed DISCOs in European countries, for example, in [7] the authors compare the results of the DEA with the SFA and the Stochastic Non-smooth Envelopment of Data (StoNED) applied to compute efficiencies scores for the electricity distribution sector in Finland. Currently StoNED is the methodology adopted by Finland's regulator, but from 2008 to 2011 SFA and DEA approaches were used to define efficiency scores. DEA, COLS and SFA methodologies are applied to investigate DISCOs in other European countries in [1], where the authors show that the choice of benchmarking techniques, model specifications, and variables can affect the efficiency scores, as well as the efficiency rank of companies. In [9], authors analyse productivity growth in the Swedish electricity distribution sector under different ownership models and different service territories. The work of [30] presents the use of the Network DEA model applied to the Turkish electric distribution companies while taking into account expansion cost for additional energy supply, undesirable outputs such as annual faults and interruptions as well as energy losses. An efficiency analysis for the largest 50 DISCOs in the United States using the DEA to assist in decision-making aspects such as how to prioritize investments to improve efficiency scores appears in [8]. The aforementioned literature focuses on DISCOs mostly located in similar regions in terms of economic development and environmental characteristics.

However, in applications where DISCOs operate in diverse areas more variables need to be incorporated to the model in order to better represent the different characteristics of the concession areas. Adding more variables will lead to improved efficiency scores since we are adding new possible combination of inputs and outputs. In this situation, it is important to avoid defining weight equal to zero to several variables or extremely high weights to a single one. One way to overcome the issue is to define production trade-offs among the variables that will be represented as additional constraints limiting the weights. The work presented in [10] compare the cost-efficiency of DISCOs in Portugal from 2002 to 2006 and concludes that the specification of production trade-offs is a difficult task. In [11] an analysis of the impact of adopting weight limits when using DEA to establish efficiency scores is performed for Norway's power distribution system. The authors show that the use of many weight restrictions can have significantly impact in DISCOs' efficiencies, because the associated benchmarking model constraints are often binding. However, previous works have analysed relatively small territorial areas and did not present a comprehensive sensitivity analysis of the weight limits in establishing efficiency scores.

The main goal of this paper is to propose improvements in the process of defining the efficiency scores for DISCOs and its particularities adopted by the Brazilian regulator, such as non-decreasing returns to scale, weight limits and negative outputs. Given the territorial extension of the country, the NDRS approach allows one to compare DMUs of different sizes within the same analysis framework. We also investigate the use of other benchmarking techniques, the cross-efficiency analysis (CEA) [12] and the ratio-based efficiency analysis (REA) [13] to define

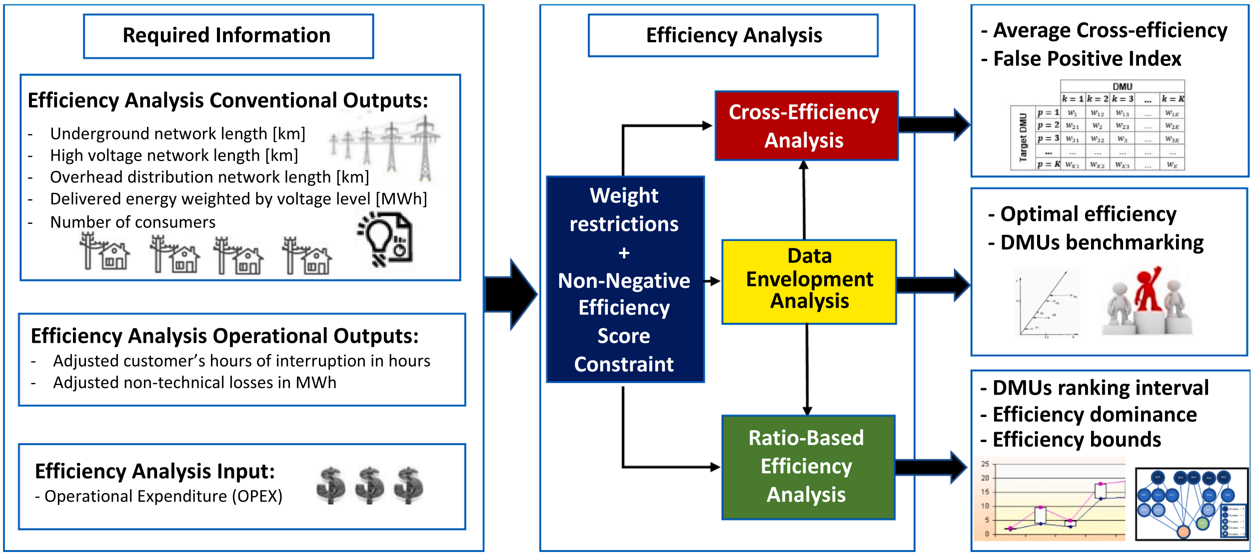

Fig. 1. Overview of the proposed efficiency analysis framework.

efficiency metrics in the electricity distribution sector. The use of CEA aims to deepen DEA by expanding the self-appraisal to a peer-appraisal analysis and establishing a cross-efficiency matrix that gives information about the maximum efficiency that can be achieved for a DMU when the efficiency of another DMU is fixed. To our knowledge, this is the first CEA application to electricity distribution sector that considers non-decreasing returns to scale (NDRS) and weight limits. The NDRS was chosen to make the alternative model results comparable to the current model adopted and in compliance with the regulator assumptions. Another important contribution of this paper is the novel application of the REA methodology for electricity distribution systems aimed to determine ranking intervals, dominance relations and efficiency bounds for the DMUs, variations of efficiency measures that are not available with the DEA method only.

The remainder of the paper is organized as follows. Section 2 describes the DEA, CEA and REA models and particularities of efficiency analysis in the context of the electricity distribution sector. Section 3 presents the case study considering the current regulatory framework of the Brazilian electric power distribution sector. Section 4 shows the results for the application of DEA, CEA and REA to the case study along with discussion and the impact of weight limits and negative outputs. Section 5 concludes the paper.

2. Efficiency analysis methods in the context of the electricity distribution sector

This paper investigates the use of three benchmarking techniques (DEA, CEA and REA) applied to define efficiency metrics for electricity DISCOs. Information related to the network characteristics, market, number of consumers, losses, quality and operational expenditure are considered when defining optimal efficiency scores for the DMUs. The efficiency analysis framework proposed in this paper is illustrated in Fig. 1. The following subsections provide details about the required information, efficiency analysis methodologies and the results obtained from applying such methodologies to the set of DMUs under analysis.

2.1. The classic DEA model

The classic DEA model is a non-parametric benchmarking technique based on linear programming used often in the literature for the evaluation of DMUs that use the same inputs and generate similar outputs [2]. The method is based on the efficiency frontier concept which identifies the best practice among a set of DMUs. In this context, the efficiency frontier is formed by the DMUs that have the best ratio of outputs produced over the amount of inputs used.

A DMU that is not located in the efficient frontier will have its efficiency measured by its distance to the efficient frontier. In that sense, the DEA model have two types of representation: oriented to inputs or oriented to outputs. The former seeks to maximize the efficiency by reducing inputs when outputs are fixed. The later maximizes efficiency by increasing the outputs level while maintaining inputs fixed. The choice of the model orientation is based on which variables (inputs or outputs) are considered manageable by the DMUs.

The DEA model is developed considering that the underlying production function is characterized either by CRS or variable returns to scale (VRS) [4]. The latter can be further classified into non-increasing (NIRS) or non-decreasing (NDRS) returns to scale. The appropriate type of return varies with the application. In the particular case of the electricity distribution sector, one can find DEA applications using CRS or VRS and variations. The authors of [14] state an empirical model like DEA that specifies cost as a function of a few outputs is far from the theorical production/cost function and variable return to scale should be used to mitigate the model limitations. Here, we assume the NDRS to define the general formulation of the DEA-NDRS model as presented by model (1).

$$w_p = \max_{u,v} \frac{\sum_{j\in J}\left(v_j y_j^p\right) + \varphi}{\sum_{i\in I} u_i x_i^p} \tag{1}$$

$$\text{s.t.} \quad \frac{\sum_{j\in J} v_j y_j^k + \varphi}{\sum_{i\in I} u_i x_i^k} \leq 1, \forall k \in K$$

$$u_i, v_j, \varphi \geq 0, \forall i \in I, \forall j \in J$$

where, K is the set of DMUs indexed by k, p represents the DMU in set K under analysis; I is the set of inputs indexed by i; J is the set of outputs indexed by j; u_i is a decision variable corresponding to the weight of input i; v_j is a decision variable corresponding to the weight of output j; x_i^k is the input i value relative to DMU k; y_j^k is the output j value relative to DMU k; w_p is the efficiency of DMU p and φ is the decision variable regarding returns to scale. If $\varphi = 0$, model (1) would correspond exactly to the DEA-CRS model representation from [2].

Model (2) is the linear version of the original nonlinear formulation presented in model (1). The linear DEA model is the most common one, because the optimal solution to linear programs can be achieved in polynomial time.

$$w_p = \max_{u,v} \sum_{j \in J} \left(v_j y_j^p \right) + \varphi \tag{2}$$

$$\text{s.t.} \ \sum_{j \in J} v_j y_j^k - \sum_{i \in I} u_i x_i^k + \varphi \leq 0, \forall k \in K$$

$$\sum_{i \in I} u_i x_i^p = 1$$

$$u_i, v_j, \varphi \geq 0, \forall i \in I, \forall j \in J$$

2.2. Modified DEA model for negative outputs consideration

The DEA model orientation toward inputs or outputs may vary depending on the application and which variables are considered to be manageable by the DMUs. Most DEA applications to DISCOs consider the input orientation given the possibility of reducing operational and capital expenditure [1,16]. Some instances of the model incorporate additional DMUs' inputs that are not manageable such as non-technical losses or reliability indexes. In that case, the work of [17] considers these non– manageable inputs as negative outputs as proposed by [18]. The negative outputs combined with variable returns to scale may lead to negative efficiency scores when calculated cross-efficiency between DMUs [2]. Negative outputs are incorporated in the model to account for variables that have a negative impact on the operational expenditure such as quality indexes, different for example from network extension and number of customers.

The objective function in model (2) corresponds to the efficient score of the DMU under analysis. Therefore, the model tries to find weights v_j and u_i that maximize the efficiency ratio in the objective function. The first set of constraints ensures that these weights are chosen in such a way that when applied to inputs and outputs of all other DMUs in set K lead to an efficient score less than or equal to 1. The only other restriction is that the weights should be non-negative. At this point it is clear that non-negative weights combined with non-negative variables (inputs and outputs) and non-negative return to scale (φ) will also ensure a non-negative efficiency ratio.

The problem arises when one or more outputs are negative, when that is the case the non-negativity of the efficiency ratio is compromised. Note that for the DMU under analysis it will still lead to positive efficiency scores, but it may be the case that in order to maximize the ratio for DMU p the model is choosing a set of weights that drives the ratio of one or more DMUs in set K below zero. One way to overcome that is by adding Non-Negative Efficiency Score Constraint to models (1) and (2) that ensures the non-negativity of the efficiency scores for all DMUs as described in Equation (3).

$$\frac{\sum_{j \in J} v_j y_j^k + \varphi}{\sum_{i \in I} u_i x_i^k} \geq 0, \quad \forall k \in K \tag{3}$$

The approach suggested here is similar to [19,20] for dealing with negative efficient scores on DEA when φ, can be negative as in the DEA VRS model [4]. However, our goal here is to use this approach to make sure that negative outputs, $y_j, \forall j \in J$, will not lead to negative efficiency scores.

2.3. Cross-efficiency analysis (CEA)

Another benchmark method commonly used in the literature is the Cross-efficiency analysis (CEA) as in [12,21]. It can be used to strengthen the DEA analysis and check sensitivity of the efficiency scores. When the DEA model is simulated, the objective is to find the maximum efficiency score for a specific DMU by varying inputs' and outputs' weights, as long as the efficiency of all other DMUs in set K do not exceed one. This approach is known as self-appraisal because it only seeks to maximize the efficiency of the DMU under analysis. The idea of CEA is to expand the analysis to a peer-appraisal by calculating the efficiency score of the DMUs when the efficiency of a given DMU p is fixed and all others are either maximized or minimized simultaneously. For a comparison of the DEA and CEA methods for analysing the electricity distribution system in Taiwan see [29].

Table 1
Cross-efficiency matrix.

		DMU				
		k = 1	**k** = 2	**k** = 3	…	**k** = **K**
Target DMU	**p** = 1	w_1	w_{12}	w_{13}	…	w_{1K}
	p = 2	w_{21}	w_2	w_{23}	…	w_{2K}
	p = 3	w_{31}	w_{32}	w_3	…	w_{3K}
	…	…	…	…	…	…
	p = **K**	w_{K1}	w_{K2}	w_{K3}	…	w_K

The DEA model is known to have multiple optimal solutions [22]. Therefore, just simulating the DEA model (2) would lead to multiple possibilities for the cross-efficiency scores. The CEA model explored here overcomes the problem of multiple solutions by simulating another optimization model to find the optimal set of weights that maintain the optimal efficiency of the target DMU p but also maximizes the sum of the efficient scores of all other DMUs in set $K \backslash p$ (set K without considering the DMU p in analysis). The CEA model is divided in two stages: the first is represented by optimization model (2) and the second is the optimization model represented in model (4), where c_p is the CEA objective function. This is a variation of the original model proposed in [12,21] to handle non-decreasing returns to scale [23].

$$c_p = \max_{u,v} \sum_{j \in J} \sum_{k \in K \backslash p} \left(v_j y_j^k + \varphi \right) \tag{4}$$

$$\text{s.t.} \ \sum_{i \in I} \sum_{k \in K \backslash p} u_i x_i^k = 1$$

$$\sum_{j \in J} v_j y_j^k - \sum_{i \in I} u_i x_i^k + \varphi \leq 0, \forall k \in K \backslash p$$

$$\sum_{j \in J} v_j y_j^p - w_p \sum_{i \in I} u_i x_i^p + \varphi = 0$$

$$v_i, u_j, \varphi \geq 0, \forall i \in I, \forall j \in J$$

Note that w_p is the DEA efficiency score for the DMU p obtained after the simulation of (2), where the decision variables obtained with model (4) are further used on Equation (5) with corresponding inputs and outputs to get the cross-efficiency values for all other DMUs in the set $K \backslash p$.

$$w_{pk} = \frac{\sum_{j \in J} v_j^p y_j^k + \varphi}{\sum_{i \in I} u_i^p x_i^k}, \quad \forall k \in K \backslash p \tag{5}$$

The score w_{pk} is the cross efficiency of DMU k with the optimal weights of DMU p; u_i^p is the optimal weight for input variable i and v_j^p is the optimal weights for output variable j obtained with model (4). The simulation of (4) for all DMUs and the computation of cross efficiency scores with (5) leads to the Cross-Efficiency Matrix (CEM) illustrated in Table 1, as described in [24].

One can compute the efficiency of DMU k by taking the average of all w_{pk}, for all $p \in K$, i.e., the average of the column of the cross-efficiency matrix. The DMU with the highest average among all observed DMUs could be considered the benchmark, or the most efficient [12]. The variation of the average efficiency given by CEA with respect to the efficiency given by DEA is called the False Positive Index (FPI). The FPI is the increment in percentage of the efficiency score when you move from the peer-appraisal to the self-appraisal and is computed for each DMU p using Equation (6).

$$FPI_p = \frac{w_p - \sum_{k\in K}\frac{w_{kp}}{|K|}}{\sum_{k\in K}\frac{w_{kp}}{|K|}} \tag{6}$$

From Equation (6), the greater the FPI_p, the greater will be the DMU efficiency score variation with respect to the DEA model (2). The FPI was important in this paper analysis because of two reasons. First, we were able to detect the sensitivity of the DMU to the set of optimal weights. And second, the CEA results have pointed out to necessity of adding the non-negativity efficient score constraint. It is important to highlight that whichever additional constraints, such as the weight restrictions or the non-negativity efficient score constraint, are added to model (2) should also be added to (4) in order to make efficiency scores comparable.

2.4. Ratio-based efficiency analysis (REA)

The Ratio-based Efficiency Analysis (REA) model has been proposed by [13] to enhance the peer-appraisal approach. Similarly, to the CEA, it also has its roots in the DEA model, but analysis other concepts such as: super efficiency, ranking intervals, efficiency dominance and efficiency bounds. According to [13], an advantage of the REA approach is that even with a small number of DMUs the results are consistent since the analysis is not simply based on the efficiency frontier, i.e. the results are less sensitive to the addition or removal of a DMU to the analysis. The REA results can be separated in 3 main groups: DMUs ranking interval, efficiency dominance and efficiency bounds.

The ranking interval establishes an efficiency ranking range for the DMUs under analysis. Recall that the efficiency score can vary according to a set of possible weights for inputs and outputs. The DEA model only cares for the set of weights that maximize efficiency. The REA explores the other sets of feasible weights. The ranking intervals are obtained through the optimization of two models. First, model (7) is optimized to find the highest efficiency ranking, i.e., the best position, that a DMU p can achieve relative to the others [13].

$$r^p_{min} = \min_{u,v,z}\left(1 + \sum_{k\in K\backslash p} z^k\right) \tag{7}$$

$$\text{s.t. } \sum_{j\in J} v_j y_j^k + \varphi \le \sum_{i\in I} u_i x_i^k + Mz^k, \forall k \in K\backslash p$$

$$\sum_{i\in I} u_i x_i^p = 1$$

$$\sum_{j\in J} v_j y_j^p + \varphi = 1$$

$$z^k \in \{0,1\}, \forall k \in K\backslash p$$

$$u_i, v_j, \varphi \ge 0, \forall i \in I, \forall j \in J$$

where M is a large numeric constant; z^k is a binary decision variable that will be 1 when DMU k has an efficiency ratio higher than DMU p; and r^p_{min} is the best position in the ranking interval. The best position is achieved when $r^p_{min} = 1$; z^k are binary decision variables, whereas the multipliers u_i, v_j and φ are still continuous non-negative decision variables. Thus, model (7) is formulated as a mixed integer linear programming.

The second model used in the REA approach is responsible to determine the worst position of DMU p, denoted by r^p_{max}. The worst ranking is obtained by doing simple modifications in model (7). The optimization objective function should be changed to a maximization and the first set of constraints should be replaced by Equation (8).

$$\sum_{i\in I} u_i x_i^k \le \sum_{j\in J} v_j y_j^k + M\left(1 - z^k\right) + \varphi, \forall k \in K\backslash p \tag{8}$$

The efficiency dominance, which compare DMUs in pairs, is a complement to ranking intervals. Imagine a case where two DMUs, A and B, have overlapping ranking intervals. When that is true the ranking intervals will fail to discriminate A and B. But it could be that for all feasible weights DMU A always has an efficiency score higher than DMU B. And that's when efficiency dominance comes into play. Recall the DEA model maximizes the efficiency ratio of DMU p with respect to the whole set K. The efficiency dominance is established by maximizing and minimizing efficiencies for DMU p with respect to a specific DMU k. Model (9) represents the maximization (and the minimization) version of the optimization model that finds the upper and lower bounds on the efficiency of DMU p when the efficiency of DMU k is equal to one.

$$\left(\underline{D}_{pk}\right)\overline{D}_{pk} = \left(\min_{u,v}\right)\max_{u,v}\sum_{j\in J}(v_j y_j^p + \varphi) \tag{9}$$

$$\text{s.t. } \sum_{j\in J} v_j y_j^k + \varphi = \sum_{i\in I} u_i x_i^k$$

$$\sum_{i\in I} u_i x_i^p = 1$$

$$u_i, v_j, \varphi \ge 0, \forall i \in I, \forall j \in J$$

If the minimum efficiency $\underline{D}_{pk} > 1$, DMU k dominates DMU p. If $\underline{D}_{pk} < 1$ dominance does not hold. If $\underline{D}_{pk} = 1$, one needs to check the maximum efficiency. If $\overline{D}_{pk} > 1$, then dominance still holds [13]. The advantage of this approach is the ability to establish a dominance among DMUs without the need of one of them being in the efficiency frontier. Another feature of the REA methodology is the relative efficiency bounds. The relative efficiency of DMU p with respect to all DMUs in set K is given by Equation (10).

$$D_{p\bar{k}}(u,v) = \min_{k\in K}\frac{w_p}{w_k} \tag{10}$$

Where w_p is the efficiency of DMU p under analysis and w_k is the efficiency of DMU k,$\forall k \in K$, both obtained with model (2). From the definition of (10) the concept of two types of efficiency can be established according to [13] as in equation (11). The first is the minimum relative efficiency, $\underline{D}_{p\bar{k}}$, given by the minimization of (10) and the second relative efficiency, $\overline{D}_{p\bar{k}}$, given by the maximization of (10).

$$\left(\underline{D}_{p\bar{k}}\right)\overline{D}_{p\bar{k}} = \left(\min_{u,v}\right)\max_{k\in K} D_{p\bar{k}} \tag{11}$$

Considering that DMU p is not benchmarking according to DEA results, its $\overline{D}_{p\bar{k}}$ is equal to the efficiency given by the DEA model. If this DMU p is benchmarking its $\overline{D}_{p\bar{k}}$ will be given by model (12), that is based on the super efficiency concept found in [25], where $l \in L$ is the set of benchmarking DMUs obtained with the DEA model application.

$$\overline{D}_{p\bar{k}} = \max_{u,v}\sum_{j\in J}\left(v_j y_j^p + \varphi\right) \tag{12}$$

$$\text{s.t. } \sum_{j\in J} v_j y_j^l - \sum_{i\in I} u_i x_i^l + \varphi \le 0, \forall l \in L \subseteq K\backslash p$$

$$\sum_{i\in I} u_i x_i^p = 1$$

$$u_i, v_j, \varphi \ge 0, \forall i \in I, \forall j \in J$$

2.5. Efficiency analysis inputs and outputs in the electricity distribution sector

There is no consensus on which variables should be considered for DEA applications to electricity distribution sector. A common practice is to consider an input orientation with operational expenditure (OPEX) and/or capital expenditure (CAPEX) as input variables or the total expenditure (TOTEX). Among common output variables those related to

Table 2
Correlation matrix between OPEX and outputs.

ρ	OPEX	nunder	nover	nHV	ncons	enavg	NTL	CHI
OPEX	1.000	–	–	–	–	–	–	–
nunder	0.384	1.000	–	–	–	–	–	–
nover	0.874	0.044	1.000	–	–	–	–	–
nHV	0.869	0.081	0.972	1.000	–	–	–	–
ncons	0.976	0.417	0.821	0.837	1.000	–	–	–
enavg	0.933	0.537	0.691	0.708	0.956	1.000	–	–
NTL	0.454	0.353	0.306	0.292	0.374	0.361	1.000	–
CHI	0.688	0.326	0.510	0.481	0.674	0.632	0.352	1.000

network and customers such as number of customers, delivered energy, distribution network extension, number of transformers, transformers capacity and service reliability indexes [16].

2.6. Weight Limits to inputs and outputs

Some applications of the DEA methodology to the electricity distribution sector consider weight restrictions [15]. The consideration of weights in the problem introduces new constraints to the optimization model, which modify the feasible region of the original formulation and therefore impact the efficiency scores. Among the reasons to incorporate weight limits are the small number of DMUs compared to the number of inputs and outputs considered and the structure of the weights in the optimal solution that should not exclude important variables from the efficiency score [10]. A common practice is to enhance model (2) by adding linked weight restrictions, i.e., imposing lower and upper limits in the weight ratios such as presented by Equation (13).

$$\alpha_r \leq \frac{v_j}{u_i} \leq \beta_r, \quad \forall r \in R \tag{13}$$

where, R is the set of weight restrictions indexed by r; α_r is lower limit for weight restriction r, β_r is upper limit for weight restriction r; v_j/u_i is the linked weight ratio being limited with respect to output j and input i. Note that it is possible to consider the weight ratio involving two outputs instead of output over input. The upper and lower bound for the linked weights are often based on production trade-offs and historical data analysis.

By rearranging (13), inequalities (14) and (15) are obtained and further added to model (2) to obtain DEA-NDRS with weight restrictions, (4) to obtain the CEA-NDRS with weight restrictions and (7)–(12) to obtain the REA-NDRS with weight restrictions.

$$-v_j + \alpha_r u_i \leq 0, \forall r \in R \tag{14}$$

$$v_j - \beta_r u_i \leq 0, \forall r \in R \tag{15}$$

3. Study case

The case study is focused on the Brazilian electricity distribution system, which contains 61 DISCOs regulated by electricity regulatory agency ANEEL. The complexity attributed to the system is directly related to the vast territorial dimension of the country and, consequently, to diversity between the different concession areas. Brazil is cut by two parallels (Ecuador and Tropic of Capricorn), has four time zones and has a vast interior and coastal region. The territory of the country is approximately equal to the territory of the whole European continent.

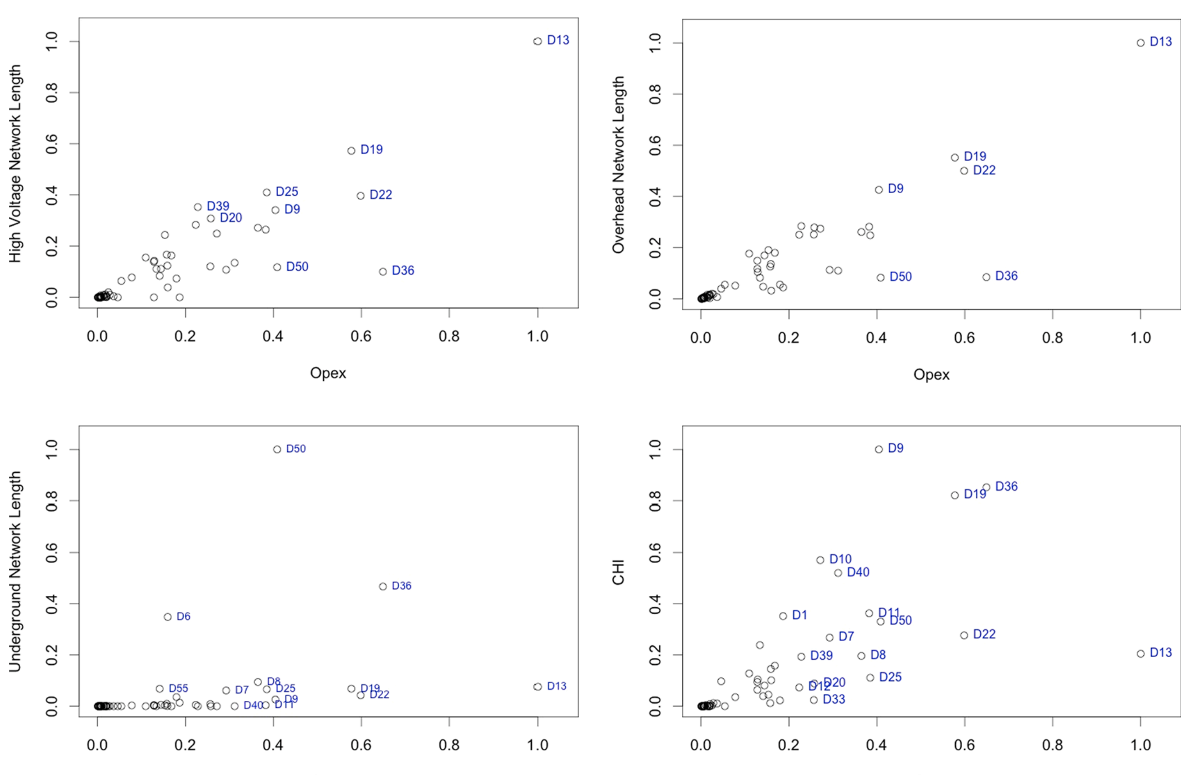

Fig. 2. Scatter plot for the DMUs input and outputs.

Table 3
Linked weight restriction added to the model.

Limited Multiplier	Linked Weight	Bounds	
OPEX **X** overhead distribution network length	$\frac{v_{nover}}{u_{OPEX}}$	Minimum	725.98
		Maximum	2753.73
underground network length **X** overhead network length	$\frac{v_{nunder}}{v_{nover}}$	Minimum	1.00
		Maximum	2.00
high voltage network length **X** overhead distribution network length	$\frac{v_{nHV}}{v_{nover}}$	Minimum	0.40
		Maximum	1.00
OPEX **X** number of consumers	$\frac{v_{ncons}}{u_{OPEX}}$	Minimum	37.55
		Maximum	181.50
OPEX **X** delivered energy weighted by voltage level	$\frac{v_{enavg}}{u_{OPEX}}$	Minimum	1.25
		Maximum	75.10
OPEX **X** non-technical losses	$\frac{v_{NTL}}{u_{OPEX}}$	Minimum	12.52
		Maximum	187.75
OPEX **X** hours of interruption	$\frac{v_{CHI}}{u_{OPEX}}$	Minimum	–
		Maximum	2.50

Therefore, the concession areas of the DISCOs have different geographical characteristics, and in addition, distinct economic characteristics (ranging from more developed to considerably poor areas). The Southeast region stands out for higher income levels, while other regions such as the Northeast and the North are characterized by lower incomes. There are also different demographic density characteristics among concession areas with highly populated states, such as Rio de Janeiro and São Paulo, and other sparsely populated states such as Minas Gerais and Amazonas.

One of the key elements of the electricity distribution regulation in Brazil is the Periodic Tariff Revision (PTR). The PTR is a regulatory mechanism, carried out by ANEEL [26] to ensure a proper and fair definition of the tariffs that will be further applied to consumers. During the PTR, the regulator establishes the allowed regulatory revenue for all DISCOs. One of the components of the required revenue is the efficient operational costs. Despite all the diversity described, ANEEL adopted the DEA model to calculate an efficiency score of all DISCOs that are further used to calculate the efficient operational costs. In an attempt to overcome heterogeneity, and consider all the 61 DISCOs in a single set, the regulator considers non-decreasing returns as in (2) and additional linked weight restrictions as in (14) and (15). The weight restrictions were incorporated based on production trade-offs and upper and lower bounds limits are based on the DISCOs data set from previous PTRs.

3.1. Variables and data sets

The data used in the case study are obtained from ANEEL database that was previously applied by regulator to calculate the efficiency of the DISCOs in the Publics Hearings 023/2014 (PH 023) and 052/2017 (PH 052) available in [27]. During the fourth PTR cycle, ANEEL established the efficiency analysis would be based on one input and seven outputs. The input is represented by DISCOs' OPEX. The outputs are divided into five conventional outputs and two outputs of operational efficiency. The conventional outputs are underground network length (nunder) in km, overhead distribution network length (nover) in km, high voltage network length (nHV) in km, number of consumers (ncons) and delivered energy weighted by voltage level (enavg) in MWh. The operational efficiency outputs are adjusted customer's hours of interruption (CHI) in hours and adjusted non-technical losses (NTL) in MWh; these outputs are considered as negative contributions of efficiency (negative outputs), i.e., non-manageable inputs. Table 2 shows the correlation between outputs and input (OPEX). By analysing the correlation matrix, one can notice a strong positive correlation between OPEX and most of the outputs.

A description of some the variables can be found in Fig. 2, for illustration the variable values were all divided by the maximum. Note that there is a concentration of dots, i.e., DMUs, in the lower left corner of the scatter plots and then other DMUs that clearly are far away from the group and could be seen as outliers. For instance, D9, D13, D19, and D22 have larger values for both high voltage and overhead network length, representing DISCOs with wide concession areas. D6, D36 and D50 with larger values of underground network represent DISCOs in more urban areas. D9, D19 and D36 have large value for the CHI variable indicating that they might operate in areas of remote access leading to longer hours of interruption.

3.2. Linked weight restriction

The current model considers linked weight restrictions to better discriminate the DISCOs and also to try to overcome the complexity added due to the diversity in the set of concession areas. The restrictions and corresponding bounds are depicted in Table 3.

4. Results and discussion

4.1. Impact of the weight restrictions

A detailed look at Fig. 3 shows the impacts of the weight restrictions in the efficiency analysis. As one can notice, the efficiency scores obtained by the application of model (2) is defined by the red squares on the chart. As all the additional constraints of the type (14) and (15) are added to (2), to represent the proper weight restrictions, the feasible region of the optimization model becomes tighter. In this situation, as the model seeks to maximize the objective function, the optimal solution obtained is now smaller than the one previously obtained and is represented by the black line on the chart. Points between red squares and the

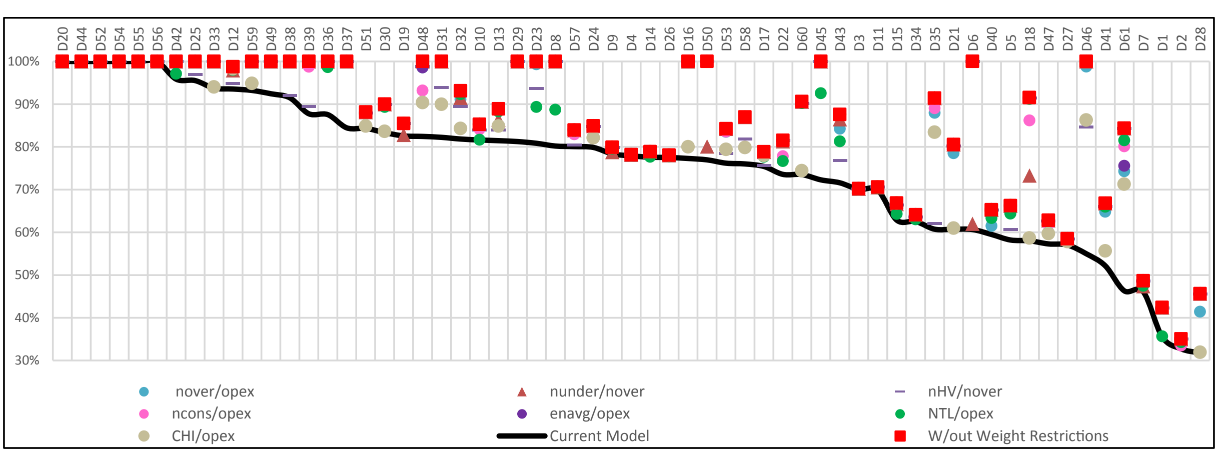

Fig. 3. Linked weight restrictions impact on efficiency scores.

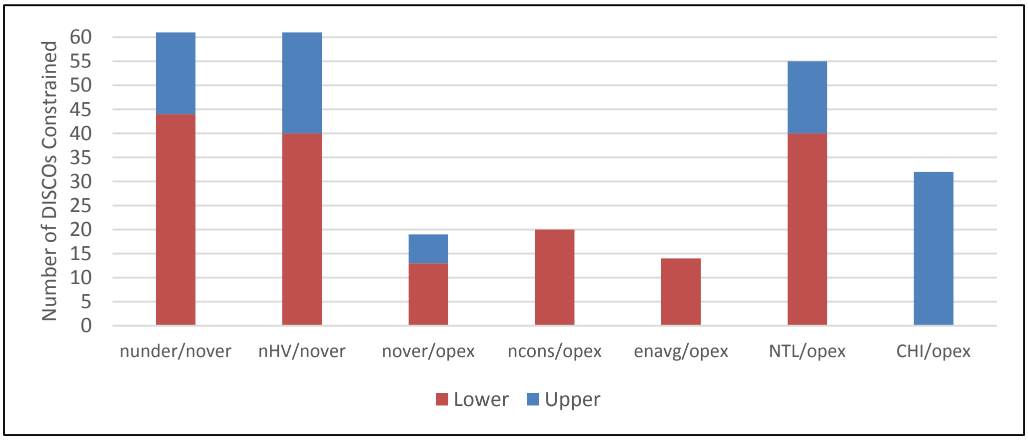

Fig. 4. Number of DMUs bounded above or below by each weight restriction.

black line represent situations with model (2) and the addition of a single weight restriction constraint of the type (14)–(15). For instance, recall from Fig. 2 that D6 and D50 are DISCOs with large underground length and benchmarks according to DEA without weight restrictions. The results of DEA with weights limits on variables underground and overhead length (v_{nunder}/v_{nover}) represented by the orange triangles, show efficiency score reductions to 0.62 and 0.81, respectively. Some DISCOs were penalized for having large underground network length. Similar conclusions can be drawn for DISCOs D25 and D39 that have larger values of high voltage network length and were penalized by the addition of the weight restriction (v_{nHV}/v_{nover}) represented by the purple dash.

Results show that without considering weight restrictions, 25 out of the 61 DISCOs are in the efficient frontier and with the added restrictions only 6 remain at the frontier. Moreover, the restrictions with higher impact on efficiency scores are the ones limiting overhead and underground multipliers (orange triangles) and hours of interruptions and OPEX multipliers (gray circles). We note that when we add all the weight restrictions applied by the Brazilian regulator the reduction in the efficiency score ranges from 0% to 45% with an average of 10%.

4.2. Analysis of the shadow prices associated with the weight restrictions

The next step is to consider the model with all weight restrictions and to analyse the shadow prices associated with each one of them. The objective is to verify if either the minimum or maximum limit on the weight are active in the optimal solution. In other words, how many DISCOs have its efficiency limited by a specific weight restriction. The results are shown in Fig. 4. Recall that there are six restrictions with upper and lower limits and one restriction v_{CHI}/u_{OPEX} with only upper limit. Thev_{nover}/u_{OPEX}, v_{ncons}/u_{OPEX} and v_{enavg}/u_{OPEX} constraints are only active for 1/3 of the DISCOs. The concern is regarding v_{nunder}/v_{nover} and v_{nHV}/v_{nover} that are active on all DISCOs and v_{NTL}/u_{OPEX} that are active on 55 DISCOs with 65% of the impact coming from lower limit restriction. The former imposes weight links among distribution grid characteristics like underground, overhead and high voltage network length. Each DISCO developed its network over the years as function of the local characteristics and often without control of the form of the expansion. In this particular study case, there are two extremes: an essentially urban DISCO with large underground network length values and another with high voltage network length because of the territorial extension of its service area. These companies would be penalized for having to serve a concession with these characteristic.

The CHI variable was incorporated to the model to account for quality of service in the efficiency evaluation. A weight restriction linked to the OPEX weight was also added. In Fig. 4, 32 DISCOs have their optimal weight for CHI falling in the upper bound limit and 25 DISCOs do not attribute any importance to the quality criterion (i.e., CHI multiplier was equal to zero in the optimal solution). The non-technical losses (NTL) variable is incorporated to consider the losses directly related to energy theft and measurement and billing errors. Similar to CHI, a weight limit linked to the OPEX is added. The number of DISCOs that have their efficiency limited by this restriction is large, with 40 DISCOs bounded by the upper limit and 15 by the lower limit.

Table 4
Linked Weight Restriction Added to the Model.

Number of Binding weight restrictions	Number of DISCOs affected
1 or less	–
2	1
3	13
4	22
5	18
6	6
7	1

Given that the optimal solution is bounded by either the upper or lower limits, the numbers of weight restrictions that are active in the optimal solution ranges from 0 to 7. Table 4 shows that all DISCOs have at least two active weight limit constraints and most of them have 4 or 5 active weight limits. Moreover, six DISCOs have their efficiencies defined by 6 boundary restrictions and one DISCO have their efficiencies defined by the all 7 constraints.

From the results of the analysis the weight restrictions have shown a significant impact on the efficiency scores and need to be further investigated with a clear and robust methodology to establish the maximum and minimum bounds on their values. An approach to deal with Brazilian electricity utilities diversity is discussed in [16] where the authors segregated the utilities concessions areas into smaller regions and considered them as DMUs in the DEA following the lead from the third PTR cycle where the DISCOs were segregated into two groups, one with smaller and other with bigger DISCOs. But the regulator went in the other direction in the fourth PTR cycle by joining all DISCOs in one set and adding the weight restrictions. The effect of weight restrictions and lack of a methodology to define the upper and lower bounds raised concerns among agents regarding to the consistency and the fairness of the approach and associated results.

4.3. Impact of adding non-negative efficiency score constraint to DEA model

Here we explore the effect of the addition of the non-negative cross-efficiency score constraint using the original DEA model represented in (2). The original DEA-NDRS results are represented in blue and the DEA-

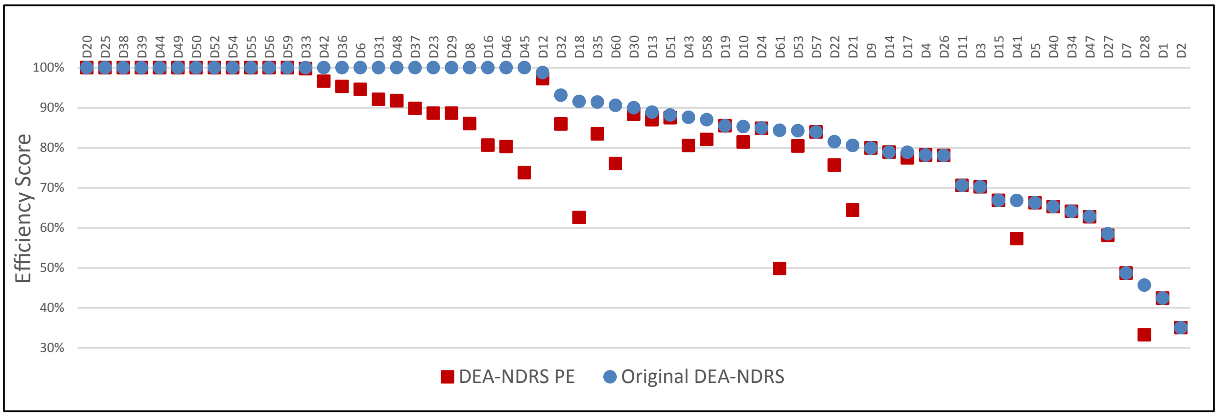

Fig. 5. Impact of non-negative efficiency constraint on efficiency scores.

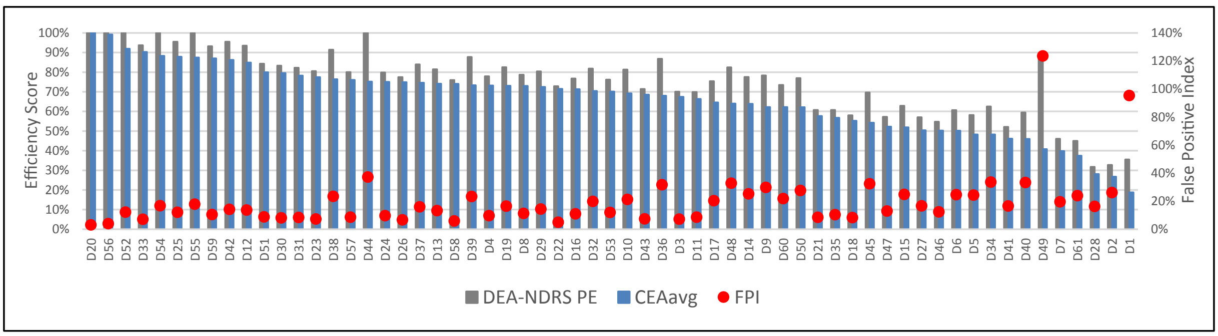

Fig. 6. CEA, DEA-NDRS and FPI results.

NDRS with non-negative efficiency score constraint results are represented in red in Fig. 5. The decrease in efficiency scores observed in the red series is proof that many DISCOs are maximizing their efficiency with a set of weights that should not be considered feasible since it drives the efficiency of others DISCOs outside the production set. Without the non-negative efficiency score constraint, 25 out of the 61 DISCOs are classified as benchmarks. Results have shown that the process of adapting the DEA model to represent negative outputs reduces the need for all the weight constraints added by the Brazilian regulator. In addition, the concern associated with the establishment of production trade-offs that lead to upper and lower bounds for the weight restrictions is solved.

4.4. Impact of adding non-negative efficiency score constraint to the CEA and REA models

All the results in this section considers the optimization models (4)–(8) augmented by the non-negative efficiency constraint (3) and the linked weighted restrictions (14)–(15) to keep consistency among the simulations with respect to DEA, CEA and REA models. Fig. 6 shows the efficiency values of DEA-NDRS, the average efficiency of CEA-NDRS and FPI produced by the model runs. The average efficiency of CEA is always less than 100% unless one DISCO can achieve 100% for all columns in Table 1. The results for CEAavg are all divided by the maximum CEA average such that at least one DISCO is benchmark which turned out to be D20 that is also benchmark in DEA. The next two most efficient DISCOs in the CEA are D56 and D52, also benchmarking DMUs in DEA. CEA ranked D33 and D25 well despite not being benchmarks in DEA, but for demonstrating greater average performance. Ideally, the efficiency

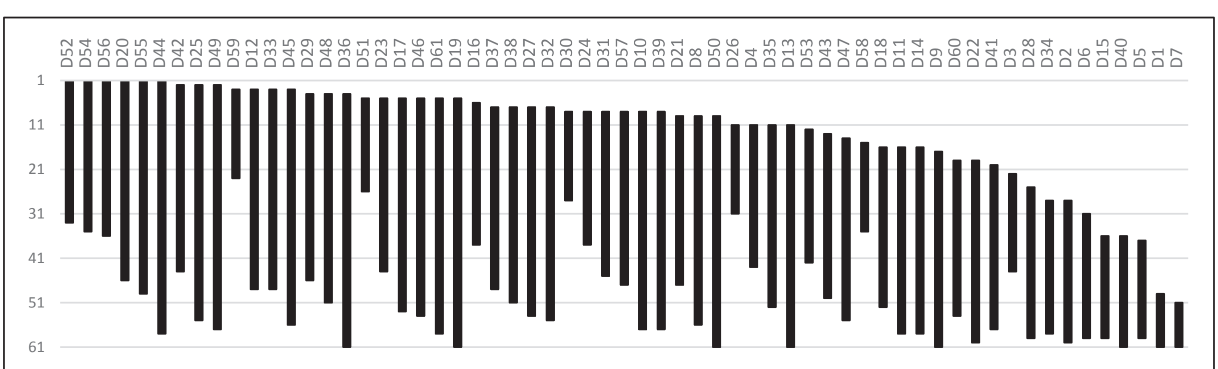

Fig. 7. REA ranking interval.

Table 5
Dominance relation.

DISCO	Dominated By	DISCO	Dominated By
D1	2–6,12,14–18,20,21,23,24,26–35,37–39,41–49,51–61	**D32**	52,55,56
D2	3,4,16–18,21,24,26–30,35,41,45–49,51–54,58–61	**D33**	56
D3	26,30,51,52,59	**D34**	3,4,16,17,24,26,29,30,51–54,58,59
D4	30,52	**D35**	51,52,54
D5	3,4,16,23,24,26,29,30,37,42,43,51–54,56,58,59	**D36**	55,56
D6	3,4,16,23,24,26,29,30,42,43,48,51–53,55–59	**D37**	–
D7	3–6,10,12,14–18,20,21,23,24,26,27,29,30–35,37–39,41–44,46–48,51–60	**D38**	54
D8	20,33,55,56,59	**D39**	20,42,54,56,
D9	10,12,20,23,33,42,54,56,59	**D40**	3,4,10,12,16,20,23,24,26,29,30–33,37,42,43,51–59
D10	12,20,42,56	**D41**	16,21,24,26,27,48,51–54,59
D11	12,20,23,33,42,56,59	**D42**	–
D12	–	**D43**	–
D13	12,20,33,42,56	**D44**	–
D14	23,37,38,42,54,56,59	**D45**	52
D15	3,4,16,17,23,24,26,29,30,37,42–44,51–54,56,58,59	**D46**	52,54
D16	52	**D47**	16,21,35,51,52,54
D17	54	**D48**	52
D18	16,21,24,51,52,54	**D49**	52
D19	12,20,42,56	**D50**	20,25,32,33,52,55,56
D20	–	**D51**	–
D21	51,52,54	**D52**	–
D22	20,25,30,33,51,55,56,59	**D53**	52,59
D23	–	**D54**	–
D24	51	**D55**	–
D25	–	**D56**	–
D26	51	**D57**	56
D27	52,54	**D58**	30,59
D28	3,16–18,21,24,26,27,29,30,35,41,45–49,51–54,59–61	**D59**	–
D29	–	**D60**	51,54,59
D30	–	**D61**	45,49,52,54
D31	56		

score from both models should be close demonstrating a greater stability in the DEA scores. This is reflected in the FPI, i.e., the DISCOs whose FPI are lower than 10% have more stable results. That is not the case of DISCOs D38, D44, D39, D36, D48, D9, D60, D50, D45, D49 and D1 (in order of appearance in Fig. 6). It is no coincidence that these are DISCOs were labelled in Fig. 2 as outliers. Once again, the complexity of concession areas is significantly influencing the stability of efficiency scores. Specifically in the case of D44, the average CEA before dividing by the maximum is 72%. The reason it is well below the DEA score is when it is benchmarked against D49 and D24, D44 score is 44% and 33%. It does not necessarily mean that D44 is inefficient compared to D24 and D49, what it means is that these DISCOs may not be comparable.

The FPI value in Fig. 6 makes it easier to understand the differences between the CEA and the DEA efficiency results, since DISCOs with high FPI values are generally those with the largest differences between the average CEA efficiency and the one obtained with DEA. The worst performance of the set was from D49 with a FPI of 127%. This is an indication that the DEA model favoured the performance of this DISCO, i.e. when the DISCO is analysed individually, the efficiency score obtained is 88%, but when we maximize the efficiency of all DMUs together the efficiency is reduced to 41%.

The REA results are presented in Fig. 7 (Raking Interval), Table 5 (Efficiency Dominance) and Fig. A.1 (Efficiency Bounds). The ranking interval analysis seeks to measure sensitivity of a DISCO efficiency score with respect to the variation of the set of feasible weights. When ranking intervals are wide, it means that one DISCO can rank well or poorly depending on the choice of input and output weights. One can notice from Fig. 7 that the choice of weights significantly influences the efficiency results. The ranking intervals vary on average by 38 positions, but it is possible to see some DMUs whose highest rank is from one to five and worst rank from 55 to 61. Note that these intervals would be even wider if we removed the weight restrictions from the analysis. The wide intervals could be associated with the large number of variables considered as opposed to the number of DMUs and the non-decreasing returns to scale assumption. Here the DISCOs were ordered by the best rank that can be achieved (r^p_{min}). DISCOs with same r^p_{min} were further order from lowest to highest r^p_{max}. D20 that was a benchmark DMU in the CEA analysis is actually in fourth place here since it can achieve lower positions than D52, D54 and D56. D49 that was among the worst performers for CEA, is in ninth place here because for some feasible set of weights it can reach rank 2. But the fact that DISCOs in the top 10 like D44 and D49 can reach positions from 1 to 57 and 2 to 55, respectively, means that DEA scores can be very sensitive to the set of DMUs, i.e., the removal of one or more DMUs from set K will have a strong impact in the results.

The efficiency dominance analysis from REA establishes how many and which DISCOs have performance superior to the DISCO under analysis. One can argue that the dominance relations have similar interpretation to the peers' concept, where DISCOs at the efficient frontier contribute, in the dual DEA model, to the efficiency composition of DISCOs outside the frontier [28]. The advantage is that with the dominance relationship from REA it is possible to identify an efficiency relationship among all the DISCOs and not only with DISCOs that are in the DEA efficiency frontier. If a DISCO is benchmark according to DEA results, it means that it cannot be dominated by any other DMU. If a DISCO p is not a benchmarking DMU, then the algorithm analyses all possible weights and identifies a set of DISCOs whose efficiency is always greater than DISCO's p. It is possible that a DISCO even if outside the efficiency frontier is not dominated by any other. In other words, this means that for some set of weights the DISCO in analysis can overcome the benchmarking DISCOs defined by the DEA model. From Table 5, we can see that although 6 DISCOs are benchmarking according to DEA a larger number of them, more specifically sixteen, are not dominated by any other. Turning our attention again to D44 and D49, they have poor performance in CEA and REA ranking intervals, but D44 is not dominated by any other DMU and D48 is only dominated by D52. DISCOs D1, D2 and D7 are the ones dominated by most DMUs, which is expected from the DEA and CEA performance results.

The relative efficiency results quantify the performance of DISCOs for all possible sets of weights relatively to the most efficient DMU of the set. The optimization model (11) defines the minimum and the maximum relative efficiency. It resembles the ranking intervals, but now we are actually defining an efficiency score instead of just a rank for the DMUs. The results of maximum possible efficiency will coincide with the DEA results. Exceptions are to be found for DISCOs that are in the efficient frontier. The DEA approach does not accept efficiencies larger than one. However, as the third stage of REA evaluates the relative efficiencies among DISCOs that are at the DEA efficient frontier, the model will lead to efficiency scores greater than one providing a ranking within benchmark DMUs. For the case study results (see Fig. A.1) from model (15) show that among the 6 benchmarks DISCOs, D54 is the most efficient followed by D52, D55, D20, D56 and D44.

4.5. Final thoughts and caveats

As a result of this research a few issues were identified: (i) existence of negative efficiency scores due to negative outputs; (ii) weight restrictions based on production trade-offs that may not be physically feasible; (iii) non-homogeneous set of DISCOs being benchmarked all together; (iv) large number of variables being considered as opposed to the number of DISCOs. To address (i), we proposed here the addition of a new set constraints to avoid negative efficiency scores and we proved that these new constraints alone improve the results by better discriminating the DISCOs even without the need for the weight restrictions. On issue (ii), we understand that the weight restrictions were added to the model to better distribute the weights in the optimal solution, but the results show that for most DISCOs four or five out of the seven weight restrictions are binding in the optimal solution. In other words, the efficient scores are being dictated by these weight restrictions.

As for issues (iii) and (iv), the regulator added more variables to the model and adopted the non-decreasing return to scale in an attempt to better represent the diversity of the DISCOs and allow for the simulation of all of them together. But CEA and REA simulation results proved that the model is benchmarking DISCOs that are non-comparable making the efficient score very sensitive to the removal of one or more DISCOs from the set. More specifically, the REA models add quantitative information to compute relative efficiency scores among the DMUs, i.e., it allows for a peer evaluation among DMUs even if they are not in the efficient frontier. But the results showed wide ranges for ranking intervals and failed to define strong dominance relationships among DISCOs.

5. Conclusion

This paper presented an analysis of different benchmarking techniques (DEA, CEA and REA) to the electricity distribution sector. The methodologies are computational efficient providing additional expedite insights to benchmark analysis in the electricity distribution sector. The CEA shows that the diversity of concession areas is significantly influencing the stability of the efficiency scores. The worst performance of the set was from D49 with an FPI of 127%, which means that there is a significant change in the efficiency scores when compared DEA to CEA. The ranking intervals from REA vary on average by 38 positions, but it is possible to see some DMUs whose highest rank is from one to five and worst rank from 55 to 61, showing that choice of weights significantly influences the efficiency results. With the dominance relationship from REA, it is possible to identify an efficiency relationship among all the DISCOs and not only with DISCOs that are in the DEA efficiency frontier. The ranking interval of REA evaluates the relative efficiencies among DISCOs that are at the DEA efficient frontier, the model will lead to efficiency scores greater than one providing a ranking within benchmark DMUs.

A sensitivity analysis for the current benchmark model adopted in Brazil to compute the efficient operational cost for the utilities during the period tariff revision process was discussed. The inclusion of weight limits for benchmarking analysis of DISCOs is interesting and make possible for one to perform benchmark analysis for a complex set of DMUs, but it is important to note that some of the weight restrictions had higher impacts than others on the optimal efficiencies. This paper has shown this by performing an analysis of each weight restriction by adding them as individual Non-Negative Efficiency Score Constraint in the DEA analysis.

As an alternative to deal with a non-homogenous set of DISCOs, a possible future research direction would be to consider a clustering analysis of the DMU set, prior to the application of the DEA model, in order to create subsets of similar DMUs to perform the benchmarking analysis. We expect that by considering homogeneous DMUs subsets the results from DEA, as well as the results from CEA and REA would be more consistent and eliminate the need to incorporate weight restrictions. This would imply policy modifications in the DISCO tariffs revision process and would potentially improve fairness in the benchmarking process and the electricity tariffs establishment. Future work could also seek to segregate the DISCO concession areas in smaller DMUs and apply the CEA and DEA, creating an idea of self-efficiency benchmarking analysis. This could help to pinpoint areas that are affecting negatively the overall score obtained by the DISCO.

CRediT authorship contribution statement

L.M.M. Lima, A.R. de Queiroz and J.W.M. Lima: Conceptualization, Methodology, Supervision, Writing - reviewing and Editing, Funding Acquisition. **G.O.S. Medeiros:** Data curation, Investigation, Software, Visualization, Writing - original draft. **L.C.B. dos Santos, M. A. Barbosa, J.E. Alvares:** Supervision, Writing - Reviewing and Editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

See Fig. A.1.

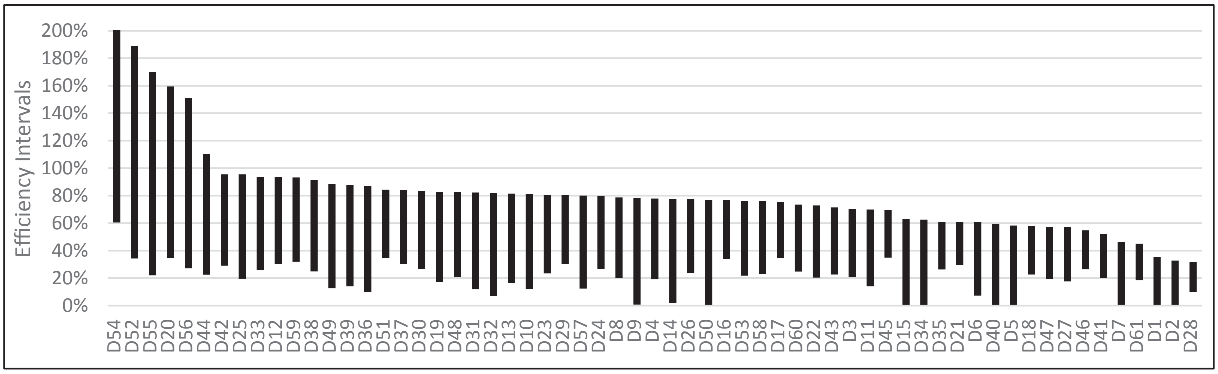

Fig. A.1. Relative efficiency bounds.

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