Renewable and Sustainable Energy Reviews 62 (2016) 382–395

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

Stochastic hydro-thermal scheduling optimization: An overview

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ARTICLE INFO

Article history:
Received 29 May 2015
Received in revised form
4 January 2016
Accepted 26 April 2016

Keywords:
Hydro-thermal scheduling
Multi-stage stochastic optimization
Renewable generation
Electrical power systems
Sampling-based decomposition algorithms

ABSTRACT

This paper presents an overview about the hydro-thermal scheduling problem. In an electrical power system power generators have to be scheduled over a time horizon in order to supply system demand. The scheduling problem consists in dispatching the available generators to meet the system electric load while minimizing the operational costs related to fuel and possible load curtailments. In a system with a large share of hydro generation, different from a thermal dominant power system, the uncertainty of water inflows play an important role in the decision-making process. In this setting the scheduling of generators has to be determined considering different future possibilities for water availability. Also, in the existence of a cascade system, the availability of water to produce electricity in hydro plants is influenced by decisions taken in upstream reservoirs. These issues complicate the hydro-thermal scheduling problem that often in the literature is modeled as a multi-stage stochastic program. In this paper we aim to give an overview about the main ideas behind this problem. We present model formulations, a solution technique, and point out to new developments related to this research.

Contents

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http://dx.doi.org/10.1016/j.rser.2016.04.065
1364-0321/

1. Introduction

Renewable energy is a key piece in the supply side of power systems as sustainable development becomes a goal of our societies. New investments in renewables are necessary in order to satisfy future energy production requirements. Moreover, the worldwide renewable generation expansion is crucial in helping to mitigate changes in climate due to global warming, while not depriving humans of access to goods and services to which they have become accustomed. In [1] the authors examine the threshold effect of the proportion of renewable energy supply for CO_2 emissions reduction, one of the main causes of global warming, for some countries. The fossil age of the last century contributes to our current way of living but we are facing the cost of that kind of energy. Therefore, the access to clean energy is part of many energy programs in European countries and others [2,3], which are engaged in making a better world to live.

Renewable resources should be part of the solution to fulfill the energy demands of seven billion people. There are many types of green energy like wind, solar, geothermal, biomass and tidal generation, which can be combined to compose a sustainable portfolio of electricity providers. The main drawbacks with these energy sources are their seasonality and storage limitations. Renewable electricity production depends on the natural resources that usually do not match the demand at the time and locations that are necessary. The present storage technologies available for renewables include hydro reservoirs, compressed air energy storage (CAES) [4], and batteries (used to store electricity). In [5], batteries with fuel cells are explored to mitigate intermittency of renewables in power systems. In many countries such storage technologies are limited because of a dependence on natural formations and/or investment attractiveness to be constructed.

There are some exceptions in terms of renewable energy controllability limitations and one example is hydro-energy resources, which reservoir storage can be constructed and used to control the timing of electricity generation. In many countries such as Canada and Norway, most of the hydroelectric generation comes from run-of-river power plants, which depend basically on the ice cycles. In the province of British Columbia, Canada, the clean energy act established that at least 93% of electricity generation has to come from renewable resources [6] and the preferable technology is run-of-river hydro plants [7]. However, in other countries like Brazil and Colombia, a relevant portion of existing hydro plants have large reservoirs, which can be used for optimizing water use. In the Brazilian power system for example, where hydroelectricity approximately corresponds to 80% of the total electricity production, hydro reservoirs have the capacity of providing energy supply for several months ahead even in periods of heavy droughts.

Given a hydro-dominant system with the possibility to optimize the water use and mix the electricity production with thermal plants, the hydro-thermal scheduling problem (HTSP) becomes extremely relevant. In such context the HTSP is one of the most important problems in power systems [8]. In the HTSP one is interested in minimizing the total operational costs related to electricity production demanded by the system during a specific time horizon. These operational costs are derived from the fuel costs that feed thermal plants and the costs of possible demand curtailments. This problem is complicated by the fact that we do not have perfect forecasts for future water inflows into system reservoirs.

Generally, the HTSP is modeled as an optimization program and solved by special-purpose algorithms. In [9] authors discuss the importance of optimization modeling and algorithms in the planning of renewable integration in power systems. A review of the role of optimization techniques in power generation and supply can be found in [10]. The work of [11] presents a review on risk-constrained hydropower scheduling in deregulated power systems from a perspective of profit maximization for generation companies. In [12] a survey on stochastic unit commitment problems for day-ahead market clearing is presented. In other perspective, we focus in a centralized dispatch scheme where the objective is aimed to minimize operational costs over a planning horizon, ranging from months to years ahead, in a system composed by hydro and thermal plants. This paper provides improvements in the knowledge basis by presenting a detailed overview about the HTSP stochastic version, by formulating precise mathematical models and by pointing out and discussing new research developments related to such challenging field.

Section II presents the basic concepts related to the HTSP. Section III gives a general description of the HTSP characteristics and mathematical model formulations to represent such problem in the context of individual hydro plants and aggregated reservoirs. Section IV presents decision-making methodologies that have been used to deal with several mathematical HTSP models and new developments related to this research area. Section V concludes this paper.

2. Basic concepts in hydro-thermal scheduling

In electric power systems where hydro and thermal power generation scheduling decisions are performed in a centralized manner, the independent system operator (ISO) may decide to use the water available at hydro plant reservoirs to produce electricity at any time. Doing so avoids economic expenses required to dispatch thermal power plants, but can risk hydro availability in future time periods. In the HTSP one is interested in minimizing electricity production costs to supply the system demand considering the operation of hydro and thermal power plants.

The water available to produce electricity at each hydro plant is bounded by the reservoir storage capacities and the future water inflows at the river basins where these reservoirs are located. Depending on the share of renewable resources and other system conditions, most of the time thermal generation must be used to complement the electricity supply in order to meet system demand. However, wise use of hydro and thermal system resources by the decision maker can reduce costs over time. The decision process faced by the ISO when operating a hydro-thermal system is presented in Fig. 1.

Hydroelectricity is inexpensive to produce, with virtually no associated costs for water usage once hydro turbines are installed.

Nomenclature

Indices

- i,j electrical region indexes
- ω stochastic water inflow scenario index
- h hydropower plant index
- k thermal power plant index
- t time period index
- m upstream hydro plant index

Sets and subsets

- I set of electrical regions of the power system
- I^* set of regions pairs that can exchange energy
- Ω set of stochastic water inflows
- H set of hydro plants
- G set of thermal plants
- T set of time periods
- H_i subset of hydro plants inside region i
- M_h subset of hydro plants immediately upstream of plant h
- G_i subset of thermal plants inside region i

Parameters

- c_k^t operational cost of thermal plant k, at stage t [\$/MW-period]
- ρ^t load curtailment cost, at stage t [\$/MW-period]
- δ_h hydro plant h productivity [MW-period/hm^3]
- $b_{h,t}$ water inflow at hydro plant h, at stage t [hm^3]
- b_{t+1}^{ω} vector that represents stochastic water inflows, and deterministic electricity demand, at stage $t+1$ at scenario ω
- d_i^t electricity demand in region i, at stage t [MW-period]
- $\overline{v}_h^t$ maximum turbine outflow at hydro plant h, at stage t [hm^3]
- $\underline{v}_h^t$ minimum turbine outflow at hydro plant h, at stage t [hm^3]
- $\overline{y}_k^t$ maximum thermal generation of plant k, at stage t [MW-period]
- $\underline{y}_k^t$ minimum thermal generation of plant k, at stage t [MW-period]
- $\overline{x}_h^t$ maximum water storage at plant h, at stage t [hm^3]
- $\underline{x}_h^t$ minimum water storage at plant h, at stage t [hm^3]

Decision variables

- y_k^t thermal generation at plant k, at stage t [MW-period]
- x_h^t water volume storage in hydro plant h reservoir, available at the end of stage t [hm^3]
- u^t load curtailment at stage t [MW-period]
- x^t decision vector of hydro plants reservoir water storage available at the end of stage t
- v_h^t turbined water outflow at hydro plant h, at stage t [hm^3]
- s_h^t water volume spilled from plant h, at stage t [hm^3]
- $p_{i,j}^t$ energy transfers from region i to region j, at stage t [MW-period]

Functions

- $h_{t+1}(\bullet)$ recursive function that represents a model like (7)–(14) where t is shifted by 1. It depends on decisions made at stage t, and random parameters that are revealed at the beginning of stage $t+1$
- $\mathbb{E}_{b_{t+1}|b_t} h_{t+1}(\bullet)$ expected future cost function

Additional Nomenclature for (15)-(22)

Parameters

- $b'_{i,t}$ energy inflow at ARR i, at stage t [MW-period]
- b'^{ω}_{t+1} vector that represents stochastic energy inflows, and deterministic electricity demand, at stage $t+1$ and scenario ω
- $\overline{v}'^t_i$ maximum hydro generation at ARR i, at stage t [MW-period]
- $\underline{v}'^t_i$ minimum hydro generation at ARR i, at stage t [MW-period]
- $\overline{x}'^t_i$ maximum energy storage at ARR i, at stage t [MW-period]
- $\underline{x}'^t_i$ minimum energy storage at ARR i, at stage t [MW-period]

Decision variables

- x'^t_i energy storage in ARR i, at stage t [MW-period]
- x'^t decision vector of ARR energy storage available at the end of stage t
- v'^t_i hydro generation at ARR i, at stage t [MW-period]
- s'^t_i energy spilled from ARR i, at stage [MW-period]

Functions

- $f_1(b'_{i,t})$ represents the controllable portion of the energy inflows (minus energy losses such as evaporation and water detour), this portion can be stored in the ARR i at time stage t
- $f_2(b'_{i,t})$ represents the uncontrollable portion of the energy inflows (minus energy losses), this portion cannot be stored in the ARR i at time stage t
- $h_{t+1}(\bullet)$ recursive function used to represent a model like (15)–(22) where t is shifted by one unit. It depends on decisions made at stage t, and random parameters that are revealed at the beginning of stage $t+1$

Actually, one possibility is to measure indirectly the value of electricity produced by hydro plants by computing the difference between the operational costs in a system containing only thermal plants and the operational costs in that same system containing both hydro and thermal plants. In the former system, thermal plants are usually dispatched in a least cost fashion to meet system demand. In the latter system, thermal plants are dispatched to complement electricity production from hydro plants. By comparing both operational costs it is possible to estimate the value of hydro generation to the system.

2.1. Hydro generation power plants

Hydro power plants play one of the main roles in HTSPs. In the simplest model of a generation-demand system that contains only thermal plants, the demand is met by the least cost approach, i.e.,

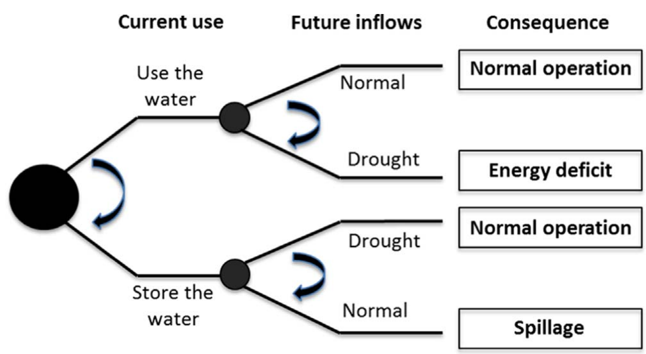

Fig. 1. Decision process for the hydro-thermal scheduling problem.

thermal plants are dispatched in ascending order of costs until demand is satisfied. In a system with hydro plants, it is possible to produce electricity with water, at no cost, and reduce the operational expenses. However, electricity production at hydro plants depends on the system storage and on the water inflow volumes that becomes available (influenced by the different seasons of the year and by other climate conditions).

There are three main types of hydro plants considered in HTSPs, they are: hydro plants with large reservoirs (HRs), run-of-river hydro plants (RRHs), and small hydro plants (SHs). It is possible to store water is HRs and the available water volume may be used to produce electricity whenever it is required. A meaningful share of hydropower comes from HRs, where the potential energy of dammed water driving a water turbine and a generator is transformed into electricity. The amount of electricity generated is proportional to the difference in height between the top of the reservoir and the water discharge level, which is called the reservoir's head.

RRHs also have reservoirs, but their capacities are small compared to other hydro plants. In a RRH, it is essentially impossible to store water, and hence the flow of water either generates electricity or is spilled depending on the plant's generation capacity. In a typical RRH, the water is captured at the intake structure and goes through a buried penstock to the powerhouse. In this scheme if the water is not captured at the intake it will continue its normal flow in the river. The dam is required to ensure enough water goes to the penstock. The penstock carries the water from an upper elevation and delivers it to a lower elevation where the water turbine is located. The difference in height characterizes the head of a run-of-river hydro plant and is responsible for the potential energy that is transformed into electricity by the hydro plant generator.

SHs vary in power output. Most commonly installed capacities range from 1 to 30 MW, but there are also plants that have power output less than 1 MW. These hydro plants are similar to RRHs, with little or no reservoir capacity. That said, SHs are usually distinguished from RRHs because there are specific equipment to simultaneously meet the requirements of sufficiently high power output, environmental restrictions and reliability. SHs do not create serious environmental impacts. This scheme of power plant does not require large flooding areas and can be installed to produce electricity in remote regions. To a greater degree than RRHs, a drawback of this scheme is that the power output of these plants is highly dependent on the natural flow of the river, making them susceptible to seasonal variations.

Generally, when an optimization model is designed for a HTSP there are three sets of decision variables that have to be considered for hydro: turbined water used for generation, storage and spillage. Depending on the horizon that one aims to solve HTSP models, hydro generation expansion may be considered in mathematical models by changing hydro plants parameters over the model time stages. The hydro generation expansion may also affect other hydro plants operational decisions in a cascade system and that also can be handled by mathematical models.

2.2. Thermal generation power plants

In contrast to hydro plants, which must be constructed on river basins, we have the flexibility to locate thermal plants near the load centers what reduce electricity transmission losses and network investment costs. A thermal plant uses fuel to transform energy from heat into electricity. The most common fuel types used by thermal plants are natural gas, coal, oil and uranium for nuclear power plants. There is a cost associated with electricity production from thermal plants, with one portion being proportional to fuel costs and the other to operational and maintenance costs. Each thermal plant has its own function that relates power output and cost. Usually these cost functions are nonlinear, but in order to simplify HTSP models one usually assumes that thermal costs are linear functions of the power output [13–16].

There are specific thermal plants that have to maintain a certain minimal power output at all times. These generators cannot be turned on and off instantly, some of them take hours or even days to start operating at normal conditions. So if we fail to satisfy these requirements we cannot count with this thermal plant during peak hours for example. Later on this paper we present mathematical model formulations for the HTSP where we consider the generation of each thermal plant as a decision variable that is restricted by the generator minimum and maximum power production capacity at each time period. Thermal generation expansion also should be considered, but different from hydro expansion this process does not affect other power plant operational decisions. Besides the addition of new thermal plants, turbines/generators parameters can be modified during the planning horizon, which may imply in fluctuations of the available thermal generation at different time periods.

2.3. Electricity demand

The electricity demand is characterized by the amount of electricity being consumed by the load during a time period. The modeling step of the power system demand is important to HTSPs. The time horizon in a HTSP usually assumes the following possible discretization: monthly, weekly, daily, or hourly. In problems with monthly discretization the electricity demand is represented by the energy amount required by the load during one month. Therefore, the unit for electricity demand considered in that case is [MW month]. Note that the electricity demand is dynamic, continuously changing in time, but instead of representing these variations several models use an average of the monthly demand curve as the demand representation.

Different load levels can be considered in order to represent different periods of the day. With different levels it is possible to have more details about the power system operation during peak and off peak hours for example. In this context, it is possible to learn about many things in the system such as: transmission system bottlenecks, thermal generation contributions, costs to produce electricity during a specific daytime and possibilities of load curtailments. By modeling different load levels it is natural to consider different durations and demand values for each level. Also, it is important to notice that decision variables related to hydro and thermal plants will suffer alterations in order to match different load levels, when that is the case.

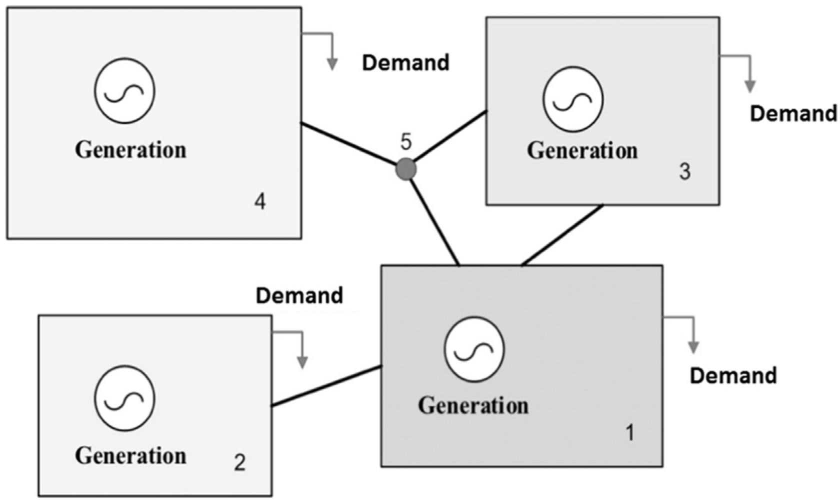

Fig. 2. Transmission lines interconnecting electrical regions.

2.4. Energy exchanges in the system

Transmission lines allow regions with excess of supply to ship electricity to regions with energy deficits. Using the available hydro generation and the transmission system it is possible to meet demand at regions far away from the river basins. Transmission capacities between regions are limited and these restrictions play an important role in the locational marginal prices [17] and consequently in the operational costs of each region. Lack of transmission capacity may lead to dispatch of expensive power plants located closer to the load or even to load curtailment.

Fig. 2 represents a small system with four regions and one virtual region. It is possible to exchange energy between regions that have direct connections. For example, we can exchange energy between regions 1 and 2 directly, but in order to transfer energy between regions 2 and 3 it is necessary that the energy pass through region 1 first. The virtual region 5 is a point where transmission lines have a connection. In this case, to transfer energy from region 3–4 the amount has to pass through the virtual region 5 first. A virtual region has no demand, so the energy that comes in must be equal the energy that comes out of that region. Inside other regions the sum of electricity production, load curtailment and energy transfers ("−" signal if going out from the region and "+" signal if going into the region) has to be equal to demand. Also, it is possible to represent energy exchanges in a finer-grain resolution when we consider a power system that contains several load and generation buses with transmission lines linking these buses.

3. Hydro-thermal scheduling problem: characteristics and model formulations

3.1. Problem characteristics

The available hydro generation capacity at a particular time period depends on the amount of water stored in the hydro plant's reservoir. If this hydro plant is part of a cascade system (there are generators upstream and/or downstream in the same river) the amount of stored water is influenced by the operational decisions applied to the generators upstream. This couples the problem in space.

Natural water inflows are responsible for a large part of the future water supply that will be available to generate electricity. These future water inflows and their stochastic nature complicate the resulting HTSP mathematical models. The HTSP is dynamic because present decisions affect the future. Fig. 1 presents the intuition behind this idea. On the one hand, if the ISO decides to use a large quantity of water to produce electricity today and in the future a drought occurs it may be necessary to dispatch more expensive thermal generation (e.g., diesel generators) in order to supply demand or even to curtail some load. This procedure would generate unnecessary expenses to the system. On the other hand, if the ISO decides to store water at present time for future use and a scenario of large water inflows realizes, it may be possible that the operator will have to make decisions to spill water volumes from hydro reservoirs. This implies a waste of potential energy and hence money. These characteristics couple the HTSP in time.

In this problem, there are multiple interconnected hydro reservoirs in the system that need to be scheduled over many time periods. This combined with stochastic inflows means that the problem may be represented by a multi-stage stochastic program [18–20]. The objective in HTSPs is to determine the optimal amount of electricity to be produced by hydro and thermal plants at each time period satisfying the problem constraints such that the expected operating costs related to the system are minimized.

3.2. HTSP Model with individual hydro plants

In formulating a HTSP model with individual hydro plants one is interested in determining generation targets for each hydro and thermal plant over multiple time periods (months, weeks, days or hours) with the objective of minimizing the total operational costs. In this setting, the parameters related to water inflows, turbine water outflows, water spillage and water storage are represented by water volumes. In general, such model would capture characteristics of an interconnected cascade system so that decisions at each generator may affect the whole cascade. Depending on the HTSP model's horizon and time discretization, water volumes that are used to produce electricity and water volumes that are spilled from upstream reservoirs are available at the same time period at the next downstream reservoir and these volumes can be used to produce electricity once again. So, besides the water inflows, the water volume available at each reservoir in a particular time stage depends on operational decisions upstream.

A hydro plant cascade system is depicted in Fig. 3. The triangles represent HRs and the circles represent RRHs. In this setting, operational decisions of generators 1 through 5 have influence on the water available for generators 6 and 7, decisions of generator 6 also influence operating conditions in 7, and so on.

A power system representation with 4 different regions is shown in Fig. 4. Each region has its own electricity demand and its own set of power generators (hydro and thermal). The hydro plants within a region are coupled by a cascade scheme, and the thermal plants are independent of each other. Fig. 4 also shows transmission links that interconnect the power system, transferring power between regions. With transmission links in a hydrothermal power system, the ISO can take advantage of the hydrological diversity between regions in order to operate the system in the best possible way.

Thermal power plants play an important role in the system robustness. During periods with unfavorable hydrological conditions, thermal plants can be dispatched to complement hydro and satisfy demand. This allows decisions to store water in the hydro reservoirs in order to secure electricity supply for the future time periods as necessary. Thus, one of the main purposes of the thermal plants and transmission links is to optimize the utilization of system resources (water and fuel).

3.2.1. General problem formulation

As mentioned earlier, the HTSP can be modeled as a multi-stage stochastic optimization problem. In this setting one of the most relevant information is the natural inflow of water that becomes available at each hydro plant at the beginning of each time period. This parameter may be considered stochastic because it depends on the random nature of the rainfall. In the literature it is common to assume that there exists a stochastic process (SP) that governs the realization of such parameters. A T-stage stochastic linear program (SLP-T) with recourse, to represent the HTSP, may be formulated as follows:

$$\min_{x_1} \; c_1 x_1 + \mathbb{E}_{b_2|b_1} h_2(x_1, b_2) \tag{1}$$

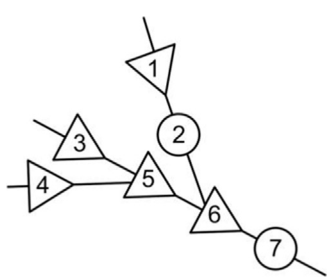

Fig. 3. Hydro plants cascade representation.

$$\text{s.t. } A_1 x_1 = B_1 x_0 + b_1 \; : \; \pi_1 \tag{2}$$

$$x_1 \geq 0 \tag{3}$$

where for $t = 2, \ldots, T$,

$$h_t(x_{t-1}, b_t) = \min_{x_t} \; c_t x_t + \mathbb{E}_{b_{t+1}|b_t} h_{t+1}(x_t, b_{t+1}) \tag{4}$$

$$\text{s.t. } A_t x_t = B_t x_{t-1} + b_t \; : \; \pi_t \tag{5}$$

$$x_t \geq 0 \tag{6}$$

The decision variables of a particular stage t are represented by the vector x_t, which includes hydro generation, thermal generation, water storage at the hydro plant reservoirs, and spilled water. The parameter vector b_t represents deterministic electricity demand and a specific realization of the stochastic water inflows at stage t. Eqs. (1) and (4) represent the objective functions of the problems at the first and t-th stage, respectively. The objective is to minimize present cost plus the expected value of the future cost. Eqs. (2) and (5) represent the model's structural constraints, which include water balance and electricity demand satisfaction requirements as well as limits on hydro and thermal generation, and on energy transfer between regions. Associated with structural constraints the model has dual variables, denoted π_t. Eqs. (3) and (6) are simple bounds on decision variables. The term $\mathbb{E}_{b_2|b_1} h_2(x_1, b_2)$ represents the expected cost function of stage 2 given decisions x_1, defined in stage 1, and the random parameter b_2 realization that affects the conditions of the system at stage 2. The term $\mathbb{E}_{b_{t+1}|b_t} h_{t+1}(x_t, b_{t+1})$ represents the expected cost function of stage $t+1$ given decisions x_t, defined in stage t, and the random parameter b_{t+1} realization that affects the condition of the system at stage $t+1$.

3.2.2. Stage-t HTSP model formulation

A detailed formulation of the problem at a particular stage t can be stated as:

$$z = \min \sum_{i \in I} \left[\sum_{k \in G_i} c_k^t y_k^t + \rho^t u^t \right] + \mathbb{E}_{b_{t+1}|b_t} h_{t+1}\left(x^t, b_{t+1}^{\omega}\right) \tag{7}$$

$$\text{s.t. } x_h^t = x_h^{t-1} + b_{h,t} - v_h^t - s_h^t + \sum_{m \in M_h} \left(v_m^t + s_m^t\right) \qquad \forall h \in H \tag{8}$$

$$\sum_{h \epsilon H_i} \delta_h v_h^t + \sum_{k \in G_i} y_k^t + \sum_{j:(i,j) \in I^*} p_{i,j}^t - \sum_{j:(i,j) \in I^*} p_{j,i}^t + u^t = d_i^t \qquad \forall i \in I \tag{9}$$

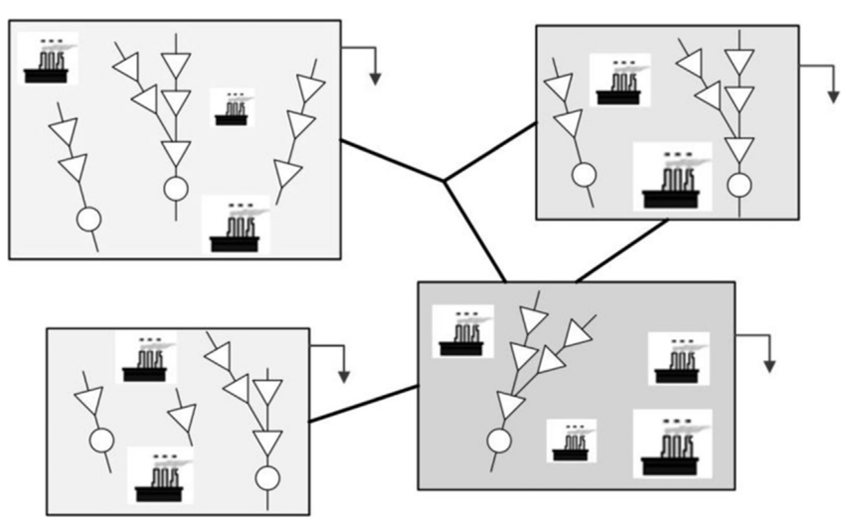

Fig. 4. Power system representation with individual hydro plants.

$$\sum_{i:(i,j)\in I^*} (p_{i,j}^t - p_{j,i}^t) = 0 \qquad \forall j \in I^* \tag{10}$$

$$\underline{v}_h^t \le v_h^t \le \overline{v}_h^t \qquad \forall h \in H \tag{11}$$

$$\underline{y}_k^t \le y_k^t \le \overline{y}_k^t \qquad \forall k \in G \tag{12}$$

$$\underline{x}_h^t \le x_h^t \le \overline{x}_h^t \qquad \forall h \in H \tag{13}$$

$$s_h^t \ge 0 \ \forall h \in H, p_{ij}^t \ge 0 \ \forall (i,j) \in I^* \tag{14}$$

Eq. (7) represents the objective function that minimizes the sum of present and future expected operational costs. Model (7)–(14) has three different sets of structural constraints at each stage and simple bounds on decision variables. Eq. (8) represents the water balance constraint for each hydro plant in the system. The purpose of this constraint is to balance the reservoirs' storage levels. This constraint ensures that the storage level at the end of stage t is equal its storage level in stage $t-1$ plus the water volume that comes into that hydro plant reservoir subtracted of the water volume that leaves that reservoir in stage t. Eq. (9) represents the set of demand satisfaction constraints, which in this particular model. is represented for each electric power system region. The demand satisfaction constraint ensures that, for each region i, the amount of electricity generated by power plants plus the unmet demand has to be equal to that region electricity demand. Eq. (10) ensures that energy exchanges from region i to region j is equal to exchanges from region j to region i, but with opposite signs. Eqs. (11)–(14) represents simple bounds on the decision variables.

3.3. HTSP Model with aggregate reservoir representation

The main goal in formulating a HTSP model with an aggregate reservoir representation (ARR) is the same as that with individual hydro plants, minimize present and expected future operational costs subjected to a set of constraints. The main difference is that in a model with ARR, the optimization model deals with all variables and parameters related to hydro in units of energy instead of water. Random water inflows and water reservoir volumes are transformed into energy inflows and energy storage for an ARR using hydro plants turbine/generator productivities along the cascade. Now instead of a solution yielding individual targets for hydro plants, a solution yields generation targets for each ARR in a specific planning horizon.

Hydro plants inside a region are aggregated into a single reservoir that has both controllable and uncontrollable energy that can be used to produce electricity. Fig. 5 depicts a region of the power system earlier introduced with all its hydro plants aggregated into a single ARR. Fig. 5 also shows some of the parameters used to represent an ARR. Note that thermal plants are represented individually. Energy inflows are divided into controllable and uncontrollable inflows. Both the controllable and the uncontrollable inflows may be used to generate electricity immediately but only the controllable inflows can be stored in the ARR for future use. We have energy losses at the ARR due to evaporation, diversion of water (e.g., for agricultural use) and water spillage.

Pierre Mass first mentioned the ARR, also known as the equivalent reservoir representation, in the mid-1940s [21]. One of the first ARR model implementation with application to the multi-reservoir hydroelectric power system is presented in [22,23] and applied to the Pacific Northwest hydropower system. The ARR is an aggregation technique used to reduce the size of the model by aggregating multiple reservoirs inside a specific region into a single aggregate reservoir. The use of ARR consequently reduces the computational effort required to solve a HTSP model. This type of representation models the total hydro generation of a power system or even of a specific region inside that system.

The ARR has been used since the 1970s decade in Brazil to model the hydroelectric power system. First the ARR was coupled with a stochastic dynamic programming approach to solve HTSPs [24]. Since the mid-1990s the same ARR model started to be used with stochastic dual dynamic programming to solve HTSPs for the Brazilian interconnected system [25–27]. We can construct an aggregate reservoir to represent as many or as few hydro plants as we want. Generally an ARR is designed for each power system region, or for a specific river basin, where random water inflows characteristics tend to be similar among hydro plants located there. In [28] it is presented a hybrid application of ARR together with individual hydro plants, where the goal is to model more precisely special hydro plants in the system. A description of the long-term hydro-thermal planning problem for the Brazilian system, along with a discussion of the required energy inflow forecasting model and a comparison of the relative merits of

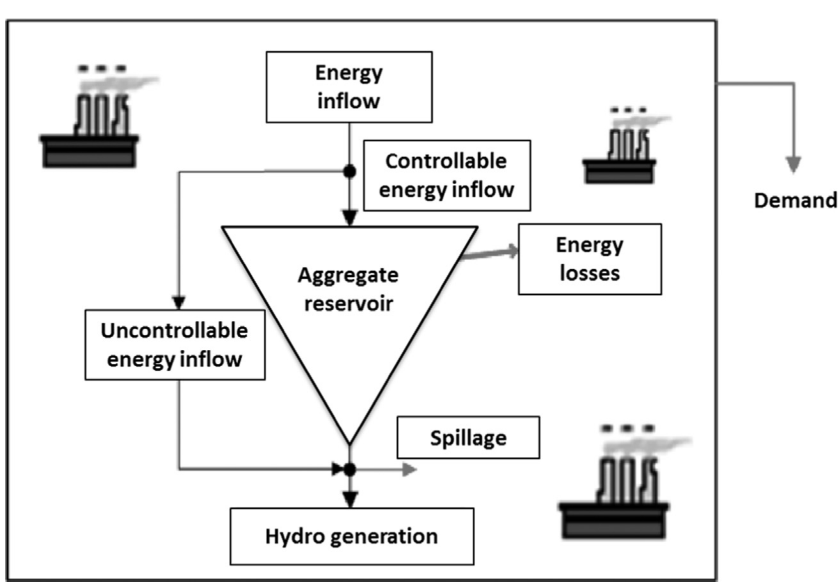

Fig. 5. Power system region represented with aggregated hydro plants.

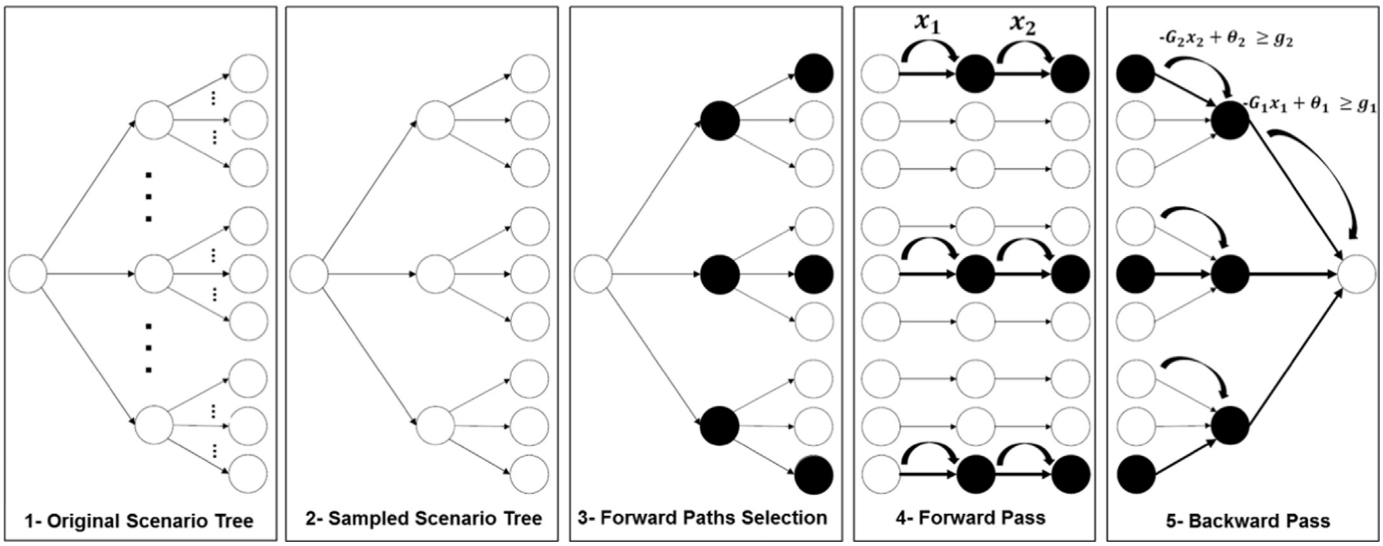

Fig. 6. Optimization solution process via SBDA.

aggregating hydro reservoirs via electrical subsystem versus aggregating them via hydrological cascade can be found in [29].

A detailed formulation at a particular stage t of the HTSP with ARR representation can be stated as:

$$z = \min \sum_{i \in I} \left[\sum_{k \in G_i} c_k^t y_k^t + \rho^t u^t \right] + \mathbb{E}_{b'_{t+1}|b'_t} h_{t+1}(x'^t, b'^{\omega}_{t+1}) \tag{15}$$

$$\text{s.t. } x_i'^t = x_i'^{t-1} + f_1\left(b'_{i,t}\right) - v_i'^t - s_i'^t \qquad \forall i \in I \tag{16}$$

$$v_i'^t + f_2(b'_{i,t}) + \sum_{k \in G_i} y_k^t + \sum_{j:(i,j) \in I^*} p_{i,j}^t - \sum_{j:(i,j) \in I^*} p_{j,i}^t + u^t = d_i^t \qquad \forall i \in I \tag{17}$$

$$\sum_{i:(i,j) \in I^*} (p_{i,j}^t - p_{j,i}^t) = 0 \qquad \forall j \in I^* \tag{18}$$

$$\underline{v}_i'^t \leq v_i'^t \leq \overline{v}_i'^t \qquad \forall i \in I \tag{19}$$

$$\underline{y}_k^t \leq y_k^t \leq \overline{y}_k^t \qquad \forall k \in G \tag{20}$$

$$\underline{x}_i'^t \leq x_i'^t \leq \overline{x}_i'^t \qquad \forall i \in I \tag{21}$$

$$s_i'^t \geq 0 \ \forall i \in I, p_{ij}^t \geq 0 \ \forall (i,j) \in I^* \tag{22}$$

Eq. (15) represents the objective function that minimizes the sum of present and expected future operational costs. Model (15)–(22) has three different sets of structural constraints at each stage and simple bounds on decision variables. Eq. (16) represents the energy balance constraint for each ARR in the system. The purpose of this constraint is to balance the ARR storage levels. This constraint ensures that the storage level at the end of stage t is equal its storage level in stage $t-1$ plus the energy that comes into the ARR subtracted of the energy that leaves that ARR in stage t. Eq. (17) represents the demand satisfaction constraints. Eq. (18) ensures that energy exchanges from region i to region j is equal to exchanges from region j to region i, but with opposite signs. Eqs. (19)–(22) represent simple bounds on the decision variables.

Note that in the case of the HTSP model with ARR representation the number of decision variables reduces significantly. For example, if we have 50 hydro plants inside a region that we want to model as an ARR, the number of decision variables that were three for each hydro plant (x_h^t, v_h^t and s_h^t) at a particular stage, and so 150 in total, reduces to three for each ARR ($x_i'^t, v_i'^t$ and $s_i'^t$). The same happens with the number of constraints, for example, we had in Eq. (8) a total of $|H|$, i.e., one for each hydro plant, and this number is reduced to one constraint Eq. (16) for each ARR. In the previous example for 50 hydro plants we would have 50 constraints like Eq. (8) and for the HTSP with ARR only one constraint like Eq. (16) is necessary for each ARR. This reduces the model size and speed-up the optimization algorithm convergence. However the ARR scheme may affect the system operational decisions with undesirable errors originated from this problem approximation.

4. Optimization algorithms and stochastic processes used in HTSP models

In the literature, there are two different problem structures used to represent the HTSP stochastic version, one is based in a modest number of scenarios and the other is based in a sampled scenario tree (the original tree associated to this sampled tree is usually too big to be represent in full size). Consequently, there are two classes of optimization algorithms that are applied to these different structures: scenario-based decomposition algorithms and sampling-based decomposition algorithms. But before going into more details about optimization algorithms we discuss the stochastics that govern water (or energy) inflows. It is important to notice that each node in a scenario tree, Fig. 6 – sub-figure 1, represents an optimization problem where decisions have to be taken. In order to be able to solve an optimization problem at each node it is necessary to have the realization of the random parameters and the state of the system at that point. Note that, at each particular time stage of the scenario tree, the state of the system depends on the decisions taken at the previous stage, represented in the classic HTSP case by the storage. In such problem, once all the necessary information is available, commercial solvers can be used to find the best solutions at each node. The challenge here is that at each stage the model needs the best possible representation of the expected future cost function in order to make optimal decisions. The expected future cost function is not available in the majority of cases and has to be constructed. Therefore, special-purpose algorithms, that we discuss further, are used to construct approximations of these expected future cost functions.

4.1. Stochastic process governing random parameters

For the purpose of this work we consider a stochastic process governing the water inflows, but we discuss randomness in other parameters in Section 4.1.3. The stochastic process that governs the water, and therefore the energy, inflows is one of the most important characteristics of HTSP models. In this section we assume that the stochastic model for b_t, from model (1)–(6), is in the same units as that of the constraints. That assumption is implicit in the $B_t x_{t-1} + b_t$ right-hand-side (RHS) of Eq. (5). So, if our flow conservation constraint is in units of water volume as in Eq. (8) then our forecasting for b_t is also in units of water. Or, if we have formulated the model using an ARR with flow conservation in units of energy then our forecasting of b_t is also in units of energy as in Eq. (16). The reader is encouraged to look at the work of [13,32] for other representations of the HTSP with water inflows. In [25,29] other and more detailed HTSP models that uses aggregate forecasts are presented, where b_t is represented in units of energy. In [20] and [40] the authors consider a variant methodology in which inflow forecasting is in units of water volume but the formulation of the stochastic program is designed in units of energy.

The multi-stage HTSP model with representation of the random variables is designed to be solved in a scenario tree. In Fig. 6 – sub-figure 1 it is presented a complete scenario tree for the problem. The first stage realization of b_t is assumed to be known. For the other stages we have probability distributions that govern the random parameters. Each of the nodes in the scenario tree represents a different realization of the random parameter associated to that stage. In the real problem cases, there are continuous probabilities density functions that represent the random inflows. This indicates that there are an uncountable number of scenario possibilities and consequently an infinite scenario tree. In practical applications, for the sake of computational tractability, it is created a sampled scenario tree from the complete scenario tree. The sampled scenario tree, Fig. 6 – sub-figure 2, represents at each stage a finite subset of possible scenario realization for the random variables. The sampled scenario tree varies in size and scenarios in this tree are constructed considering assumptions of independence or dependence among stages for random variables.

4.1.1. Interstage independent case

The simplest way to represent the random variables related to random inflows in a scenario tree is to assume that vectors b_t, $t=2,\ldots,T$, are interstage independent. When we assume independence from one period to the next we mean that the realization of the random variable at a future stage has no relationship with the realization of random variables from previous stages. That means that for each node in a particular stage the set of descendant nodes in the scenario tree has to be the same, for more details see [20].

4.1.2. Interstage dependent case

In the interstage dependent case, we assume the random inflow vector satisfy Eq. (23), where η_t for $t=2,\ldots,T$ are independent.

$$b_t = R_{t-1} b_{t-1} + \eta_t, t=2,\ldots,T \tag{23}$$

The matrices $R_j^t, j=1,\ldots,t-1, t=2,\ldots,T$, are assumed known, presumably because they have been estimated using historical data. Dependency model (23) generalizes the periodic autoregressive model (PAR) [33,34] in which R_j^t exhibits seasonality or can be defined in the context of a dynamic linear model [36]. For example, referring to [29], we can appropriately define the R_j^t matrix and η_t random vectors to have: (i) 12 seasons in a model with monthly time increments, (ii) the length of the lag depending on the month (because some matrices satisfy $R_j^t = 0$), (iii) the PAR model use centered terms $b_j - \mathbb{E} b_j$ in place of b_j (because the associated deterministic terms can be absorbed in η_t), and (iv) the distribution of η_t can be that of a multivariate shifted lognormal. Note that if we have a lag of a specific order (e.g., six monthly periods) then we can alter Eq. (23) so that in the initial periods the inflow vector depends on the "prehistory" of the optimization model (e.g., the inflows in the six months predating the optimization model's first month).

In the general statement of the stage $t-1$ problem, i.e., a version of Eq. (23) shifted by one stage, the expectation operator is $\mathbb{E}_{b_t|b_1,\ldots,b_{t-1}}$. In light of the autoregressive-style dependency process specified above, the first term on the RHS of Eq. (23) is deterministic given that we condition on $b_1,\ldots,b_{t-1}$. So, given these values of the inflows in stages $1,\ldots,t-1$, the expectation amounts to integrating with respect to the distribution of η_t. In other words, under Eq. (23) we can rewrite the conditional future cost function in model (4)–(6) as:

$$\mathbb{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x_t, b_{t+1}) = \mathbb{E}_{\eta_{t+1}} h_{t+1}\left(x_t, b_{t+1}\left(b_1,\ldots,b_t,\eta_{t+1}\right)\right) \tag{24}$$

4.1.3. Representation of other uncertainties in the problem

In terms of other uncertainties, most of the literature related to HTSPs represents the stochasticity only in the water or energy inflows. However other uncertainties can be incorporated. For example, power system demand can be considered as uncertainty and no modification is required in the model or in the solution strategy, further discussed in this Section, except by creating a scenario tree using this information. A realization of vector b_t^ω, defined in model (7)–(14) or (15)−(22), would in this case represent a realization of the demand and the water (or energy) inflows at each location. Recently, increase in penetration of wind farms and solar photovoltaic plants in power systems is changing the representation of other uncertainties. In the HTSP a representation of wind and solar resources uncertainties can be implemented by altering b_t^ω as well. In this configuration the future scenarios represented in the scenario tree will take into consideration specific realizations of water inflows, wind and solar power. In such context, one possibility is to represent the power system demand to be discounted by the wind power and/or the solar power realizations. By representing uncertainties in this way there is no need to increase the dimensions of the mathematical model vectors and matrices. Another option is to increase the dimension of the decision vector x_t and the number of structural constraint by changing the dimension of matrix A_t and the vector b_t^ω. In this alternative it is possible to represent wind and solar resources with more details. The work presented in [35] shows a representation of stochastic wind-HTSP following the last idea.

Other types of uncertainties related to thermal power plants can be represented in HTSP models and handled by the further discussed solution strategy. For example, different scenario realizations for future fuel prices can be considered and embedded in the scenario tree as proposed by [45]. Besides the representation of the stochastic b_t^ω vector it is also necessary to incorporate a representation of uncertainty in the cost vector c_t^ω. In terms of power plants and transmission lines reliability, deterministic parameters have been used to represent future maintenance of such assets in HTSPs. However, every addition of new stochastic variables also increases the future uncertainty related to the original problem and as a consequence solution time to solve such modified models will also increase. It is necessary to have a sense judgment to choose which uncertainties are the most important ones to represent in the problem and drive decisions regarding to the scheduling of power generators. This is not an easy task and it depends on the problem's horizon, problem's discretization, uncertainties variabilities, and on the share of each power generation source.

4.2. Classical optimization methods

Until the mid-1980s, most research on HTSP under uncertainty used the stochastic dynamic programming (SDP) technique as solution method for multi-stage stochastic programs. The work presented in [30] shows a review of dynamic programming techniques applied to water resources problems and in [31] the authors present the application of SDP to hydro-thermal generation scheduling and its comparison with its deterministic counterpart. The main drawback of SDP is the "curse of dimensionality" of dynamic programming (DP) that makes the problem intractable when the dimension of the state vector is medium or large. DP algorithm constructs the future cost function by discretizing the state variables at each stage into a set of finite values. Then the algorithm proceeds backward in time using Bellman's recursion; see [37,38]. Methods to overcome DP's "curse of dimensionality" and solve real-size instances of this type of problem were necessary. Benders' decomposition algorithms [39] lead the way. Benders' decomposition algorithm is at the core of the solution methods for multi-stage stochastic linear programs. The reader is encouraged to look at [13] for a scenario-based decomposition algorithm applied to a HTSP model and at [14,29,40,41,42] for applications of sampling-based decomposition algorithms (SBDAs). Recently, new enhancements of the SDP method combined with the convex hull algorithm have being used as well for solving HTSPs [86].

4.2.1. Scenario-based decomposition methods

Scenario-based methods first select a modest number of scenarios to represent the probability distribution of the random variable. The problem, after the scenarios are chosen, is considered as a large deterministic linear program. The optimal solution obtained for this problem is exact, but it is only an approximation of the true original problem, assuming that the probability distribution was approximated. Two of the most well-known scenario-based decomposition algorithms are the L-shaped method of [43] for two-stage stochastic linear programs, and the nested Benders' decomposition algorithm [44] for problems with more than two stages. According to [45], one advantage of this method is that more uncertainties can be modeled and represented at the same time. In a HTSP, for example, one can consider randomness not just on the water inflows, but also in other parameters such as on electricity demand, fuel prices and other parameters, as long as a modest number of overall scenarios are considered.

An important application of a Benders-style algorithm to a HTSP is presented by [13]. Using a scenario-based method, the authors attempt to solve three- and five-stage HTSP models with two possible random realizations at each stage. The authors present the dynamic dual programming (DDP) algorithm that later was revised in [46] to be valid for problems with more than two stages. In [47] the author presents enhancements to the nested Benders' decomposition algorithm, and an application of this algorithm to HTSP at Pacific Gas and Electric Company can be found in [48].

4.2.2. Sampling-based decomposition methods

SBDAs for multi-stage stochastic linear programs are the state of the art for solving stochastic HTSP models since the origin of the first algorithm of the class. SBDA avoids the DP "curse of dimensionality" by constructing an approximation of the future cost function. The algorithm approximates the future cost function with piecewise linear functions (Benders' cuts) that are iteratively added as the algorithm proceeds. An SBDA differs from a scenario-based decomposition algorithm in that it handles scenario trees whose size is too large for a scenario-based algorithm. Sample observations of the random variables are drawn, at each time stage. An SBDA proceeds, pursuing convergence in some probabilistic sense, until it finally reaches a stopping criterion [49].

The idea to introduce sampling methods into the nested Benders' decomposition algorithm gave origin to SBDA. The first SBDA to appear in the literature is called stochastic dual dynamic programming (SDDP), presented in [14,41]. SDDP is one of the most used and well-known SBDAs, and the motivation for its development was the HTSP. Since the early 1990s SBDAs have received considerable attention from the stochastic programming community. SDDP-related algorithms such as abridged nested decomposition (AND) by [50,51], the convergent cutting-plane and partial-sampling (CUPPS) algorithm of [49] and the dynamic outer approximation sampling algorithms (DOASAs) by [42,52] were developed to improve SDDP's computational efficiency. In [51] the authors present a different sampling scheme and computational results for AND and SDDP applied to a dynamic vehicle allocation problem with uncertain demand. The AND algorithm also was applied to a HTSP in the Colombian Power System [53]. In [8] the authors present computational studies for the long-term HTSP in the Brazilian power system. A total of 10 simulation cases were solved using an SDDP implementation and different time series model considerations. The DOASA, first presented in [52] to deal with a HTSP model, was also used to solve a production planning problem for the dairy industry in New Zealand [54].

4.2.3. SBDA Optimization Process and Simulation Flow Chart

SBDAs use a master program, as the one defined in Eqs. (25)–(28), at each stage of the problem. These master programs accumulate, at each iteration of the algorithm, new cuts represented by Eq. (27) to approximately represent the future cost function. During a typical iteration of a SBDA, Benders cuts have been accumulated at each stage. These represent a piecewise linear outer approximation of the expected future cost function, i.e., $\mathbb{E}_{b_{t+1}|b_t} h_{t+1}(x_t, b_{t+1})$, at each time stage. In the master program, the Benders' representation considers θ_t for the future cost as a decision variable, which will change its value as the algorithm proceeds, due to improvement in the representation of the future cost function with the addition of new cuts. In Eq. (27), $\vec{G}_t$ and $\vec{g}_t$ represents the cut gradient matrix and the cut intercept vector that are obtained using the solution of the future stage problems, and α_t represent the dual variables associated with the cuts, for more details see [19,20].

$$\min_{x_t,\theta_t} c_t x_t + \theta_t \tag{25}$$

$$\text{s.t. } A_t x_t = B_t x_{t-1} + b_t : \pi_t \tag{26}$$

$$-\vec{G}_t x_t + e\theta_t \geq \vec{g}_t : \ \alpha_t \tag{27}$$

$$x_t \geq 0 \tag{28}$$

It is depicted in Fig. 6 a visualization of how SBDAs work. It is important to keep in mind that HTSP scenario tree sizes of interest are much larger than the one presented in Fig. 6. For example, in [29] a tree with 20 scenarios per stage with 120 stages and in [40] trees with 24 stages with up to 2000 scenarios per stage are used to represent the sampled problem.

Once a sampled scenario tree is available the SBDA starts by sampling forward paths in this tree as depicted in Fig. 6 – sub-figure 3. After the selection of paths the algorithm proceeds to the forward pass phase (Fig. 6 – sub-figure 4). In the forward pass a sequence of problems represented by Eqs. (25)–(27) at each time stage along a single forward path is solved, the cuts that have been accumulated so far are used to form decisions at each stage. In this way, the sample mean of the costs incurred along all the forward sampled paths through the tree form an estimator of the expected cost we incur by following the policy specified by the current set of cuts. Note that those sampled forward paths should be selected independently at each SBDA iteration. At the end of the forward pass we have an estimation of the total expected cost and a true lower bound for the sampled problem (solution from the first stage problem) that we can compare, test a stopping criteria and decide if the SBDA needs to run for another iteration or not.

If the algorithm does not stop at the end of the forward pass it proceeds to the backward pass presented in Fig. 6 – sub-figure 5. In the backward pass the algorithm add cuts to the collection defining the current approximation of the expected future cost function at each stage. We do this by solving the descendent nodes of each node in the linear paths solved in the forward pass phase, except in the final stage, T. In Fig. 6 – sub-figure 5 the black nodes correspond to the nodes that were selected in this iteration's forward pass. The white nodes are the additional nodes we solve as part of the backward pass in order to construct Benders cuts. Fig. 6 also shows sets of cuts corresponding to all the nodes on each stage when the scenario tree is interstage independent. It is important to mention that not all the forward paths selected before have to be solved in the backward pass, the SBDA can select a subset of them depending on the algorithm design. The number of paths selected to be solved at the backward pass can speed-up or speed-down the SBDA convergence. Once the backward pass is finished the SBDA go back to the forward pass selection in order to choose other paths in the sampled scenario tree and continue with the optimization process. Besides the SBDA solution process it is important to have in mind a process flowchart for the HTSP as the one presented in Fig. 7.

In order to construct the master programs, first it is necessary to read from a database all the power system data and perform the appropriate calculations to define de cost vector c_t, the constraint matrices A_t and B_t and the relationship between decision variables in the objective function and constraints for each stage t. From the water inflows database, the information of historical water inflows available for hydro plants are used to construct an appropriate time series model. The time series model is then to generate future scenarios of random inflows that will be used in the sampled scenario tree construction. The sampled scenario tree and the master programs are used by the SBDA to solve the stochastic problem. In Fig. 7 general steps of a SBDA-type are described, for a formal description one can refer to [19,20,41].

4.2.4. SBDAs computational time and power

The computational time required for solving HTSP models using SBDAs varies significantly depending on the test case. A solution can be found in matter of minutes for very small instances to several days for large size instances. The number of optimization problems in a scenario tree as the one depicted in Fig. 6 – sub-figure 1 increases exponentially with the number of stages and with the number of scenarios (or branches) per stage. For example, if we want to enumerate the different optimization problems in a tree with 3 stages and 2 scenarios per stage, except for the first stage which is deterministically known and represented as a single scenario, the scenario tree will have a total of 7 different optimization problems. If we increase the number of stages from 3 to 4, the number of optimization problems goes to 15; instead if we increase the number of scenarios to 3 and maintain the 3 stages the scenario tree will end up with 13 different optimization problems. In general terms we have $n = \sum_{t=0}^{T} ns^t$ as the total number of optimization problems in a scenario tree, where ns is the number of scenarios (or branches) per stage and T is the total number of stages considered in the multi-stage problem. For real practical size problems with T ranging from 12 to 120 and ns ranging from 20 to 2000 [29,40] the enumeration of all optimization problems is practically impossible and as mentioned earlier solution strategies such as SBDAs have to be used in order to approximately solve such problems. However, as T and ns grows in size the time for SBDAs to solve the problem also grows. In [40] more discussion about computational time for HTSPs of different sizes using SBDAs can be found.

Computational time to solve HTSPs also depends on the number of decision variables and constraints represented in each optimization model. The number of decision variables and constraints depends on the modeling choices, and in some sense this was earlier discussed in Section 3. Another aspect that affects the computational time to obtain a solution with SBDAs is the representation of the uncertainties in the model by the chosen stochastic process (refer to Section 4.1 and [19] for more details). Also, it is important to mention that efficient implementations of SBDAs are constructed with parallel programing technologies in order to exploit the problem structure as presented in [40,55,56] in high end modern cluster computers, which can handle a large number of processes at the same time. For example, it is possible to set the number of parallel processes to be equal to the number of forward paths selected in the scenario tree (Fig. 6 – sub-figure 3) [40].

4.2.5. Simulation, convergence properties and stopping criteria for SBDAs

Since the appearance of the first SBDA, the SDDP, convergence properties of these algorithms have been studied in the literature.

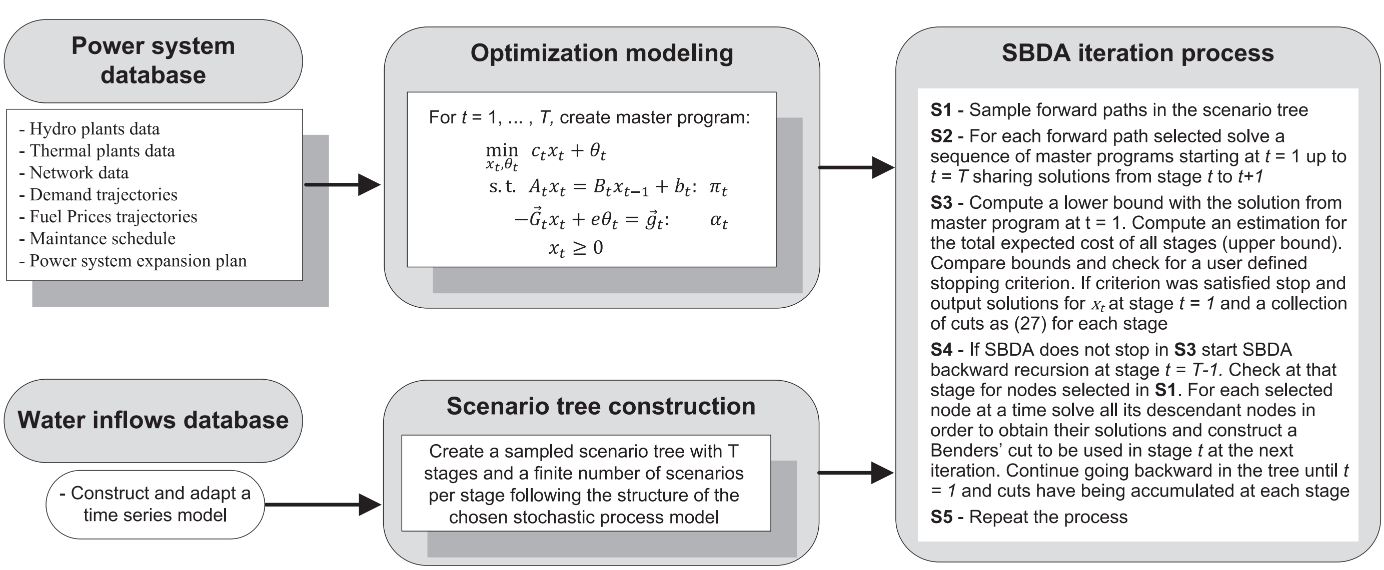

Fig. 7. Simulation flowchart for HTSP with stochastic water inflows and SBDA.

Convergence analyses and statements are shown in [49,50,52,57]. A convergence proof was presented by [49] for the CUPPS algorithm. Later, [57] extend the proof for SBDA related algorithms in general. In [52] it is presented a simpler and robust convergence proof for SBDA that holds in general, based on an idea of [50] that states: "*Finite convergence of this algorithm follows from the finite convergence of the Nested Decomposition algorithm, since the scenarios from which the optimality cuts are generated are re-sampled at each iteration*".

The work of [58] discusses statistical properties of the SDDP algorithm. In his work, considering stage-wise independence, a sample from the distribution of the original problem is taken to create a finite sample average approximation (SAA) of the true problem. SDDP is studied for this SAA problem and an extension for the risk-averse case, using conditional value at risk [59] is presented. The author states that the stopping criteria proposed by [41] and used in [51], does not guarantee correct stopping or reasonable solution quality for SBDA. He presents another stopping criterion and argues that it is more meaningful from a statistical point of view.

Stopping criteria for SDDP were also studied by [60]. The authors present the stopping criterion proposed in [41] as a hypothesis test. They argue that the original stopping criterion allows the algorithm to terminate sooner than it should, depending on the sample size chosen, without achieving good solutions. The authors then suggest a modification of the original criterion to alleviate the premature stopping issue.

Monte Carlo methods usually define the sampling schemes for SBDA [41,49,51,52]. In [60] the authors present two different sampling schemes for SBDA, randomized quasi-Monte Carlo (QMC) and the Latin hypercube sampling (LHS) schemes. The authors apply SDDP with these alternative sampling schemes to a three-year horizon HTSP and achieve more consistent operational policies than with SDDP with traditional Monte Carlo methods.

4.3. Heuristic methods

In [61] it is presented a scheme based in fuzzy decision-making methodology to decide the generation scheduling for a long-term HTSP model that considers uncertainty in system production costs, greenhouse emissions, system demand and water inflows. In [62] the HTSP is modeled as a deterministic nonlinear optimization problem, and artificial neural networks (ANN) are used in a cascade hydropower system to perform scheduling decisions for each hour of the day. A two-phase ANN optimization method is employed to decompose and solve a nonlinear version of the HTSP in [63]. An approach that combines Hopfield ANN and a heuristic rule based search algorithm is proposed for the short-term HTSP is presented in [64]. ANN is employed to solve other HTSP mathematical models as the work of [65] and to other related problems such as the hydro plant dispatch problem [66].

In [67] it is presented an algorithm based in a combination of Tabu search and generalized Benders decomposition to solve a HTSP that considers nonlinearities such as startup costs of thermal plants. An improved quantum-behaved particle swarm optimization (QBPSO) method was applied to solve a multi-objective short-term HTSP in [68]. The authors tested the QBPSO efficacy compared to other methods reported in the literature, such as differential evolution, with their HTSP model that considers active power balance constraints in addition to water balance constraints. In [69] the hydroelectric generation scheduling is solved by an ant colony system (ACS). The ACS is applied after a search space is determined for the multi-stage problem, through a collection of cooperative agents in order to obtain near optimal solutions for their model. A comparison between the application of genetic algorithm and particle swarm optimization to a fixed head short-term hydro-thermal scheduling that considers transmission losses can be found in [70].

4.4. Other topics related to the hydro-thermal scheduling problem

Research related to HTSP has produced over the years remarkable models and methodologies that can be applied to several types of problems.

4.4.1. Risk measures and optimization methods

One recent advance in the field related to HTSPs is the introduction of risk measures within SBDAs. In [58], the author proposes the addition of conditional value at risk (CVar) to multi-stage stochastic optimization problems that employs SBDAs as solution technique. The goal is to find a compromise solution between minimizing the average cost and trying to control the upper limit of the future cost functions every stage of the process. The idea presented in [58] was applied in [42] together with the DOASA to solve a HTSP model in the New Zealand power system and in [72,73] with the SDDP to solve a HTSP model in the Brazilian power system respectively. The goal of these papers is to test different risk aversion levels and compare results obtained by risk neutral policies.

One of the challenges regarding to the use of the risk-averse formulation is related to the characterization of the upper bound for the sampled problem, which is not well defined as discussed in [74,75]. The authors in [74] propose a new approach based in importance sampling in order to improve the poor performance of the upper bound estimator; the methodology is tested for an asset allocation problem. In [75] it is presented an approach that combines CVar with SDDP in the context of long-term power generation planning problem for the Brazilian system. The goal to use CVar in such framework is to avoid large amounts of load curtailment in critical inflow scenarios. The authors present a case study of the modeling/solution procedure application in order to specify CVar parameters and obtain a reasonable trade-off between system security and generation costs.

4.4.2. Other constraints and modeling developments

In [15] the author presents a linear programming approach to consider multiple uses of the water in the short-term HTSP by designing river stream level constraints and river-routing effects. These constraints restrict the minimum and maximum values or maximum hourly/daily variations in the level of the river at specific points. By using stream level constraints things such as ship navigation, fishing, and other environmental concerns may be represented in river courses. In the modeling part it is necessary to include new variables to HTSP models such as river levels. Other recent development related to the stochastic HTSP is the consideration of CO_2 emission constraints in [76] with a study case considering Guatemala power system. The authors consider costs related to CO_2 emissions originated from the electricity production of a thermal plant in the objective function to minimize these contributions. Emission constraints were designed in order to capture annual restrictions and therefore new decision variables had to be created to represent a pseudo reservoir for emissions (new state variable). This new formulation impacts the Benders cuts formation from SBDAs because of this new state variable existence. The work of [77] presents an addition of transmission network constraints in HTSP modeling due to the distance of several hydro plants in a power system from the load center. Their work considers Lagrangian relaxation method to solve a model with nonlinearities between hydro generation and tail racing levels and between forebay level of reservoirs and hydro reservoirs volumes. In [71,78] similar nonlinearities are considered in the

HTSP and DC power flow formulation is used in the modeling process.

The integration of other renewable sources such as wind and solar in power systems also demand new developments in the study of HTSP. The wind integration in hydro-thermal systems has already been subject of several studies [77,79-81]. In the HTSP literature, wind farms are considered to be similar to the run-of-river hydro plants, with both encompassing variable generating sources and thus being dependent on the availability of their respective primary energy source [82]. In [35] the authors study the wind-HTSP with stochastic water inflows and wind speed to exploit the complementarity behavior of these two sources in the scheduling of a small power system. It is proposed in [83] the addition of a constraint related to wind power uncertainty in a HTSP nonlinear model; the authors use a particle swarm optimization algorithm to solve such model.

There are only a few studies in the literature considering solar generation together with HTSP. The power output from a photovoltaic solar plant is intermittent and depends on the randomness of sunlight. Similar to wind farms generation, the generation from solar plants present challenges to the operation of power systems but also present benefits. It is presented in [84] the use of pumped storage hydro to improve the reliability in a system with large share of photovoltaic generation. The authors propose a cooperative scheduling method for the pumped hydro, solar and thermal power plants that make possible to improve both reliability and cost minimization in a system. In [85] it is presented a stochastic optimization model and Quasi-Monte Carlo simulation method to represent scenarios for the variability of wind and solar in the medium-term generation planning problem.

5. Conclusion

We have presented an overview about the hydro-thermal scheduling problem. The HTSP is a challenging problem that is often modeled in the literature as a large-scale multi-stage stochastic program. Basically, there are two main representations of such a problem; one representation of the HTSP is with the modeling resolution of individual hydro plants and stochastic inflows of water, the other approach uses an energy-based ARR with stochastic inflows of energy. The representation with individual hydro plants can be more precise since we can better represent the relationship of the hydro plants in each river basin. Also, this representation uses forecasting models that can exploit local predictors to forecast the stochastic water inflows. But the computational effort to solve such a model grows with the number of hydro plants, the level of representation of the system details, the number of stages and the branches in the scenario tree. On the other hand, the representation of the problem using the aggregate reservoir scheme is more appealing from the computational point of view, since the number of decision variables and constraints shrinks considerably for large systems. However applications of the ARR are often tied to the forecasts of energy inflows that may not represent well the behavior of the water inflows at each hydro plant.

The class of sampling-based decomposition algorithms that are used to solve HTSP in the literature was described in this work. We presented the main ideas behind the iteration process of SBDA and discussed specific characteristics of such class of algorithms that still to the date the state of the art for solving multi-stage stochastic optimization models. It was mentioned about the risk-averse methodologies that recently became to be used together with SBDAs. Several branches of current research related to HTSP were pointed out such as: integration of other renewables, addition of CO_2 emissions and multiple water uses.

Acknowledgment

A portion of this work is part of the first author's Ph.D. dissertation research, which was funded by the Capes/Fulbright Program through process 2800-06-3/2007 and Grant 15073707. The author would like to thank Professor David P. Morton for relevant discussions and for his contributions to the author's research. The author also would to thank the Editor-in-Chief Lawrence L. Kazmerski and the anonymous referees for their comments on the initial version that helped to construct this improved manuscript.

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