ON THE SOLUTION QUALITY ASSESSMENT IN MULTI-STAGE STOCHASTIC OPTIMIZATION UNDER DIFFERENT MODEL REPRESENTATIONS

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Overview

- Introduction
- Power Generation Scheduling & Optimization
  - Hydro-thermal scheduling (HTSP)
  - Remarks about model formulation
  - HTSP as T-stage stochastic linear program (SLP-T)
- SBDA & Solution Quality Assessment
  - A Sampling-based Decomposition Algorithm
  - Solution Quality Assessment in SLP-T
- Case Study
- Future Directions & Final Comments

Introduction

Motivation

- **Renewable power** sources became a key aspect around the world by **disrupting old frontiers**
- These energy sources are linked to **sustainable development** that is one of the main goals of the modern society these days
- **The raise of renewable power installed** capacity demand studies about its effects
- **Power generation scheduling** is one of such studies and is our focus

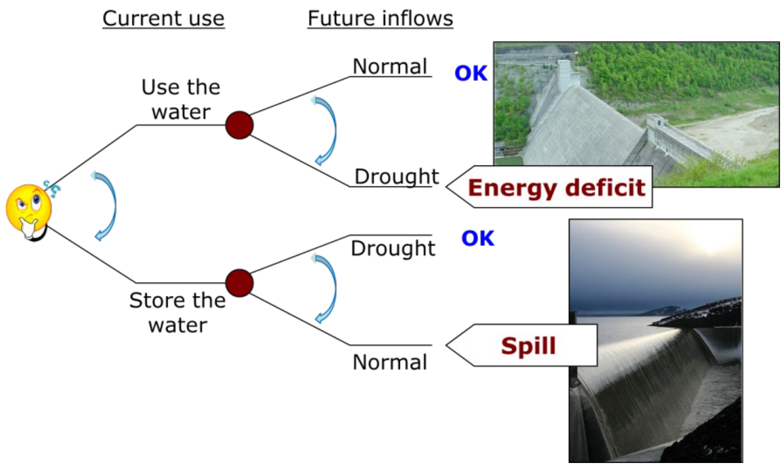
Background & Goals

- The main problem with renewable power is its dependence on natural resources (**may not be available when necessary**)
- Hydropower is an exception of these restrictions, since **reservoirs can store water and control generation**
- We present the idea behind the classical **hydro-thermal scheduling problem (HTSP)** with ≠ model formulations
- We describe a **Sampling-based Decomposition Algorithm (SBDA)** and apply it to approximately solve multi-stage stochastic programs
- In this case, it is important to **assess the solution quality** that can be obtained from the resulting policy applied to out-of-sample paths and scenario trees (**under ≠ model formulations & sizes**)

Power Generation Scheduling & Optimization

Hydrothermal Scheduling Problem

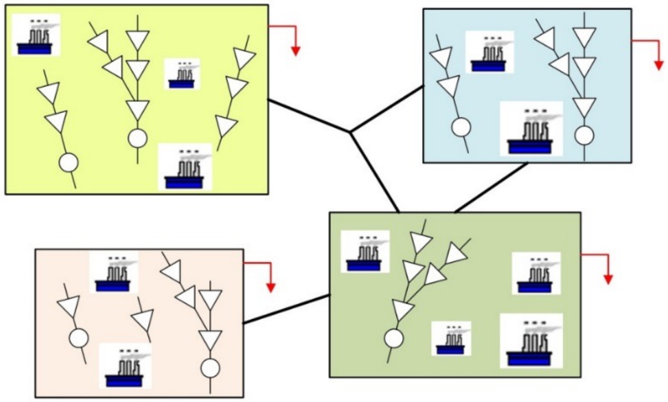
- Find the sequence of **hydro releases** and **thermal plant dispatches** for a planning horizon in order to match system demand
  - Resource management
  - Input variable forecasting
  - Operational aspects



- Basic economic criterion
  - **Minimize operational costs** (present + expected future)
- Multi-stage Stochastic Linear Program (SLP-t)

Variables & Parameters

- Objective is to minimize total expected cost to operate the system:
  - Fuel costs for generating thermal power
  - Penalties for failure to meet demand
- Decision variables for each hydro plant, includes:
  - **Hydro generation** GH_i^t
  - **Spilled volumes** S_i^t
  - **Storage** (water or energy) x_i^t
- Other decision variables:
  - **Thermal generation** GT_ℓ^t
  - **Energy transfers between regions** $\mathrm{F}_{r\,r'}^t$
  - **Load curtailment** GD_k^t
- Uncertainty:
  - Future water inflows $b_t, b_{t+1}, \ldots, b_T$



HTSP Model Formulation for Stage-t

Present Cost **Expected Future Cost**

$$h_t(x^{t-1}, b_t^{\omega}) = \min \underbrace{\sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t}_{\text{Present Cost}} + \underbrace{\frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\dots,b_t} h_{t+1}(x^t, b_{t+1})}_{\text{Expected Future Cost}}$$

Water Balance

$$\text{s.t.} \quad x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} \left(GH_j^t + S_j^t \right) = x_i^{t-1} + b_{t+1}^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr} \quad \forall r \in R$$

Simple Bounds

$$\underline{x}_i^t \leq x_i^t \leq \overline{x}_i^t \quad \forall i \in I$$

$$0 \leq GH_i^t \leq \overline{GH_i^t} \quad \forall i \in I$$

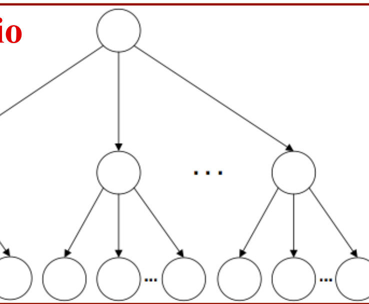
$$0 \leq S_i^t \quad \forall i \in I$$

$$\underline{GT_\ell^t} \leq GT_\ell^t \leq \overline{GT_\ell^t} \quad \forall \ell \in L$$

$$0 \leq GD_k^t \quad \forall k \in K$$

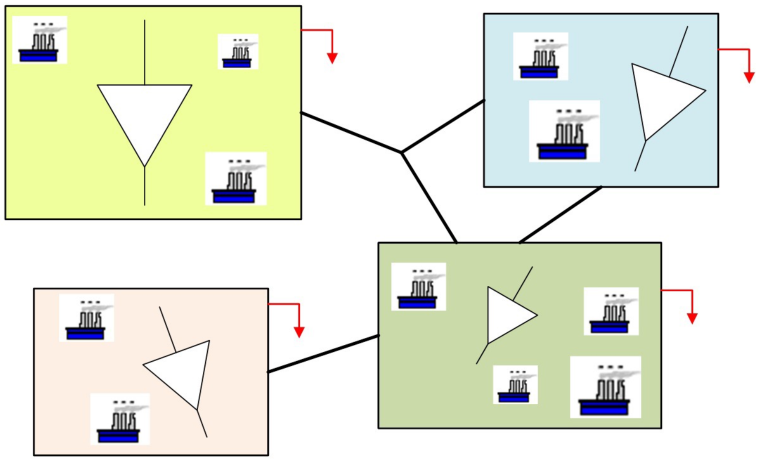
$$0 \leq F_{r\,r'}^t \leq \overline{F_{r\,r'}^t} \quad \forall (r, r') \in R$$

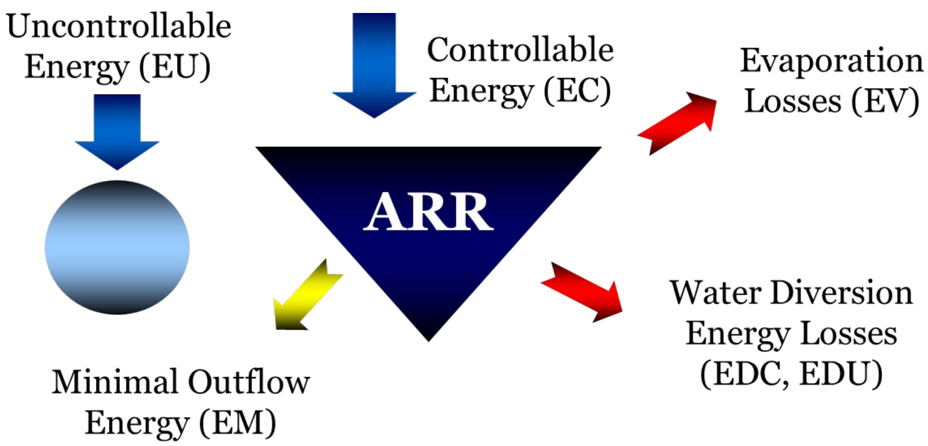
Scenario Tree



Remark 1: Formulation and Model's Size

- The model's size at each stage t and branch ω depends on:
  - **# of hydro plants** → **For each hydro plant:**
    - **3 sets of decision variables**
    - **1 set of structural constraints**
  - # of thermal plants
  - # of electrical regions (subsystems)
- In order to reduce model's size → **Aggregate Reservoir Representation**





In Terms of HTSP with ARR:

- Original HTSP:

Water Balance

$$x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} (GH_j^t + S_j^t) = x_i^{t-1} + b_{t+1}^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr} \quad \forall r \in R$$

- ARR by Subsystem:

Energy Balance

$$x_r^t + GH_r^t + S_r^t = x_r^{t-1} + b_{t+1}^{\omega} \quad \forall r \in R$$

Demand Satisfaction

$$GH_r^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr}^{Net} \quad \forall r \in R$$

- ARR by River basin:

Energy Balance

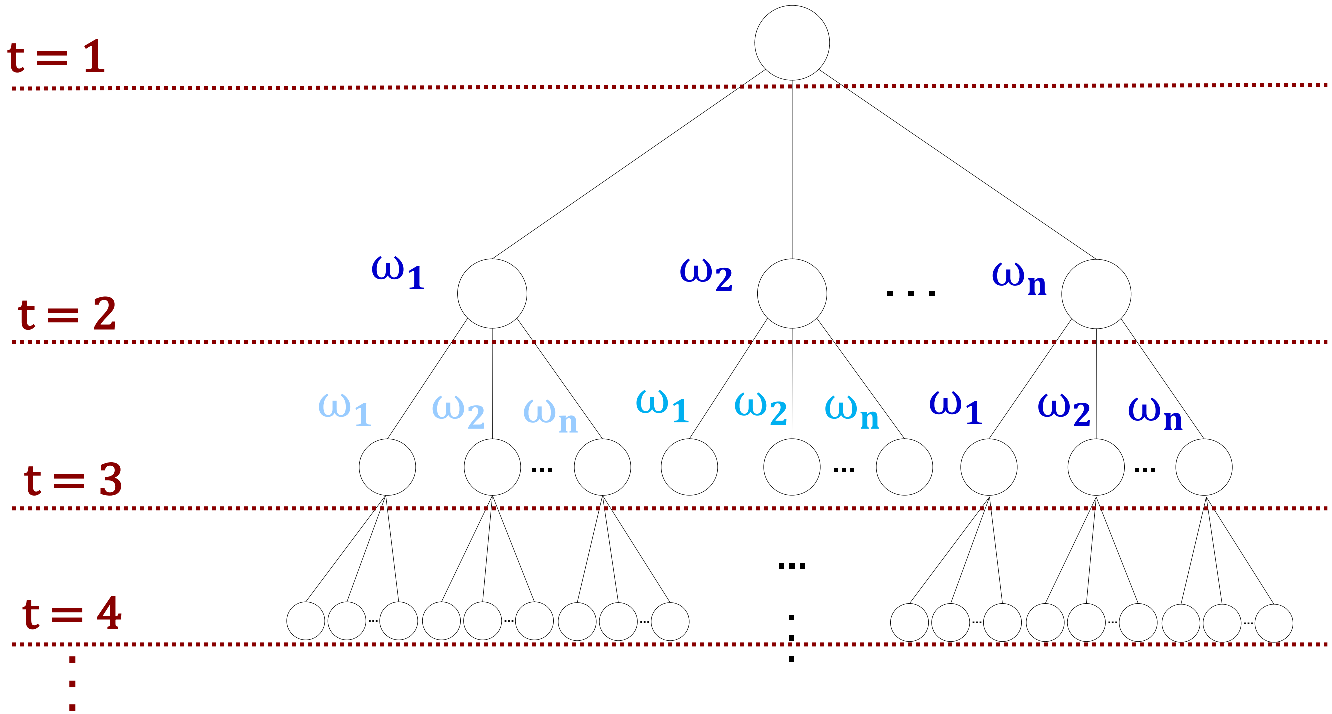
$$x_n^t + GH_n^t + S_n^t = x_n^{t-1} + b_{t+1}^{\omega} \quad \forall n \in N$$

Demand Satisfaction

$$\sum_{n \in N_r} GH_n^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D'^{Net}_{tr} \quad \forall r \in R$$

Remark 2: Tree Density and Model's Size

- The model's size for the whole time horizon depends on:
  - **# of time stages**
  - **# of scenarios (branches) per stage**



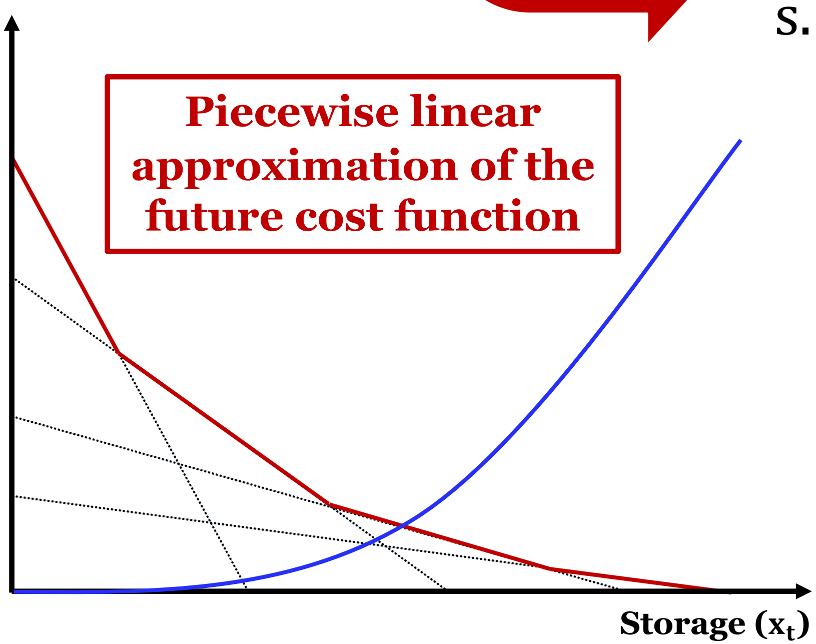
HTSP as SLP-t

We consider a general model that uses water inflow forecasts

$$\min_{x_1} \quad c_1 x_1 + E_{b_2|b_1} h_2(x_1, b_2)$$
$$\text{s.t.} \quad A_1 x_1 = B_1 x_0 + b_1$$
$$x_1 \geq 0$$

where, for $t = 2,\ldots,T$

$$\min_{x_t} \quad c_t x_t + E_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x_2, b_{t+1})$$
$$\text{s.t.} \quad A_t x_t = B_t x_{t-1} + b_t$$
$$x_t \geq 0$$



x_t: stage t decision variables including: hydro generation, storage, spillage, thermal generation, energy transfers, load curtailment

A_t: constraint matrix including mass balance, demand satisfaction

b_t: stochastic water inflow at each hydro plant and deterministic demand

$B_t x_{t-1}$: storage from last stage

A Sampling-based Decompostion Algorithm

Sampling-based Decomposition Algorithms

- Introduce sampling methods into nested Benders' decomposition
- Algorithm first presented by Pereira & Pinto 1991
- SDDP evolved from Pereira & Pinto 1985 work
  - 1985: **3- and 5-stage problems with 2 inflow realizations per stage**
  - 1991: Monte Carlo sampling → create a SAA with inflow sequences and solve it. SDDP shown for a **10-stage problem with 2 inflow realizations per stage**. Total of **$2^9 = 512$ possible inflow sequences**
- **Related algorithms** (Philpott & Guan 2008, Philpott & de Matos 2010, Chen & Powell 1999, Donohue & Birge 2006); **Convergence analysis** (Chen & Powell 1999, Linowsky & Philpott 2005, Philpott & Guan 2008); **Cut-sharing** (Infanger & Morton 1996, de Queiroz & Morton 2013); **Alternative sampling schemes** (Homem-de-Mello et al. 2011); **Risk aversion** (Shapiro 2010, Philpott & de Matos 2010, Guigues & Sagastizabal 2012, Komizk & Morton 2015, Maceira et al. 2015); **Solution Quality Assessment** (Chiralaksanakul & Morton 2004, de Queiroz 2011, de Mattos et al 2016), **Other Modeling Issues** (Rebennack et al. 2012, Diniz & Souza 2014) …

Stage-t Benders' Master Problem

- Suppose we are at stage t under ω and we have:

$$\min_{x_t,\theta_t} \; c_t x_t + \theta_t$$
$$\text{s.t.} \quad A_t x_t = B_t x_{t-1} + b_t \; : \pi_t$$
Benders' cuts → $$-\vec{G}_t x_t + e\,\theta_t \geq \vec{g}_t \; : \alpha_t$$
$$x_t \geq 0$$

$$\max_{\pi_t,\alpha_t} \; \pi_t (B_t x_{t-1} + b_t) + \alpha_t \vec{g}_t$$
$$\text{s.t.} \quad \pi_t A_t - \alpha_t \vec{G}_t \leq c_t$$
$$e^T \alpha_t = 1$$
$$\alpha_t \geq 0$$

$$b_t = R_{t-1} b_{t-1} + \eta_t$$

$\text{vec}(\eta_t, c_t, B_t, A_t), t = 2, \ldots, T$ **are** $\perp\!\!\!\perp$

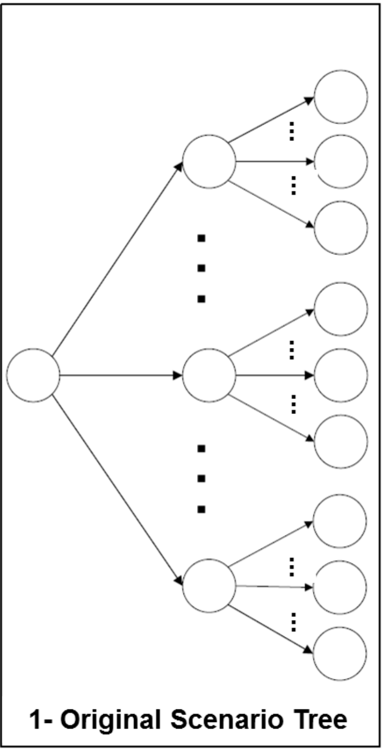
$$G_t = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} B_{t+1}$$ → **cut-gradient matrix**

cut-intercept vector ↓

$$g_t^{\omega_t} = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} b_{t+1}^{\omega_{t+1}} + \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \alpha_{t+1}^{\omega_{t+1}} \vec{g}_{t+1}^{\omega_{t+1}}$$

may have interstage depedency

SBDA Optimization Process

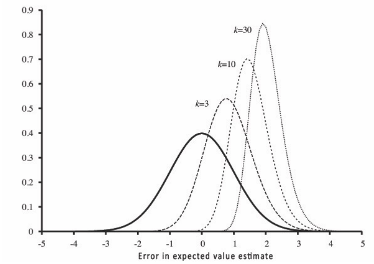


Solution Quality Assessment

- We use Monte Carlo simulation to assess if a candidate solution (i.e., policy) is near optimal

cannot solve the SP exactly

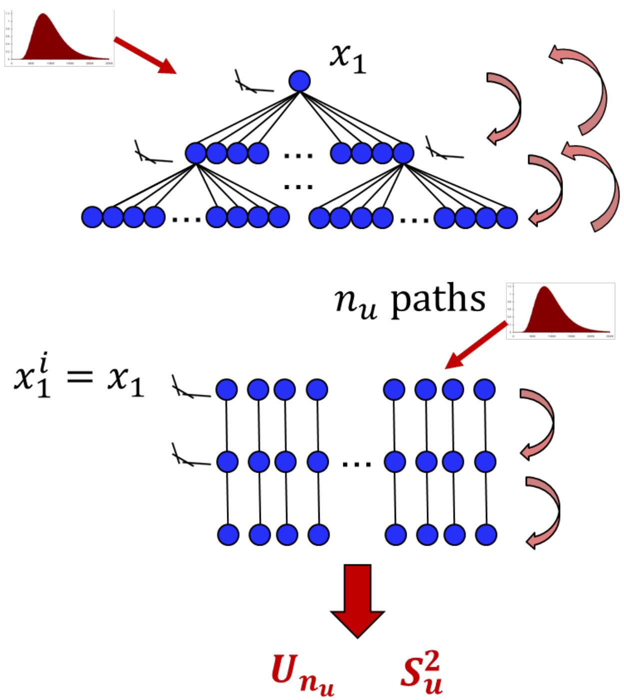
- When optimizing a sample-mean estimator we get an optimistic bound for **the solution**



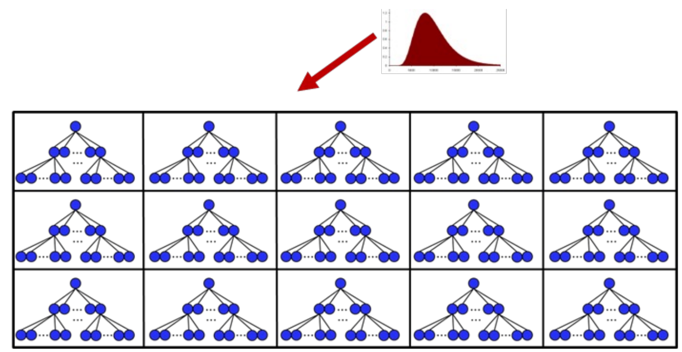
- This implies a weak statement regarding quality of a candidate solution ⟶ **Estimate may have large bias**
- When bias is large it is not possible to be sure if a candidate solution is near optimal

Confidence Interval Construction

Upper bound estimator (UBE)



Lower bound estimator (LBE)



- do $i = 1, \ldots, n_\ell$

Run SBDA and compute $\underline{z}^i$

Let $\epsilon_\ell = t_{n_\ell - 1} S_\ell / \sqrt{n_\ell}$ and $\epsilon_u = z_\alpha S_u / \sqrt{n_u}$

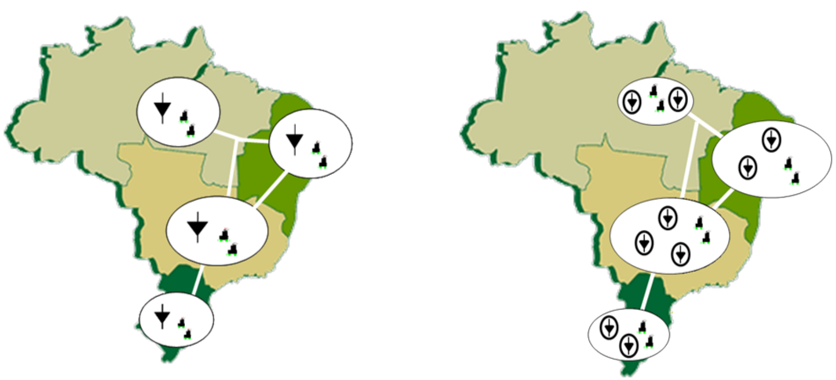
Output one-sided **CI** on $\mathbb{E}\boldsymbol{U} - \boldsymbol{z}^*$, $\left[0\,,\left(U_{n_u} - L_{n_\ell}\right)^+ + \epsilon_\ell + \epsilon_u\right]$

Based on: Chiralaksanakul & Morton 2004, de Queiroz 2011, de Mattos et al 2016

Case Study

Applied to a Portion of the Brazilian System

- Optimization over 6, 12 & 24 monthly stages
- Aggregated by subsystem (Cepel 2011 & de Queiroz 2011) & by river basins (de Mattos 2008 & Pietrafesa 2015)



For more details: Pietrafesa 2015

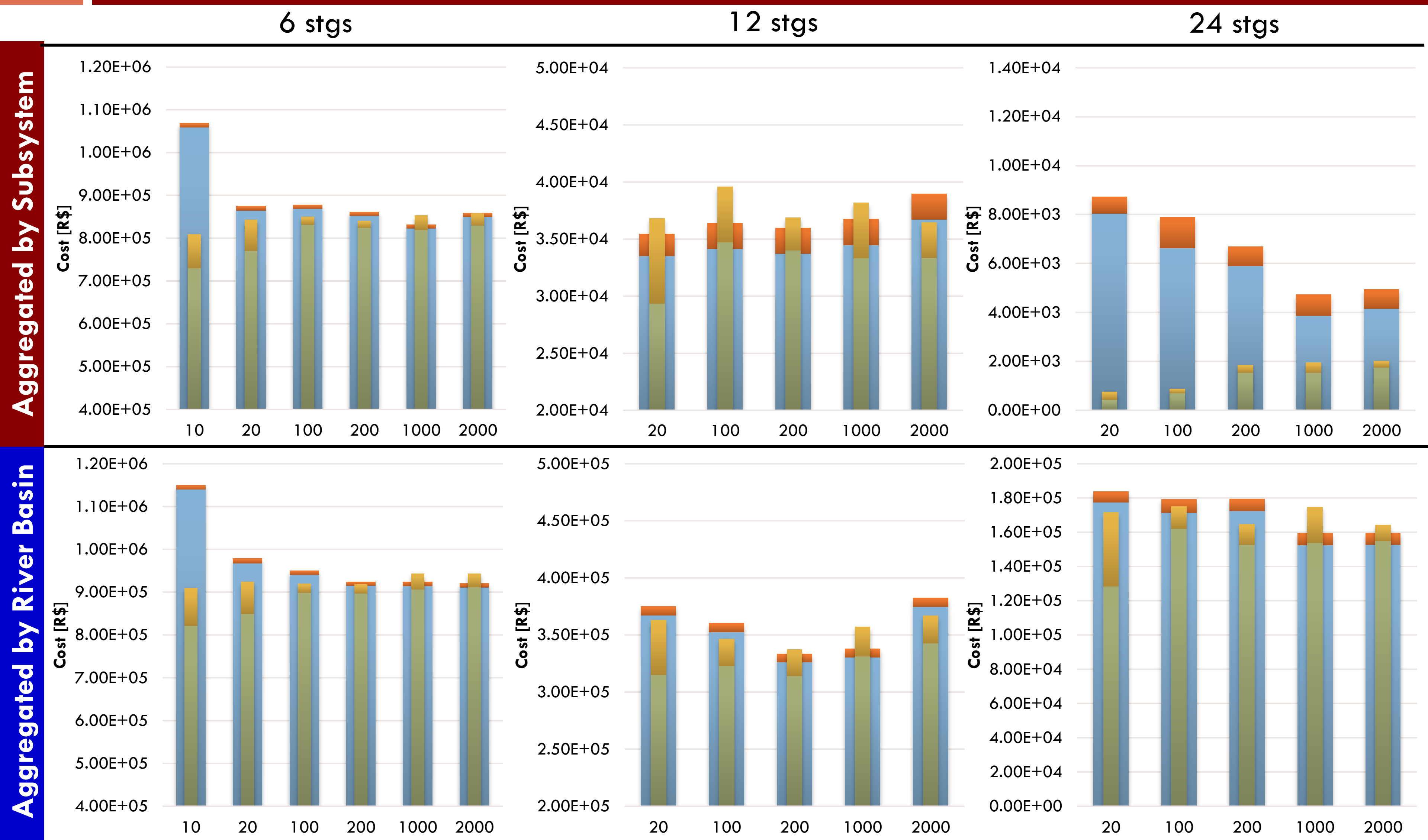
	SE/Central	South
Maximum Energy Storage [aGW]	**Tietê** 7.2	**Iguaçu** 10.1
	Grande 30.3	**Uruguai** 5.1
	Paranaíba 39.0	
Max Hydro Gen	21.4 [GW-month]	11.0 [GW-month]

- 64 hydro and 19 thermal plants (with 5.6 [aGW], where 8 are located in the SE and 11 in the South)
- Time- & spatial-dependent water inflow forecasts produced by a DLM (Marangon Lima et al. 2014)
- We consider different sample sizes for the same problem instance to analyze solution results

Simulation Assumptions

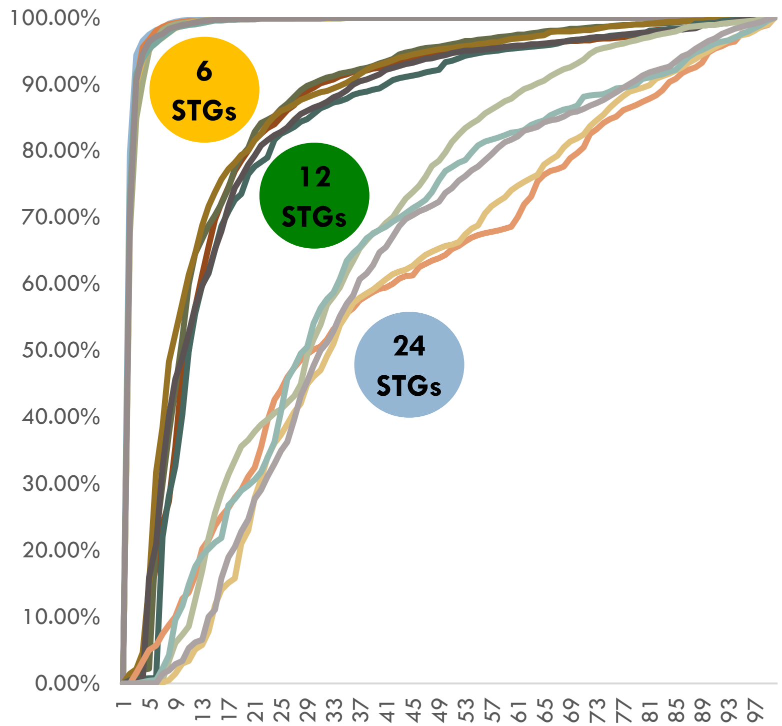
- We run SDDP until when the LB obtained in the first tree stabilizes or for a maximum number of iterations
- We vary the number of scenarios per stage
- We consider 32 cuts to be computed at each iteration
- We use 15 trees to assess the LBE
- We consider 12800 forward paths to evaluate UBE
- Initial reservoir levels: 60%
  - This is also a requirement as end of time constraint

UB & LB Results

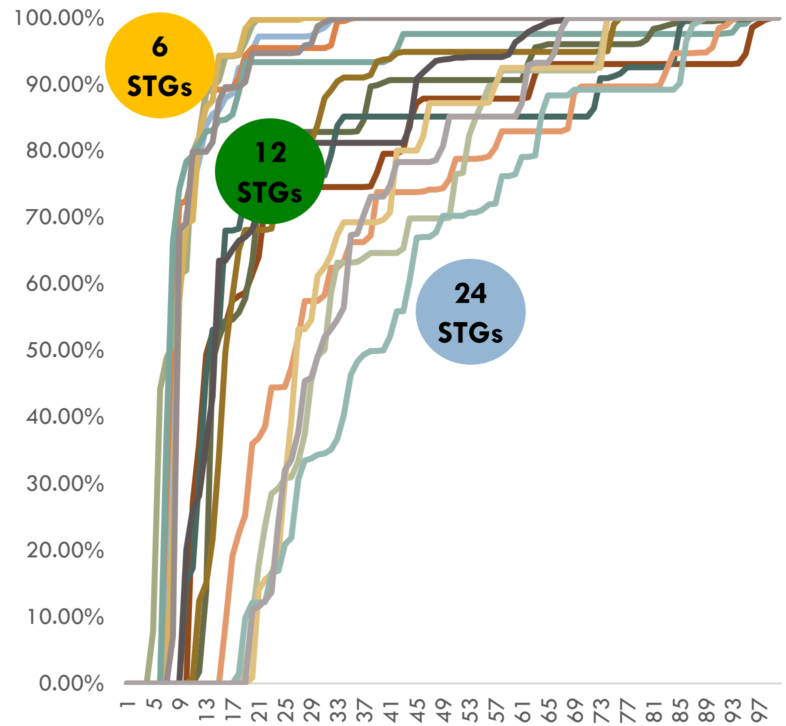


1st Tree Lower Bound Values

Aggregated by Subsystem



Aggregated by River Basin



Lower bound of the 1st tree stabilizes earlier in the system with ARR for each subsystem

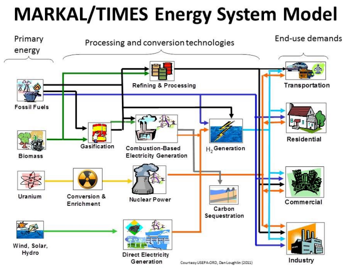
Final Thoughts and Future Directions

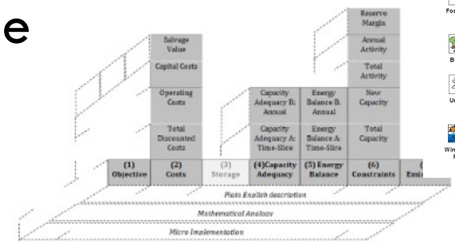
Final Thoughts & Future Directions

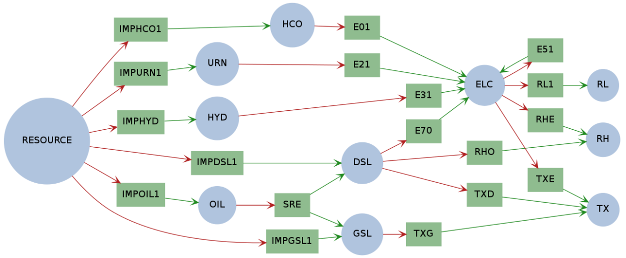
- We presented the idea behind the **HTSP** along with a discussion about model formulations
- The main **structure of a SBDA** was discussed
- **Solution quality** assessment in SLP-t's was addressed and it is an important research in stochastic programming
- The results presented here and in the literature shows the benefits of using such procedure to obtain better solutions
- There is motivation to explore the use of **SBDA and solution quality assessment in other** multi-stage stochastic optimization **problems**

SDBA & Sol. Quality Assessment in Capacity Expansion Models for Energy Systems

- Models for conducting energy system analysis:
  - Markal/Times, OSeMOSYS, Message
- TEMOA (Hunter et al. 2013)
  - Energy economy optimization model
  - Technology assessment and policy analysis at ≠scales
  - Model is implemented in a general algebraic formulation combined with
  - Stochastic Programming capabilities (extensive LP and Progressive Hedging)
  - Represents a multi-stage problem in a network with multiple technologies and multi-commodities







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THANK YOU !

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