On a Sampling-based Decomposition Algorithm Under Aggregate Inter-stage Dependency Model

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Outline

- Hydrothemal Scheduling Problem
- Model Formulation
- SBDA Multi-stage Scheme
- Cut-Sharing Under Dependency Models
- Future Work

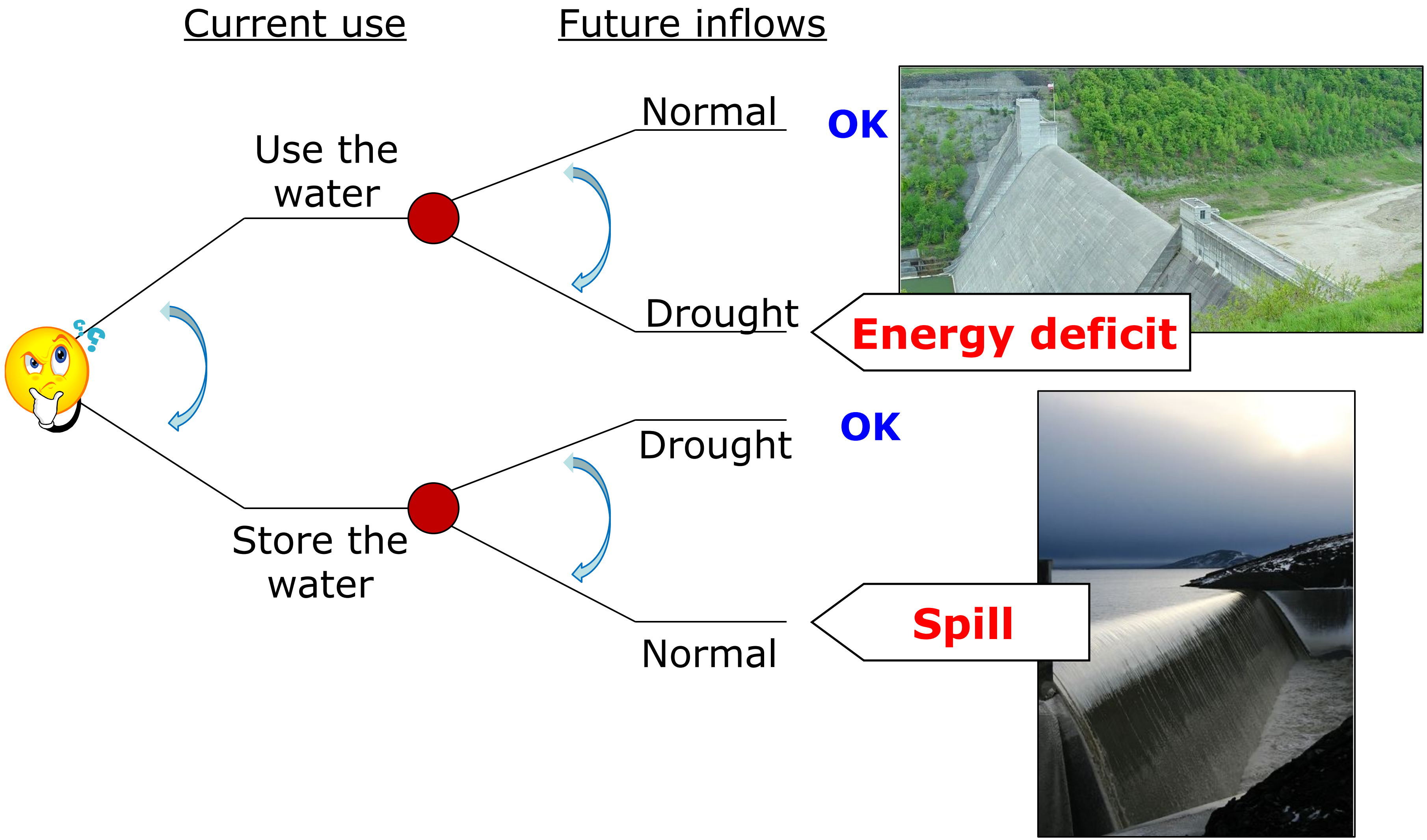
Introduction

- Hydroelectricity is inexpensive to produce
- Depends on the supply of water (stochastic)
- Present decisions affect future conditions of the system and also future decisions (dynamic)
- Multiple interconnected reservoirs, transmission constraints and multi-period optimization (large-scale)

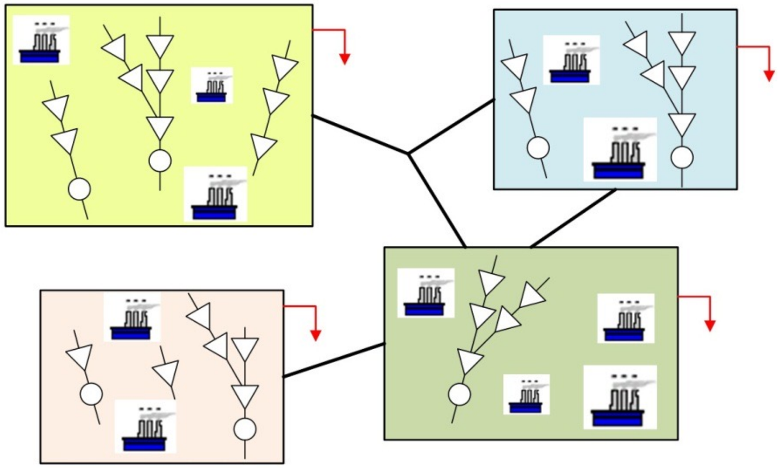
Hydrothermal Scheduling Problem

- Find the sequence of hydro releases and thermal plant dispatches for a planning horizon in order to match system demand
  - Resource management
  - Input variable forecasting
  - Operational aspects
- Basic economic criterion
  - Minimize operational costs (present + future)

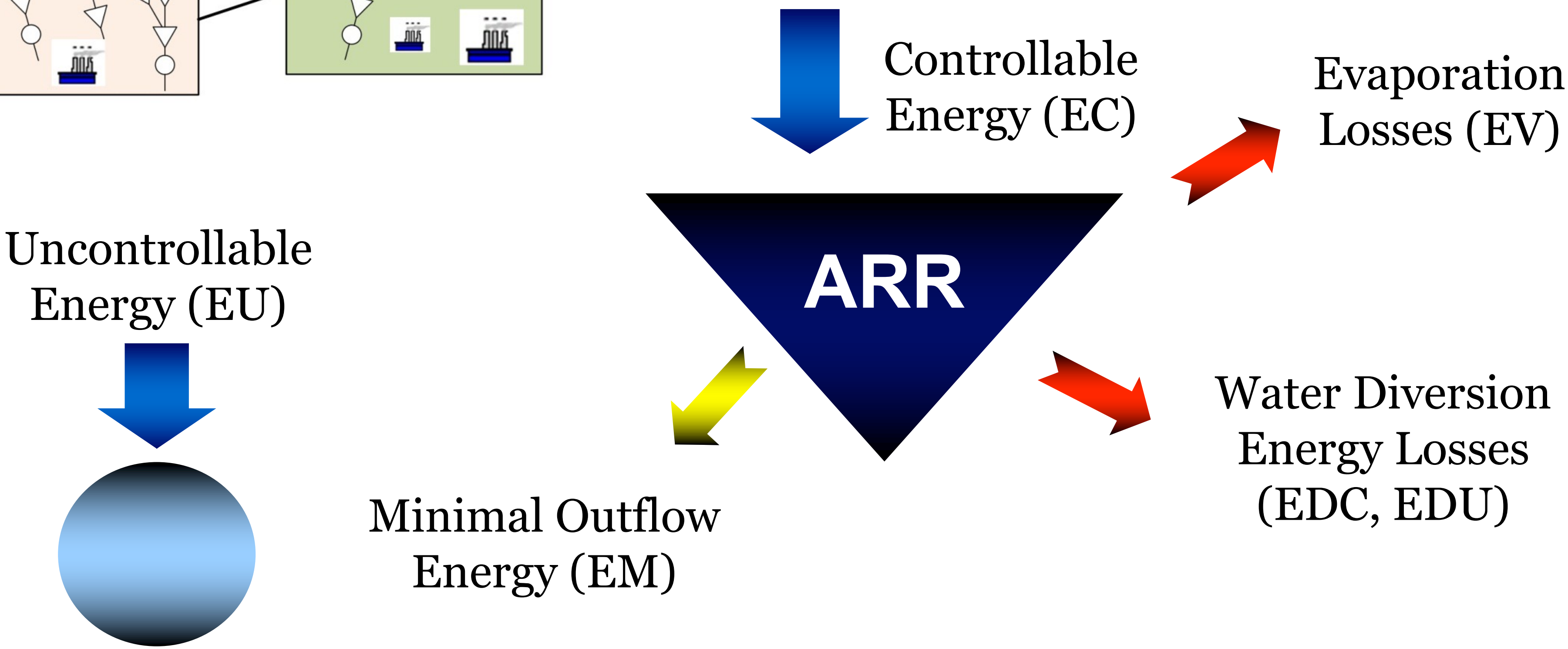
Decision Tree



Aggregate Reservoir Representation



- Aggregate hydro plants in a region to create unique ARRs
- Reduce model's size



Water Inflow Vs. Energy Inflow

- Arguments for forecasting **water inflows**:
  - **Exploit local predictors**
    - Precipitation
    - El ninõ
    - Soil use
  - Are **measurable**
  - **Unaffected** by the hydro system **configuration**
- Problems when forecasting **energy inflows**
  - Ties model of a natural process to the decision process
  - **Harder** to validate
  - **Affected** by the hydro system **configuration**

Variables & Parameters

- Objective is to minimize total expected cost to operate the system:
  - *Fuel costs for generating thermal power*
  - *Penalties for failure to meet demand*
- Decision variables for each ARR, includes:
  - ***Hydro generation*** $gh_{i,k}^{t}$
  - ***Spilled volumes*** s_i^t
  - ***ARR energy storage*** x_i^t
- Other decision variables:
  - ***Thermal generation*** $gt_{g,k}^{t}$
  - ***Energy transfers between regions*** $p_{i,j,k}^{t}$
  - ***Load curtailment*** $u_{l,k}^{t}$
- Uncertainty: future water inflows b_t , b_{t+1} , ... , b_T

Stage t Model Formulation with ARR

$$h_t(x^{t-1}, b_t^\omega) = \min \sum_{i \in I} \sum_{k \in K} \left[\sum_{g \in G_i} c_g^t gt_{g,k}^t + \sum_{\ell \in L_i} \rho_\ell^t u_{\ell,k}^t \right] + (1+\beta)^{-1} \mathbb{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x^t, b_{t+1})$$

energy balance

$$\text{s.t.} \quad x_i^t + \sum_{k \in K} gh_{i,k}^t + s_i^t = \boxed{f_1^t(x_i^{t-1}, b_{i,t}^\omega)} \qquad \forall i \in I$$

$$gh_{i,k}^t + \sum_{g \in G_i} gt_{g,k}^t + \sum_{\ell \in L_i} u_{\ell,k}^t - \sum_{j:(i,j) \in E} p_{i,j,k}^t + \sum_{j:(i,j) \in E} p_{j,i,k}^t - y_{i,k}^t = \boxed{f_{2,k}^t(x_i^{t-1}, b_{i,t}^\omega)} \quad \forall k \in K, \forall i \in I$$

demand satisfaction

$$gh_{i,k}^t - y_{i,k}^t \le \boxed{f_{3,k}^t(x_i^{t-1}, b_{i,t}^\omega)} \qquad \forall k \in K, \forall i \in I$$

max hydro generation

$$\sum_{i:(i,j) \in E} (p_{i,j,k}^t - p_{j,i,k}^t) = 0 \qquad \forall k \in K, \forall i \in I^+ \setminus I$$

$$gh_{i,k}^t,\ s_i^t,\ y_{i,k}^t \ge 0 \qquad \forall k \in K, \forall i \in I$$

$$0 \le gt_{g,k}^t \le \overline{GT}_{g,k}^t \qquad \forall k \in K, \forall g \in G$$

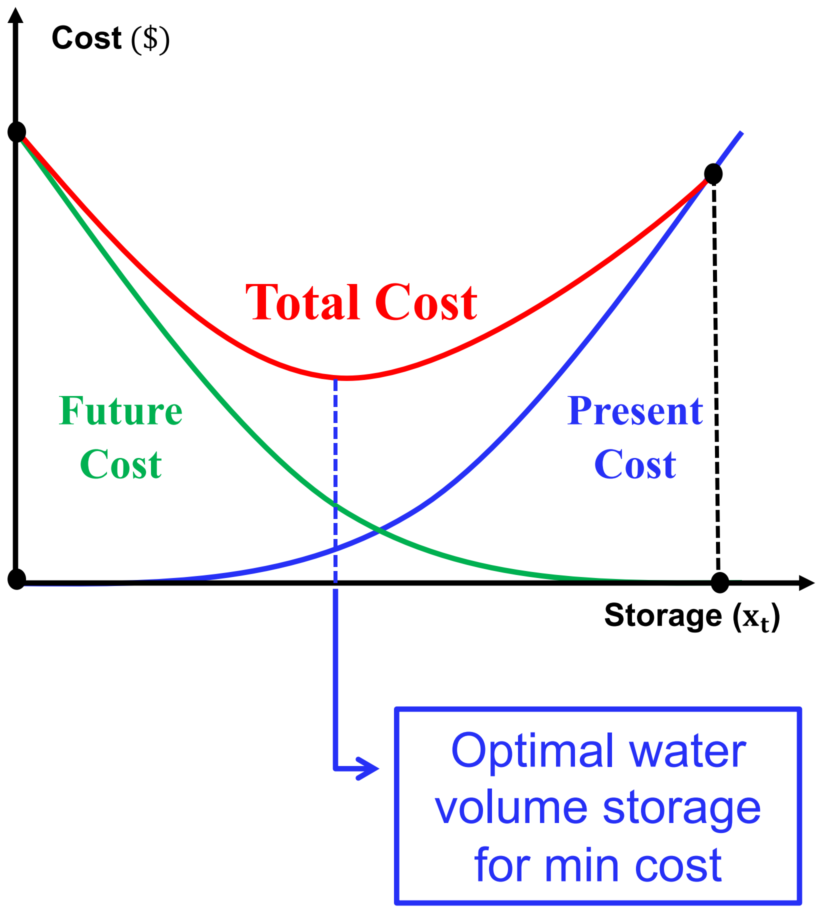
$$\underline{p}_{i,j,k}^t \le p_{i,j,k}^t \le \overline{p}_{i,j,k}^t \qquad \forall k \in K, \forall (i,j) \in E$$

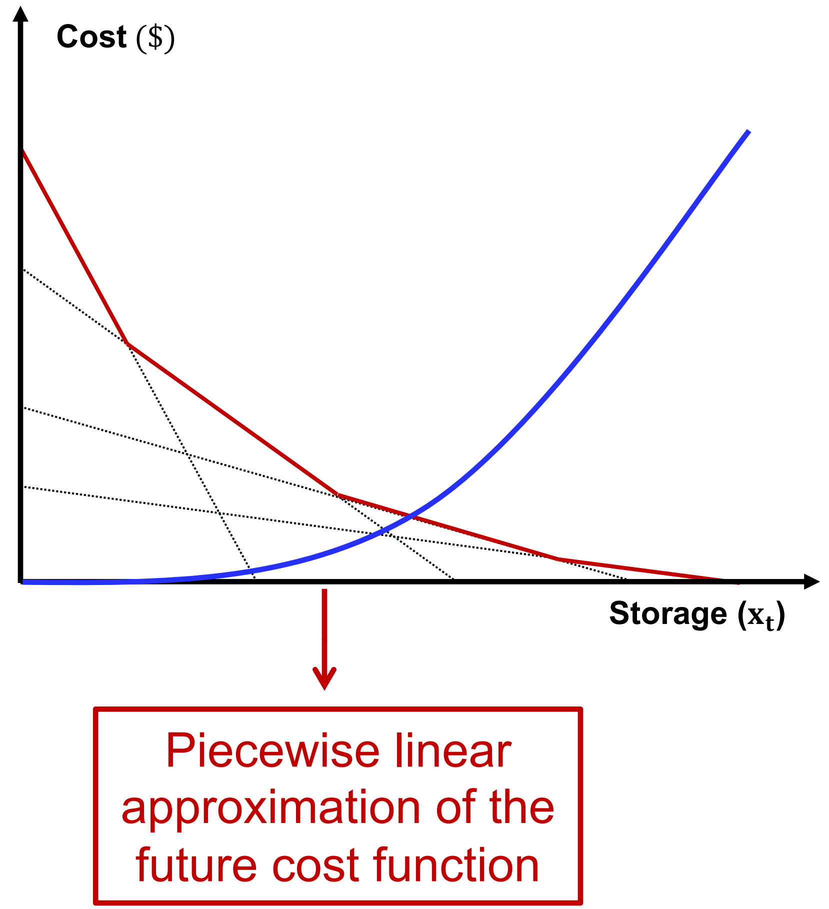
$$0 \le u_{l,k}^t \le \overline{u}_{l,k}^t \qquad \forall k \in K, \forall l \in L$$

$$\underline{x}_i^t \le x_i^t \le \overline{x}_i^t \qquad \forall i \in I$$

Problem Objective

- Minimize total operational costs (present + future)





Brief Survey

- Introduce sampling methods to nested Benders' decomposition algorithm created the first Sampling-based decomposition algorithm **(SBDA)**, the **SDDP (Pereira & Pinto 91)**
- Since then SBDA has received considerable attention, **DOASA**, **CUPPS**, **Abridged Nested Decomposition**
- **Cut sharing** procedure for inter-stage dependency models **(Infanger & Morton 1996)**
- **Statistical properties** & **risk measures** **(Shapiro 2010)**
- **Alternative sampling** **(Homem-de-Mello et al. 2011)**

A General SLP-T

We consider a model that uses water inflow forecasts instead of energy

$$\begin{aligned} \min_{x_1} \quad & c_1 x_1 + \mathrm{E}_{b_2|b_1} h_2(x_1, b_2) \\ s.t. \quad & A_1 x_1 = B_1 x_0 + \boldsymbol{\rho_1} b_1 + k_1 \\ & x_1 \geq 0 \end{aligned}$$

where, for $t = 2,\ldots,T$

$$\begin{aligned} \min_{x_t} \quad & c_t x_t + \mathrm{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x_2, b_{t+1}) \\ s.t. \quad & A_t x_t = B_t x_{t-1} + \boldsymbol{\rho_t} b_t + k_t \\ & x_t \geq 0 \end{aligned}$$

x_t: all stage t decision variables including: hydro generation, hydro storage, spillage, thermal generation, energy transfers, ...

A_t: constraint matrix including energy balance, demand satisfaction, ...

b_t: stochastic water inflow at each hydro plant

$\boldsymbol{\rho_t}$: matrix to transform water into controllable and uncontrollable energy inflows

$B_t x_{t-1}$: storage from last stage, energy parameters that depend on storage

$\boldsymbol{k_t}$: deterministic demand, constant energy parameters

Stage t Benders' Master Problem

- Suppose we are at stage t under ω_t and we have:

$$b_t = R_{t-1} b_{t-1} + \eta_t$$

$$\begin{aligned}
\min_{x_t, \theta_t} \quad & c_t x_t + \theta_t \\
s.t. \quad & A_t x_t = B_t x_{t-1} + \rho_t b_t + k_t \quad : \pi_t \\
& -\vec{G}_t x_t + e\,\theta_t \ge \vec{g}_t \quad : \alpha_t \\
& x_t \ge 0
\end{aligned}$$

where,

(# of individual hydro plants)

ρ_t: matrix with m_t rows and q_t columns

R_t: matrix with q_t rows and q_t columns

η_t: column-vector with q_t elements

b_t: column-vector with q_t elements

$vec(\eta_t, c_t, B_t, A_t), t = 2, \dots, T$ are $\perp\!\!\!\perp$

$$G_t = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} B_{t+1}$$

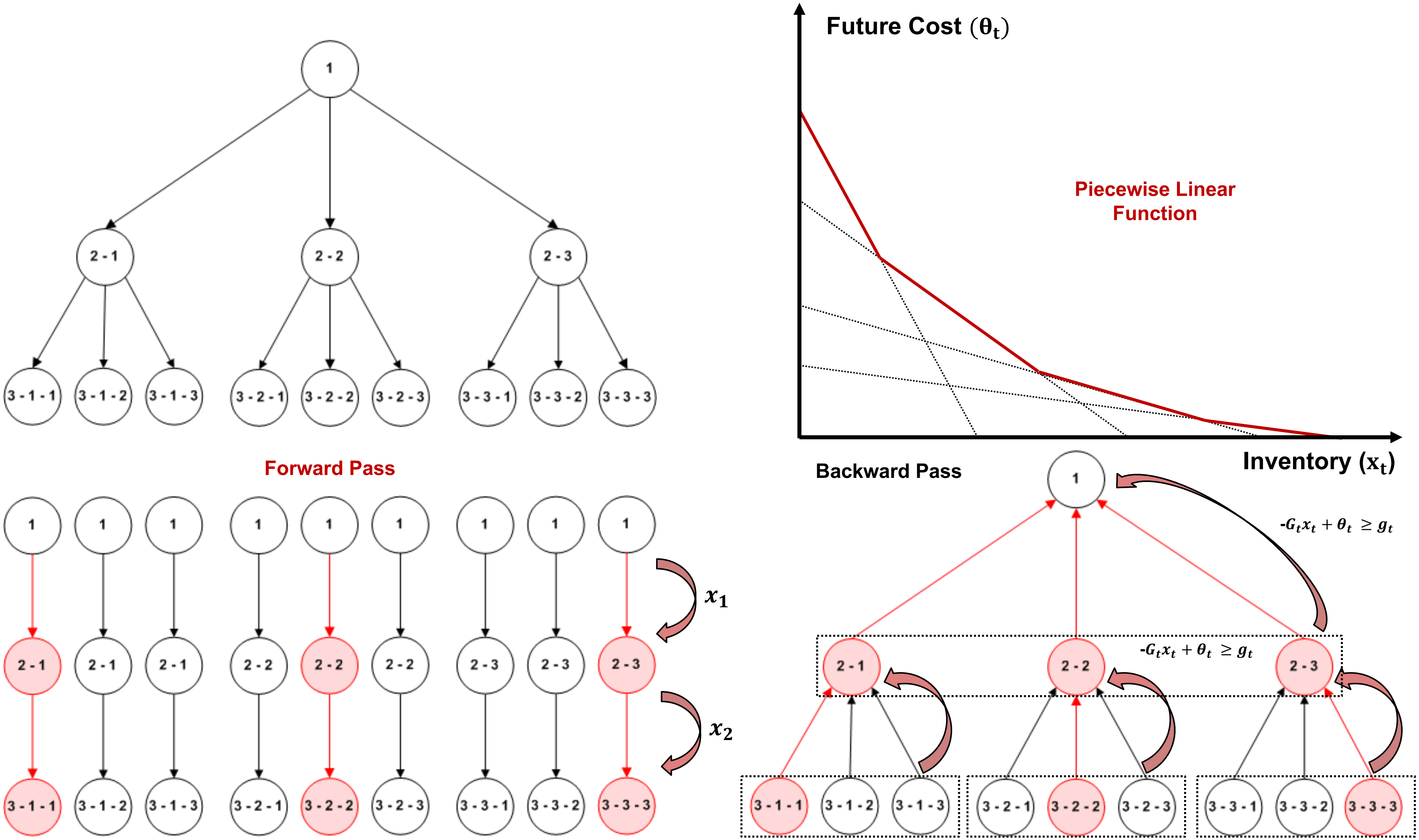
→ cut-gradient matrix

cut-intercept vector

$$g_t^{\omega_t} = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} \left(\rho_{t+1} b_{t+1}^{\omega_{t+1}} + k_{t+1}\right) + \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \alpha_{t+1}^{\omega_{t+1}} \vec{g}_{t+1}^{\omega_{t+1}}$$

have interstage depedency

Sampling-based Decomposition Algorithm



Interstage Dependency Models

- Under interstage independence the future cost function does not depend on the current scenario
- Interstage dependencies usually appear in forecasting:
  - Water inflow
  - Wind speed
  - Electricity demand
- Because of that, the ability to share cuts is necessary in practical SBDA implementations

Cut-Sharing Under Aggregate Dependency Model

- Previous scheme handles the case where the random parameters are individual values
  - Model with individual plants: forecast natural inflows
  - Model with ARR: forecast energy inflows
- Our goal: forecast natural inflow at each hydro plant and then form energy inflow for the ARR
- For SBDA we need to develop an extension of the cut-sharing procedure to be able to handle this case

Notation Index

- Let $\sigma_t \in \Sigma_t$ index the realization for stage t
- A full index would be $\omega_t(\omega_{t-1}, \sigma_t)$
- But the only parameter that requires the ω_t index is:

$$b_t^{\omega_t} = R_{t-1} b_{t-1}^{\omega_{t-1}} + \eta_t^{\sigma_t}$$

Expanding the State

- Suppose we add an auxiliary set of variables to capture the inflow history $y_t = b_t = R_{t-1} b_{t-1} + \eta_t$

$$
\begin{array}{llr}
\min\limits_{x_t, y_t, \theta_t} & c_t x_t + \theta_t & \\
s.t. & A_t x_t = B_t x_{t-1} + \rho_t y_t + k_t & : \pi_t^B \\
& -\vec{G}_t^x x_t - \vec{G}_t^y y_t + e\,\theta_t \geq \vec{g}_t & : \alpha_t \\
& y_t = R_{t-1} y_{t-1} + \eta_t & : \pi_t^S \\
& x_t \geq 0 &
\end{array}
$$

$$G_t^{\ x} = \sum_{\sigma_{t+1} \in \Sigma_{t+1}} p^{\sigma_{t+1}} \pi_{t+1}^{\sigma_{t+1}} B_{t+1} \qquad G_t^{\ y} = \sum_{\sigma_{t+1} \in \Sigma_{t+1}} p^{\sigma_{t+1}} \pi_{t+1}^{S,\sigma_{t+1}} R_t$$

$$g_t^{\omega_t} = \sum_{\sigma_{t+1} \in \Sigma_{t+1}} p^{\sigma_{t+1}} \left(\pi_{t+1}^{B,\sigma_{t+1}} k_{t+1} + \pi_{t+1}^{S,\sigma_{t+1}} \eta_{t+1}^{\sigma_{t+1}} \right) + \sum_{\sigma_{t+1} \in \Sigma_{t+1}} p^{\sigma_{t+1}} \alpha_{t+1}^{\sigma_{t+1}} \vec{g}_{t+1}^{\sigma_{t+1}}$$

$$\pi_t^B = \pi_t \qquad \pi_t^S = \pi_t \rho_t$$

Cut-Sharing Under Aggregate Dependency Model

- With the expanded formulation we can share cuts among different subproblems with SBDA using the interstage independent cut-sharing procedure
- The model's size is larger and we believe that it will require more time to be solved
- Because of that we extended the previous work from (**Infanger & Morton 1996**) to address the aggregate dependency model

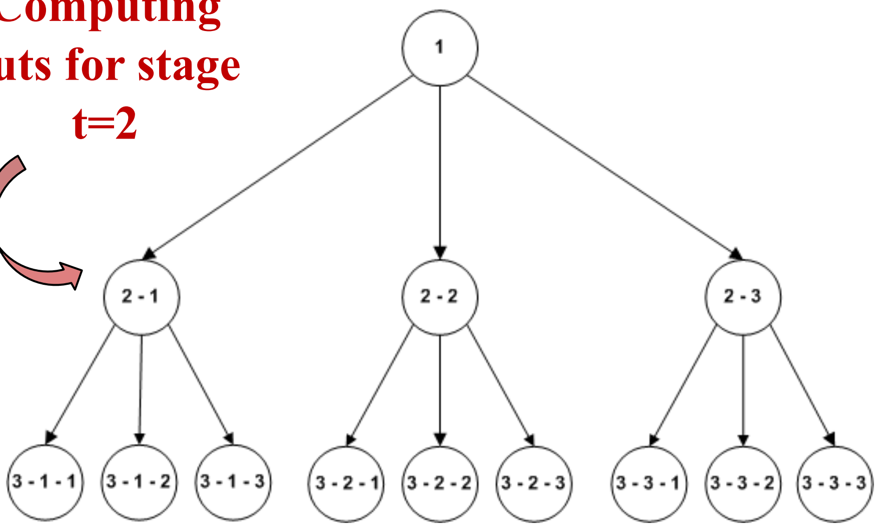
Cut-Sharing Under Aggregate Dependency Model

- Linear lag-one dependency model: $b_t = R_{t-1} b_{t-1} + \eta_t$, for $t = 2, \dots, T$

SLP-3

where, η_t is a random vector and R_t is a known matrix for $t = 2, \dots, T$

Computing cuts for stage t=2



$$G_2 = \sum_{\sigma_3 \in \Sigma_3} p^{\sigma_3} \pi_3^{\sigma_3} B_3$$

$$g_2^{\omega_2} = \sum_{\omega_3 \in \Delta(\omega_2)} p^{\omega_3|\omega_2} \pi_3^{\omega_3} (\rho_3 b_3^{\omega_3} + k_3)$$

$$g_2^{\omega_2} = g_2^{ind} + g_2^{dep}(\omega_2)$$

has interstage depedency

$$g_2^{ind} = \sum_{\sigma_3 \in \Sigma_3} p^{\sigma_3} \pi_3^{\sigma_3} (\rho_3 \eta_3^{\sigma_3} + k_3)$$

$$g_2^{dep}(\omega_2) = \underbrace{\left[\sum_{\sigma_3 \in \Sigma_3} p^{\sigma_3} \pi_3^{\sigma_3} \right]}_{\bar{\pi}_3} \rho_3 R_2 b_2^{\omega_2}$$

Cut-Sharing Under Aggregate Dependency Model

SLP-T

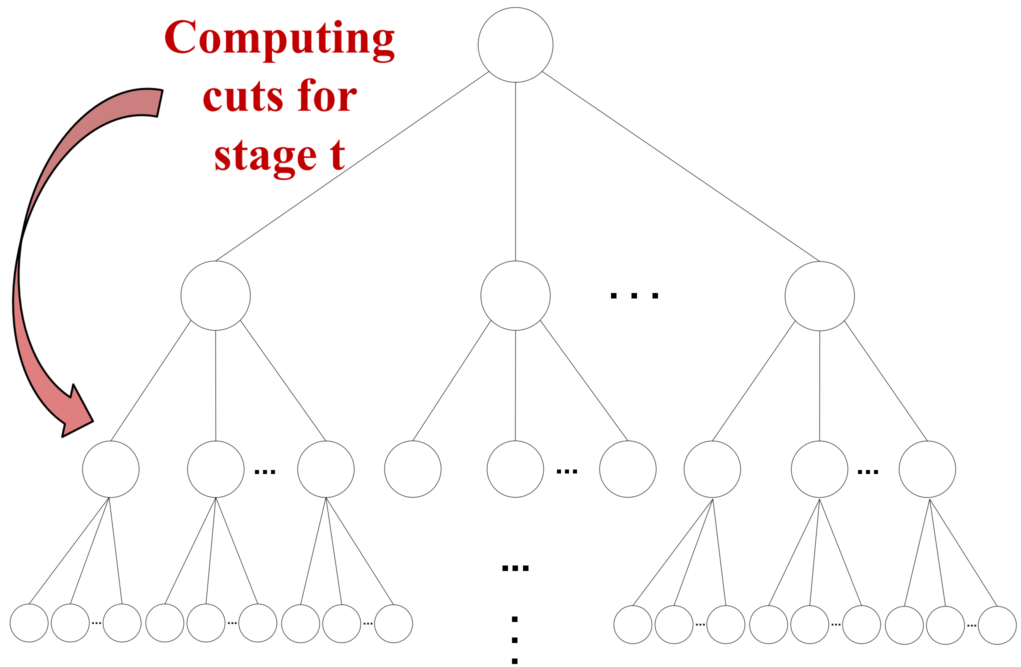
$$b_t = R_{t-1} b_{t-1} + \eta_t$$

$$g_t^{\omega_t} = g_t^{ind} + g_t^{dep}(\omega_t)$$

where for $t = 2, \ldots, T$

$$D_t = [\boldsymbol{\mathcal{P}}_{t+1} \rho_{t+1} + \boldsymbol{\mathcal{A}}_{t+1} D_{t+1}] R_t$$

$$D_T = 0$$



$$g_t^{dep}(\omega_t) = [\bar{\pi}_{t+1} \rho_{t+1} + \bar{\alpha}_{t+1} D_{t+1}] R_t b_t^{\omega_t}$$

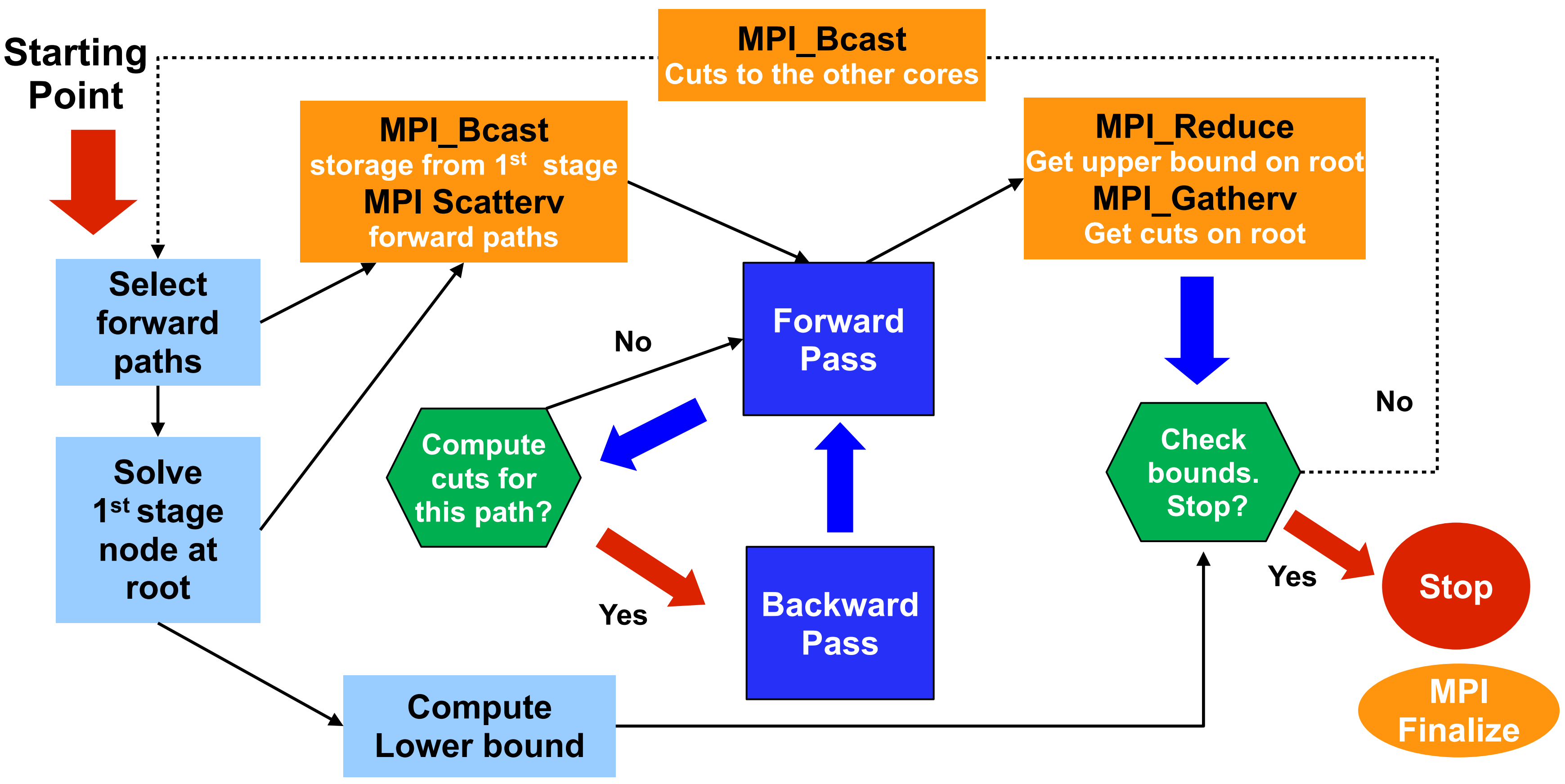
$$g_t^{\omega_t} = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} \left(\rho_{t+1} b_{t+1}^{\omega_{t+1}} + k_{t+1}\right) + \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \alpha_{t+1}^{\omega_{t+1}} \vec{g}_{t+1}^{\omega_{t+1}}$$

$\mathcal{P}_t$: is a matrix with dimensions $l_{t-1} \times m_t$, its rows contain $\bar{\pi}_t$

$\mathcal{A}_t$: is a matrix with dimensions $l_{t-1} \times l_t$, its rows contain $\bar{\alpha}_t$

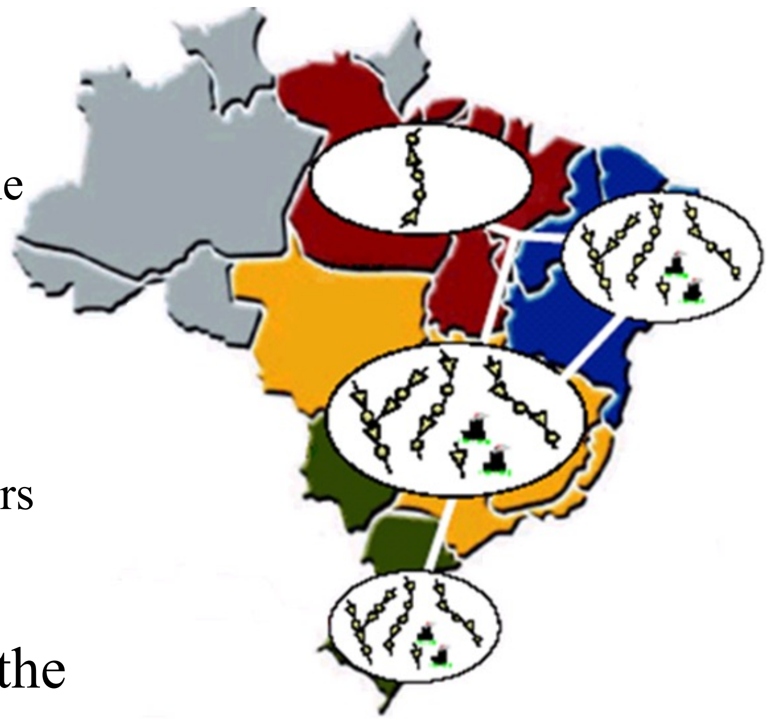
SBDA Parallelization

- MPI to communicate with the different cores
- Synchronize using blocking collective communication calls



Application to the Brazilian System

- 80% of generation capacity → hydro
- Model Characteristics
  - Optimization over 24 stages to determine the generator dispatches
  - Aggregated reservoir scheme
  - Water inflow forecasts produced by a DLM
  - 150 hydro generators, 150 thermal generators



- We consider different sample sizes for the same problem instance to analyze computational time

Iteration Time in Minutes

# of paths

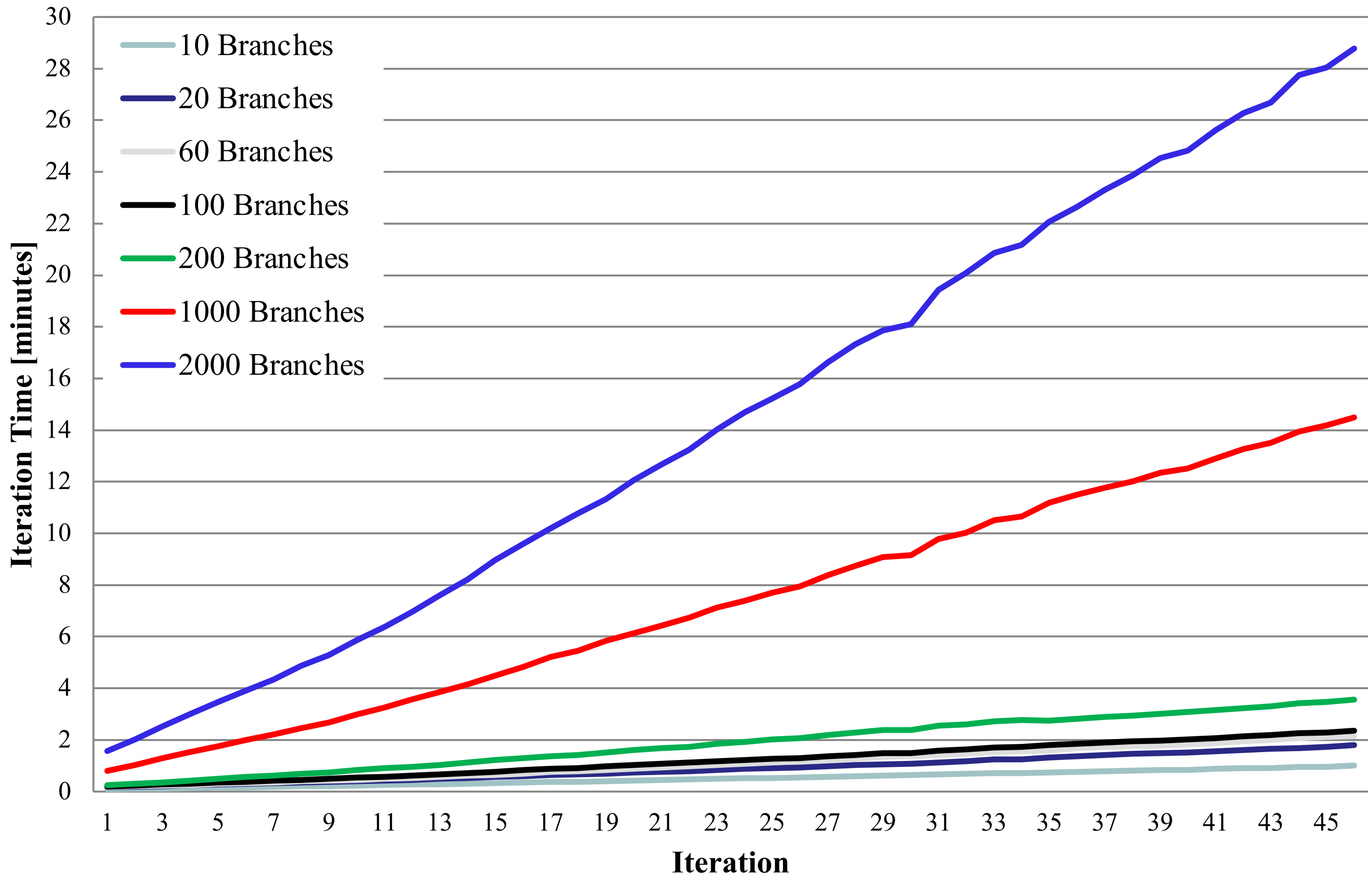
- 128 F
- 32 B

# of iter

- 45

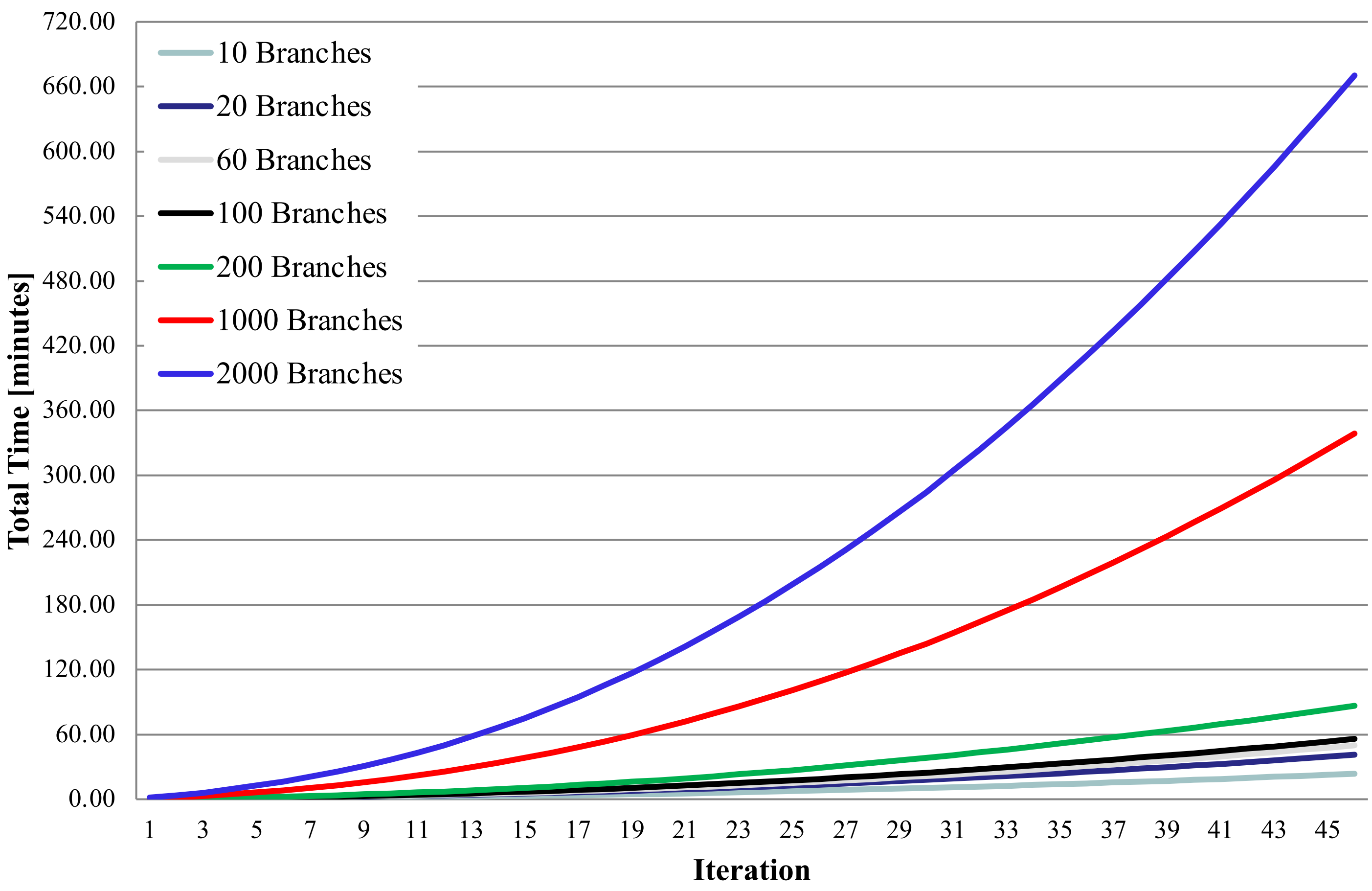
# of cores

- 128



$$n(t) = \max\{\rho^{t-1} n(1), n_{\min}\} \qquad \text{for } t = 2, \ldots, T-1$$

Total Time in Minutes



Final Remarks & Future Step

- The hydro-scheduling problem is a challenging multi-stage stochastic optimization problem
- SBDA handles the problem and avoids the known "curse of dimensionality" of DP
- We presented an extension of the cut-sharing procedure to deal with aggregate interstage dependency models
- Perform a computational study in order to analyze the computational efficiency of both formulations as the problem size scales large

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Thank you!