On a Sampling-based Decomposition Algorithm Applied to Hydrothermal Scheduling: Solution Quality and Bounds

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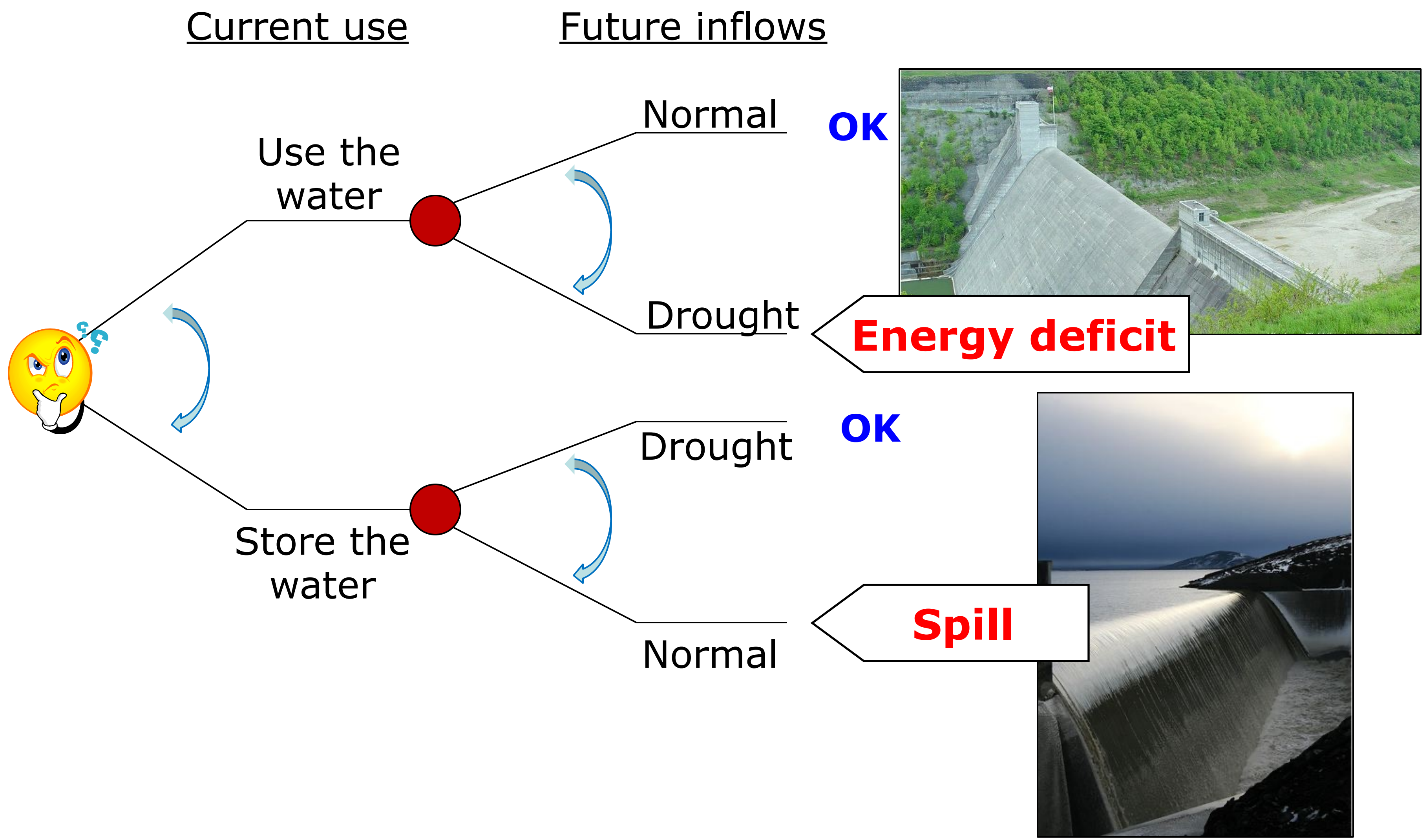
Outline

- Hydrothemal Scheduling Problem
- Model Formulation
- SBDA Multi-stage Scheme
- Solution Quality Evaluation in a Multistage Stochastic Program
- Future Work

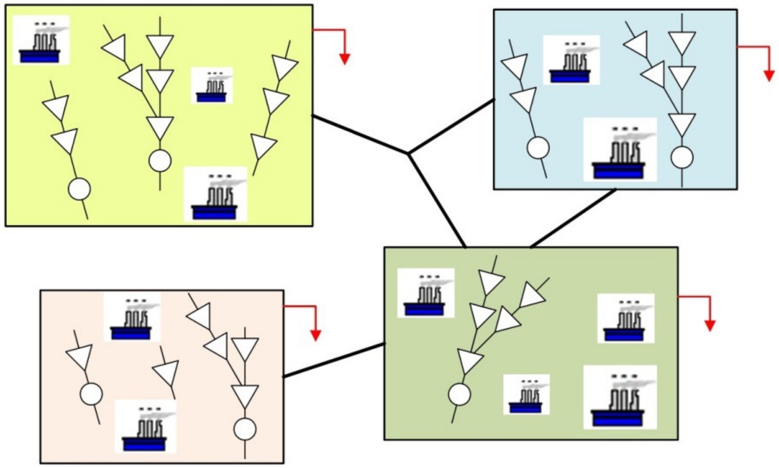
Introduction

- Hydroelectricity is inexpensive to produce
- Depends on the supply of water (stochastic)
- Present decisions affect future conditions of the system and also future decisions (dynamic)
- Multiple interconnected reservoirs, transmission constraints and multi-period optimization (large-scale)

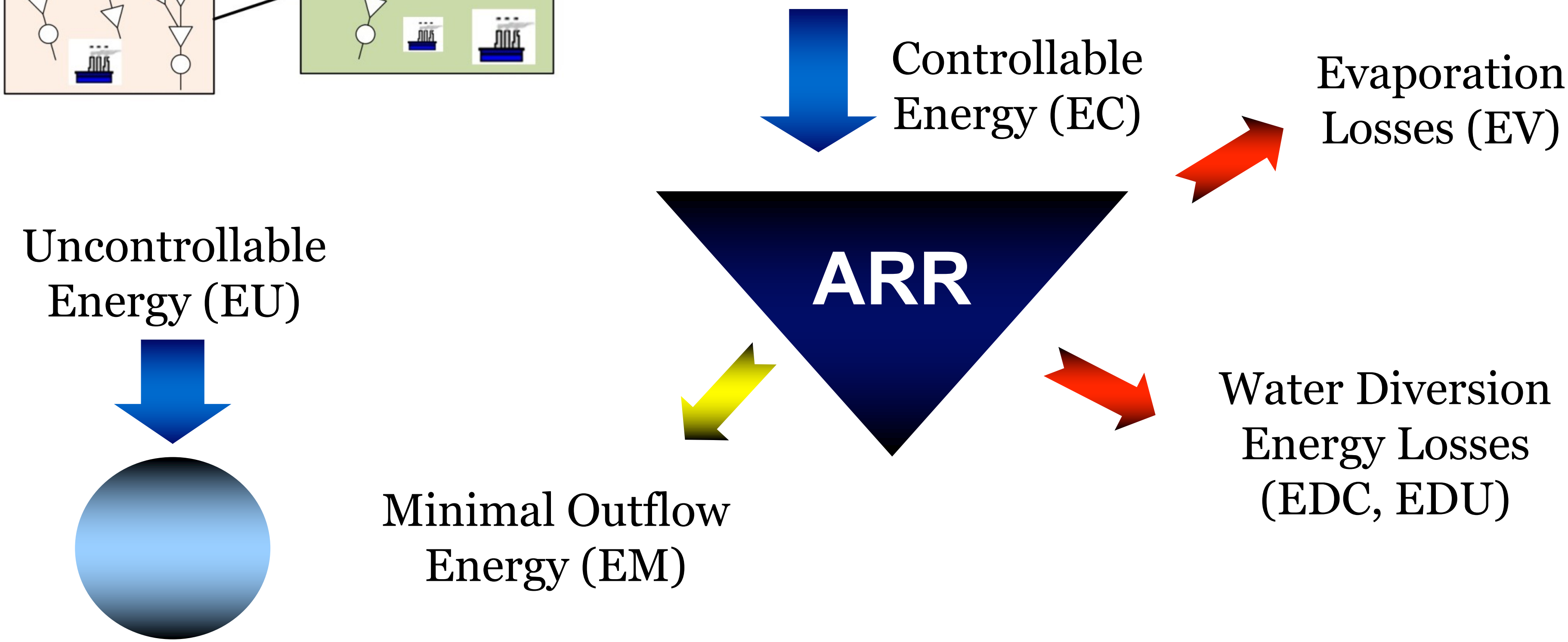
Decision Tree



Aggregate Reservoir Representation



- Aggregate hydro plants in a region to create unique ARRs
- Reduce model's size



Water Inflow Vs. Energy Inflow

- Arguments for forecasting **water inflows**:
  - **Exploit local predictors**
    - Precipitation
    - El ninõ
    - Soil use
  - Are **measurable**
  - **Unaffected** by the hydro system **configuration**
- Problems when forecasting **energy inflows**
  - Ties model of a natural process to the decision process
  - **Harder** to validate
  - **Affected** by the hydro system **configuration**

Brief Survey

- Introduce sampling methods to nested Benders' decomposition algorithm → Sampling-based decomposition algorithm **(SBDA)**, the **SDDP** **(Pereira & Pinto 91)**
- Since then SBDA has received considerable attention, **DOASA**, **CUPPS**, **Abridged Nested Decomposition**
- **Cut sharing** procedure for inter-stage dependency models **(Infanger & Morton 1996)**
- **Solution quality** **(Chiralaksanakul & Morton 2004)**
- **Statistical properties** & **risk measures** **(Shapiro 2010)**
- **Alternative sampling** **(Homem-de-Mello et al. 2011)**

A General SLP-t

We consider a model that uses water inflow forecasts instead of energy

$$\begin{aligned} \min_{x_1} \quad & c_1 x_1 + \mathrm{E}_{b_2|b_1} h_2(x_1, b_2) \\ s.t. \quad & A_1 x_1 = B_1 x_0 + \boldsymbol{\rho_1} b_1 + k_1 \\ & x_1 \geq 0 \end{aligned}$$

where, for $t = 2,\ldots,$T

$$\begin{aligned} \min_{x_t} \quad & c_t x_t + \mathrm{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x_2, b_{t+1}) \\ s.t. \quad & A_t x_t = B_t x_{t-1} + \boldsymbol{\rho_t} b_t + k_t \\ & x_t \geq 0 \end{aligned}$$

x_t: all stage t decision variables including: hydro generation, hydro storage, spillage, thermal generation, energy transfers, ...

A_t: constraint matrix including energy balance, demand satisfaction, ...

b_t: stochastic water inflow at each hydro plant

$\boldsymbol{\rho_t}$: matrix to transform water into controllable and uncontrollable energy inflows

$B_t x_{t-1}$: storage from last stage, energy parameters that depend on storage

$\boldsymbol{k_t}$: deterministic demand, constant energy parameters

Stage t Benders' Master Problem

- Suppose we are at stage t under ω_t and we have:

$$b_t = R_{t-1} b_{t-1} + \eta_t$$

$$\min_{x_t, \theta_t} \quad c_t x_t + \theta_t$$
$$s.t. \quad A_t x_t = B_t x_{t-1} + \rho_t b_t + k_t \; : \pi_t$$
$$-\vec{G}_t x_t + e\,\theta_t \geq \vec{g}_t \quad : \alpha_t$$
$$x_t \geq 0$$

where,

(# of individual hydro plants)

ρ_t: matrix with m_t rows and q_t columns

R_t: matrix with q_t rows and q_t columns

η_t: column-vector with q_t elements

b_t: column-vector with q_t elements

$vec(\eta_t, c_t, B_t, A_t), t = 2, \ldots, T$ are $\perp\!\!\!\perp$

cut-intercept vector

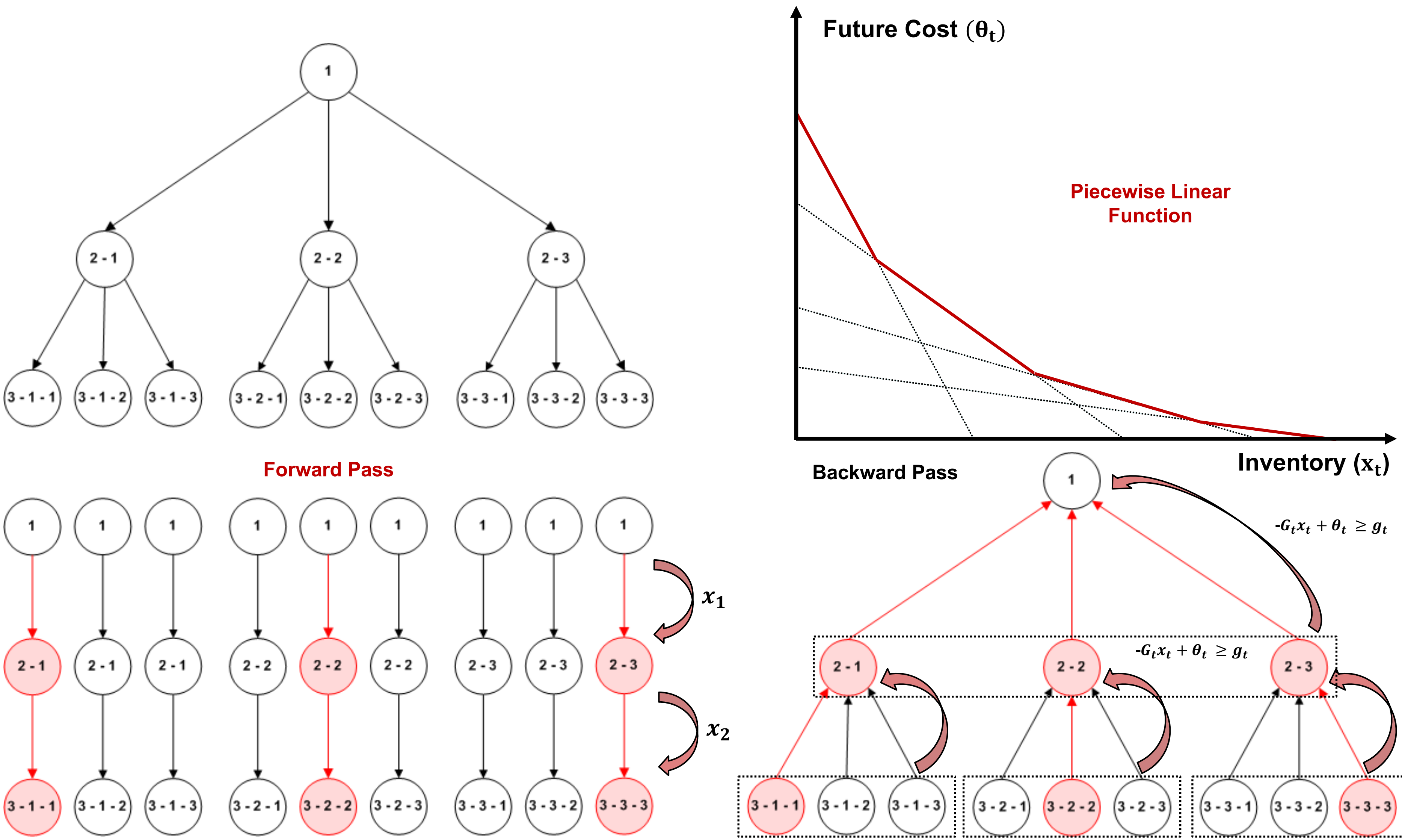
$$G_t = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} B_{t+1}$$

cut-gradient matrix

$$g_t^{\omega_t} = \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \pi_{t+1}^{\omega_{t+1}} \left(\rho_{t+1} b_{t+1}^{\omega_{t+1}} + k_{t+1}\right) + \sum_{\omega_{t+1} \in \Delta(\omega_t)} p^{\omega_{t+1}|\omega_t} \alpha_{t+1}^{\omega_{t+1}} \vec{g}_{t+1}^{\omega_{t+1}}$$

may have interstage depedency

Sampling-based Decomposition Algorithm



Policy Generation Procedure

- A solution to a multistage stochastic program is defined by a policy, which specifies what decision to take at each stage, given the history of the stochastic process

- **Input:** Sample size $\boldsymbol{n_u}$ and Bender's Master Problem with **SDDP outputs**: cuts computed for stages $t = 2, \ldots, T$, and 1st stage solution, x_1

- **Output:** Sample mean estimator $\boldsymbol{U_{n_u}}$ and variance $\boldsymbol{S_u^2}$ for expected cost of policy

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- 1. Let $x_1^i = x_1,\ \mathrm{i} = 1, \ldots, n_u$
- 2. Sample i.i.d. paths from $\Omega_T,\ b_1^i, \ldots, b_T^i,\ \ i = 1, \ldots, n_u$

  do $i = 1, \ldots, n_u$

      do $t = 2, \ldots, T$

          form RHS of the problem:

  $$B_t x_{t-1}^i + \rho_t b_t^i + k_t$$

          solve and obtain x_t^i

      end do

      let $z^i = \sum_{t=1}^{T} c_t\, x_t^i$

  end do

  3. Compute $\boldsymbol{U_{n_u}}$ and $\boldsymbol{S_u^2}$

Lower Bound Estimation

- $\widehat{\boldsymbol{\Omega}}_{\boldsymbol{T}}$ denotes the sample space of a finite scenario tree and $\boldsymbol{\Omega}_{\boldsymbol{T}}$ represents the sample space of the true stochastic process
- We want to form a **lower bound on z^***. In this case $\hat{\mathbf{z}}^*$ or its bounds play an important role in achieving that
- As shown in **Chiralaksanakul and Morton (2004),** we have that $\mathbb{E}\hat{\mathbf{z}}^* \leq \mathbf{z}^*$, which is clear when the branch size $n(t) = 1$

- **Input:** Instance of **SLP-t**, branch size $\boldsymbol{n'(t)}$, $t = 2, \ldots, T$, and sample size $\boldsymbol{n_\ell}$
- **Output:** Sample mean estimator $\boldsymbol{L_{n_\ell}}$ and variance $\boldsymbol{S_\ell^2}$ for lower bound on optimal value z^*

1. **do** $i = 1, \ldots, n_\ell$
   - Create a sample tree with $\boldsymbol{n'(t)}$ branches at stage t, independent from previous
   - Run SDDP to obtain a lower bound on the optimal value, $\underline{\boldsymbol{z}}^{\boldsymbol{i}}$

   end do
2. Compute $\boldsymbol{L_{n_\ell}}$ and $\boldsymbol{S_\ell^2}$

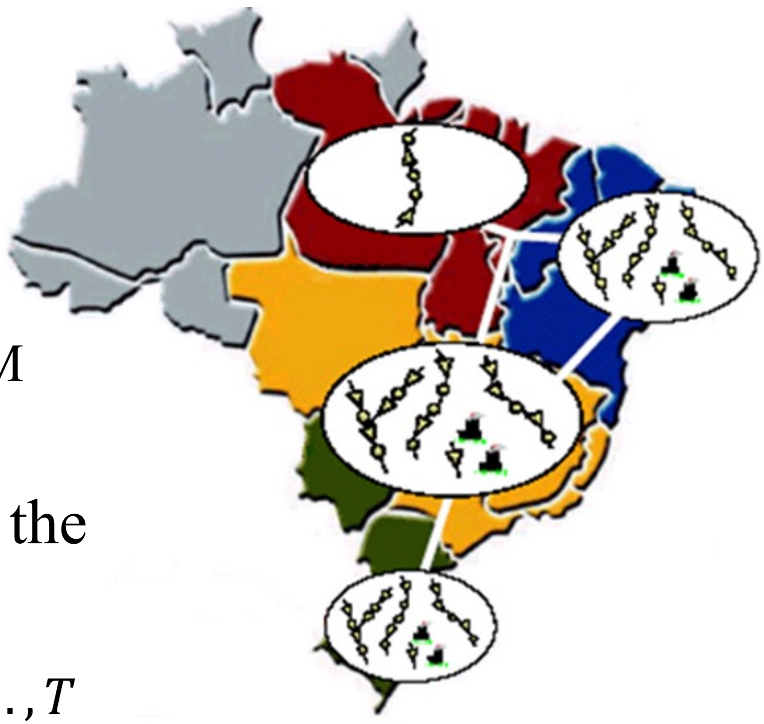
Confidence Interval Construction

- **Input:** Instance of **SLP-$\boldsymbol{t}$**, branch size $\boldsymbol{n(t)}$, $t = 2, \dots, T$, for policy construction and $\boldsymbol{n'(t)}$, $t = 2, \dots, T$, for lower bound estimation, sample sizes $\boldsymbol{n_u}$ and $\boldsymbol{n_\ell}$, and $\alpha \in (0,1)$
- **Output:** Approximate $(1 - \alpha)$-level confidence interval on optimality gap $\mathbb{E}\boldsymbol{U} - \boldsymbol{z}^*$

1. Form a sample scenario tree with branches size $n(t)$
2. Run SDDP to approximately solve SLP-$\boldsymbol{t}$ defined on the sampled scenario tree to obtain cuts for all stages, first stage solution, x_1
3. Run PGP with sample size $\boldsymbol{n_u}$ to obtain $\boldsymbol{U_{n_u}}$ and $\boldsymbol{S_u^2}$
4. Run LBE with branch size $\boldsymbol{n'(t)}$, $t = 2, \dots, T$ and sample size $\boldsymbol{n_\ell}$, to obtain $\boldsymbol{L_{n_\ell}}$ and $\boldsymbol{S_\ell^2}$
5. Let $\epsilon_\ell = t_{n_\ell - 1} S_\ell / \sqrt{n_\ell}$ and $\epsilon_u = z_\alpha S_u / \sqrt{n_u}$. Output one-sided **CI** on $\mathbb{E}\boldsymbol{U} - \boldsymbol{z}^*$, $\left[0\,, \left(U_{n_u} - L_{n_\ell}\right)^+ + \epsilon_\ell + \epsilon_u\right]$

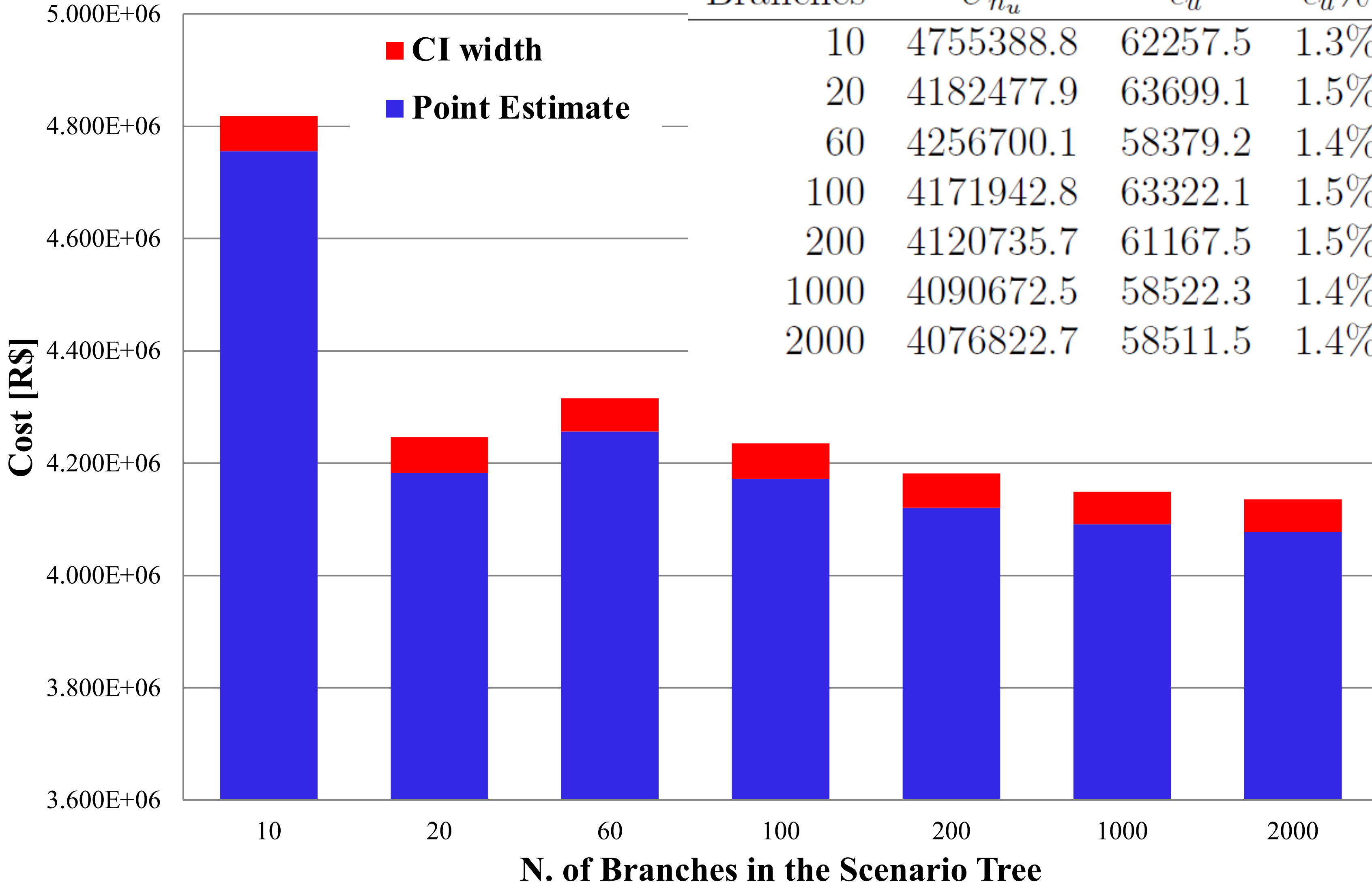
Application to the Brazilian System

- 80% of generation capacity → hydro
  - 150 hydro generators, 150 thermal generators
- Model Characteristics
  - Optimization over 24 stages
  - Aggregated reservoir scheme
  - Water inflow forecasts produced by a DLM **(Marangon Lima, 2011)**
- We consider different sample sizes for the same problem instance
  - $n(t) = \max\{\rho^{t-1} n(1)\,, n_{min}\}$ for $t = 2, \dots, T$
  - $n_u = 12800$ for PGP
  - $n_\ell = 15$ for LBE



Upper Bound Estimator Analysis

Branches	U_{n_u}	ϵ_u	$\epsilon_u\%$
10	4755388.8	62257.5	1.3%
20	4182477.9	63699.1	1.5%
60	4256700.1	58379.2	1.4%
100	4171942.8	63322.1	1.5%
200	4120735.7	61167.5	1.5%
1000	4090672.5	58522.3	1.4%
2000	4076822.7	58511.5	1.4%



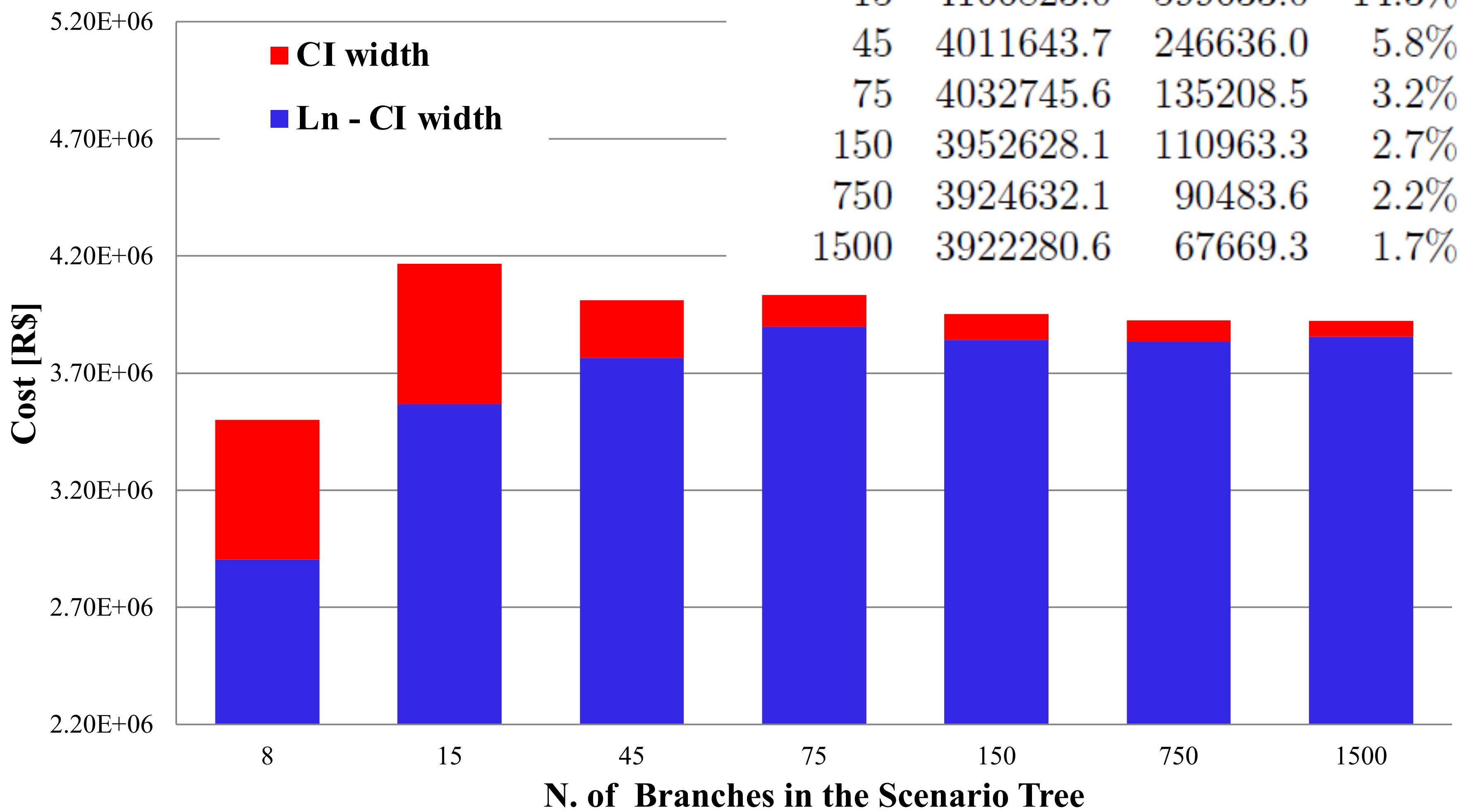
Upper Bound Estimator Analysis *(cont.)*

Branches		10	20	60	100	200	1000
20	Pt	572910.9					
	HW	23540.8					
60	Pt	498688.8	-74222.1				
	HW	17604.6	13660.4				
100	Pt	583446.1	10535.1	84757.3			
	HW	23811.1	9626.8	13067.4			
200	Pt	634653.2	61742.2	135964.4	51207.1		
	HW	23365.3	10191.1	10923.7	7531.9		
1000	Pt	664716.3	91805.4	166027.5	81270.2	30063.1	
	HW	23698.3	11212.3	10373.0	9103.9	5905.1	
2000	Pt	678566.1	105655.2	179877.3	95120.1	43913.0	13849.8
	HW	24116.9	11058.8	10742.0	8959.6	6019.8	4668.8

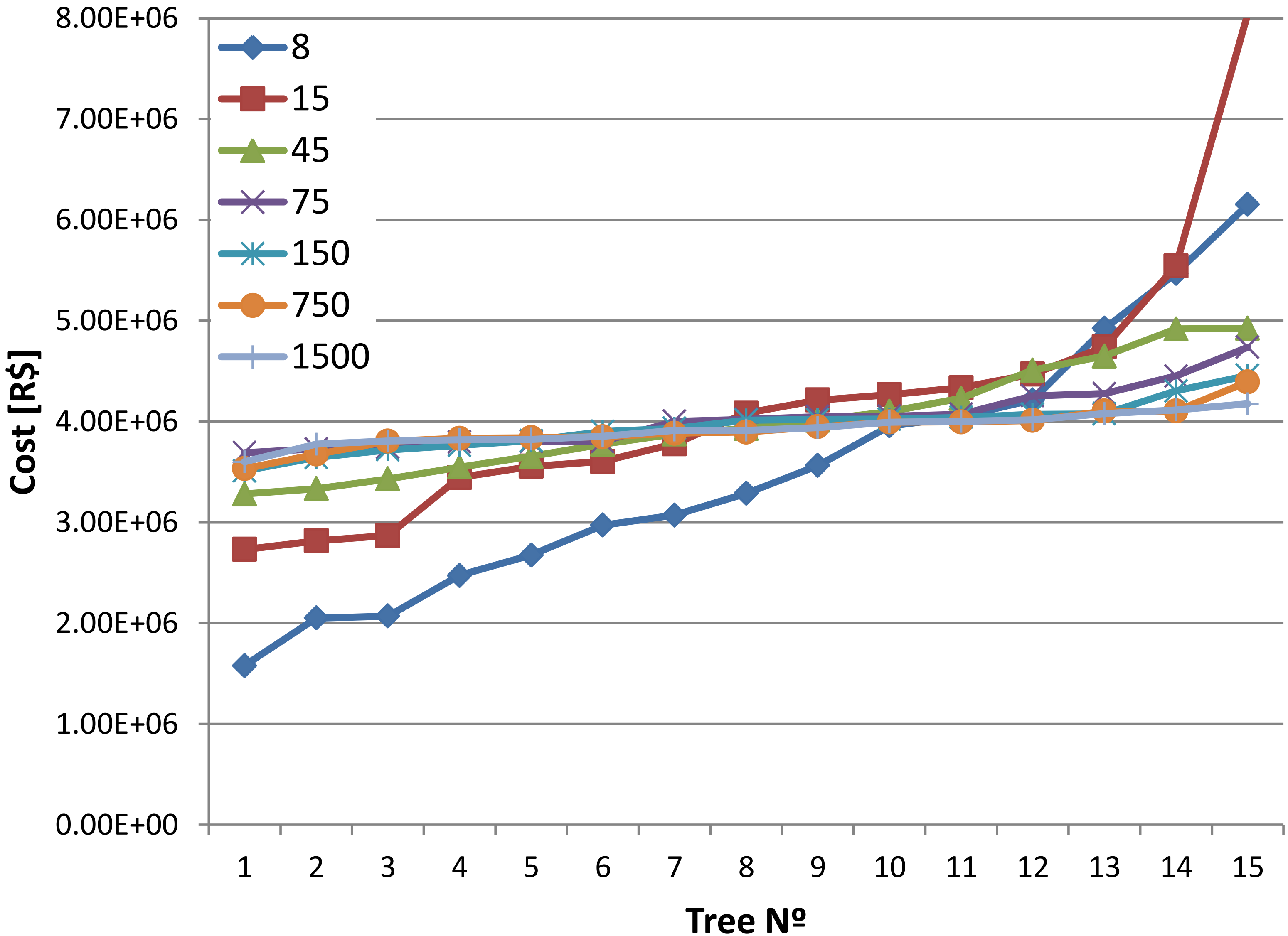
Table 4.2: Paired Student-t Test for PGP with Different Scenario-Tree Sizes Paired Student-t tests using common random numbers and a 90% level with a sample size of 12800. The table contains confidence intervals for the column entry less the row entry; e.g., the first entry is 572910.9 ± 23540.8 is a confidence interval for $U_{n_u}(10) - U_{n_u}(20)$, where $U_{n_u}(10)$ and $U_{n_u}(20)$ denote the point estimates from scenario trees with $n(1) = 10$ and $n(1) = 20$ branches at each stage, respectively.

Lower Bound Estimator Analysis

Branches	L_{n_ℓ}	ϵ_ℓ	$\epsilon_\ell\%$
8	3500672.5	595878.0	12.5%
15	4166823.0	599633.0	14.3%
45	4011643.7	246636.0	5.8%
75	4032745.6	135208.5	3.2%
150	3952628.1	110963.3	2.7%
750	3924632.1	90483.6	2.2%
1500	3922280.6	67669.3	1.7%

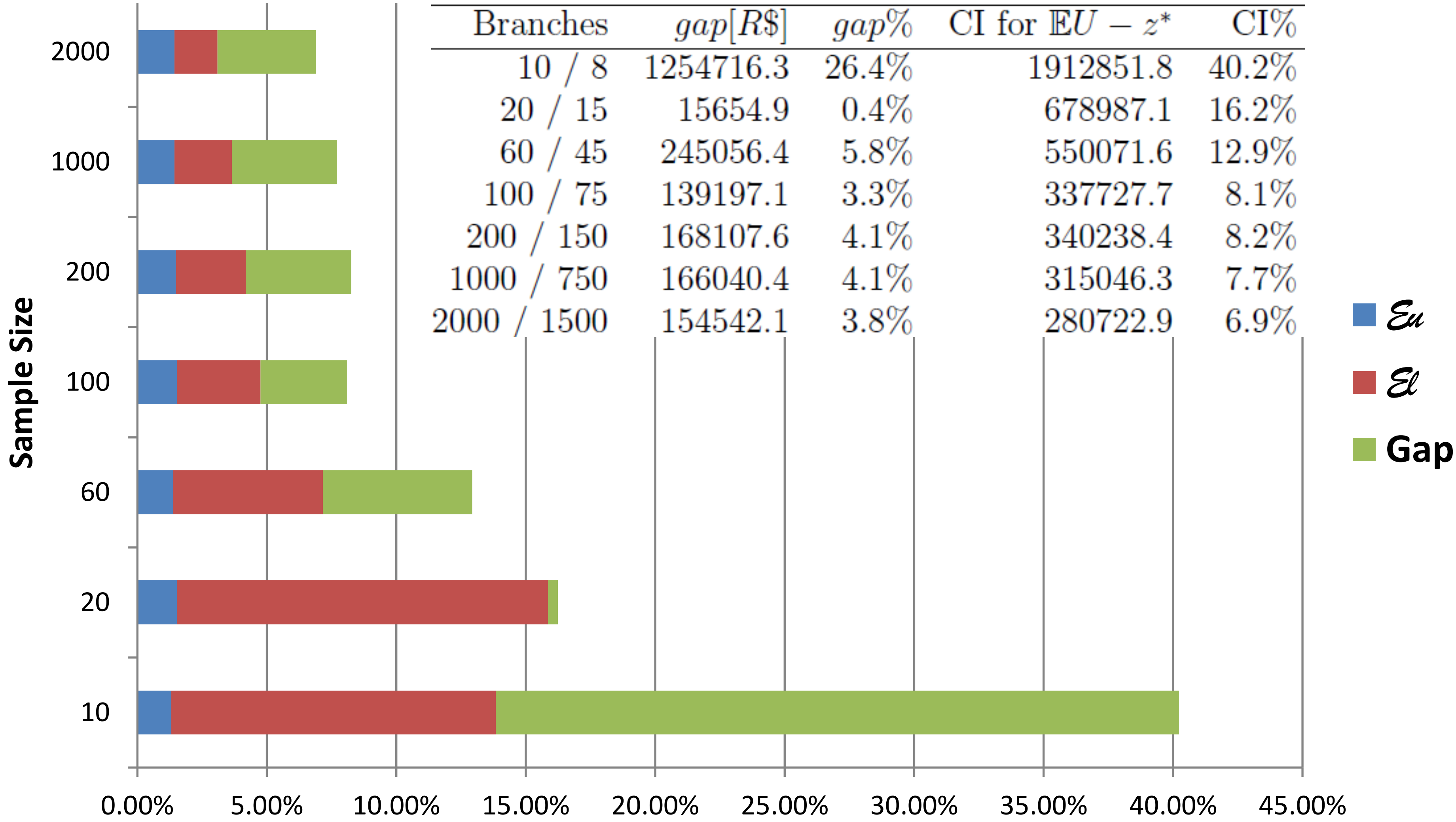


Lower Bound Estimator Analysis *(cont.)*



Confidence Interval Analysis

Branches	$gap[R\$]$	$gap\%$	CI for $\mathbb{E}U - z^*$	CI%
10 / 8	1254716.3	26.4%	1912851.8	40.2%
20 / 15	15654.9	0.4%	678987.1	16.2%
60 / 45	245056.4	5.8%	550071.6	12.9%
100 / 75	139197.1	3.3%	337727.7	8.1%
200 / 150	168107.6	4.1%	340238.4	8.2%
1000 / 750	166040.4	4.1%	315046.3	7.7%
2000 / 1500	154542.1	3.8%	280722.9	6.9%



Confidence Interval Analysis *(cont.)*

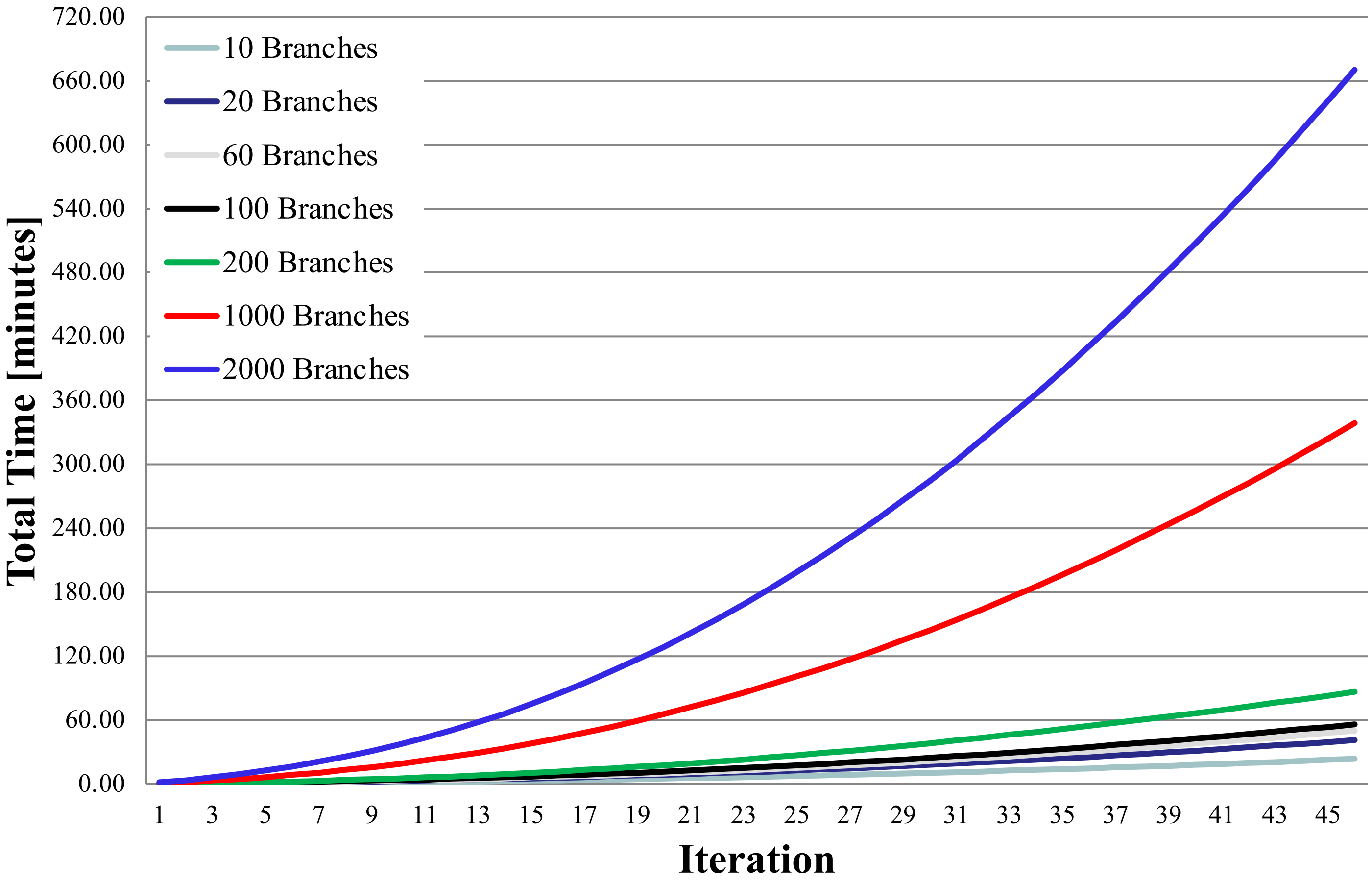
- The **CI error reduces** as the sample size becomes larger
- The **CI width shrinks** as the scenario trees grow in size
- As the **scenario tree used to form the policy grows** the **point estimate** associated with the upper bound tends to **decrease**
- As the **scenario trees used on the LBE grow** the **lower bound tends to grow** and the sampling error reduces
- Together this means that the **gap estimate tends to shrink**

Total Time in Minutes

# of paths
- 128 F
- 32 B

# of iter
- 45

# of cores
- 128



Final Remarks & Future Steps

- The hydro-scheduling problem is a challenging multi-stage stochastic optimization problem. SBDA handles the problem
- We presented a procedure to assess the quality of the solution with respect to the true problem in a multi-stage setting
- Assess the solution quality in multi-stage stochastic programs using smart sampling ideas to better select the scenarios to create the sampled scenario trees
- Assessment of the policy quality as the time horizon grows
- Employ risk measures such as CVaR within the SDDP algorithm. Assessing solution quality in such a setting would require extension of the current techniques

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Thank you!