Solution Quality and Jackknife Estimators in Large Scale Hydroelectric Scheduling

Anderson Rodrigo de Queiroz   **GESIS**

**David P. Morton**   **IEMS**

November 8th, 2014

Outline

- Hydrothemal Scheduling Problem

- Model Formulation

- SBDA Multi-stage Scheme

- Solution Quality Evaluation in Multistage Stochastic Programs

Hydrothermal Scheduling Problem

- Find the sequence of **hydro releases** and **thermal plant dispatches** for a planning horizon in order to match system demand

  - Resource management

  - Input variable forecasting

  - Operational aspects

- Basic economic criterion

  - **Minimize operational costs** (present + expected future)

- Multi-stage Stochastic Linear Program
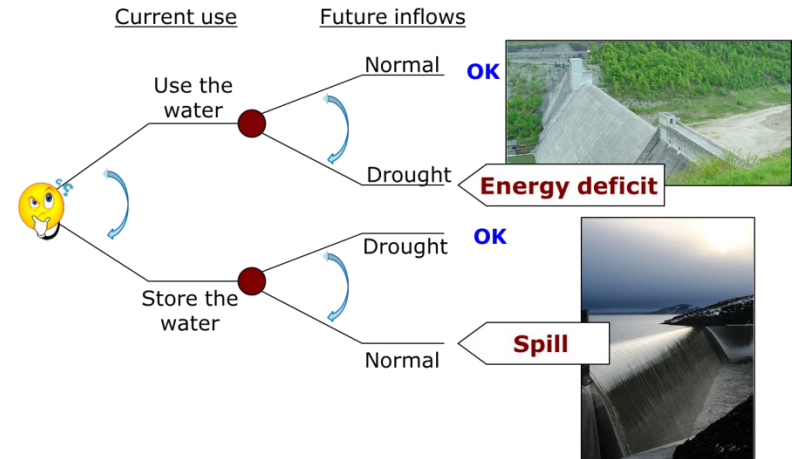
Stage t Benders' Master Problem

- Suppose we are at stage t under ω_t and we have:

$$vec(\eta_t, c_t, B_t, A_t), t = 2, \dots, T \text{ are } \perp\!\!\!\perp$$

$$\min_{x_t, \theta_t} \quad c_t x_t + \theta_t$$

$$s.t. \quad A_t x_t = B_t x_{t-1} + \rho_t \, b_t + k_t : \pi_t \longrightarrow \textbf{\textcolor{red}{Structural constraints}}$$

$$-\vec{G}_t x_t + e\, \theta_t \geq \vec{g}_t \qquad\qquad : \alpha_t \longrightarrow \textbf{Benders' cuts}$$

$$x_t \geq 0$$

$$b_t = R_{t-1} b_{t-1} + \eta_t$$

x_t: stage t decision variables including: hydro generation, hydro storage, spillage, thermal generation, energy transfers, load curtailment

A_t: constraint matrix including energy balance, demand satisfaction
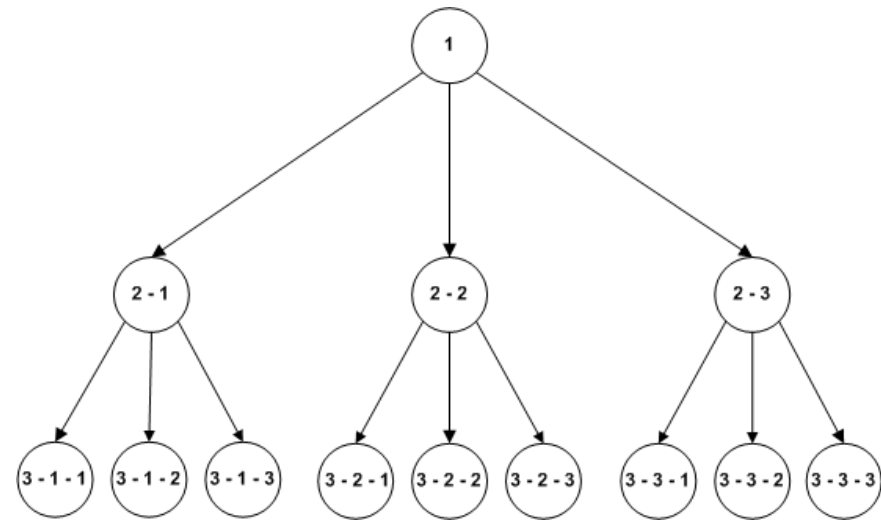
b_t: stochastic water inflow at each hydro plant

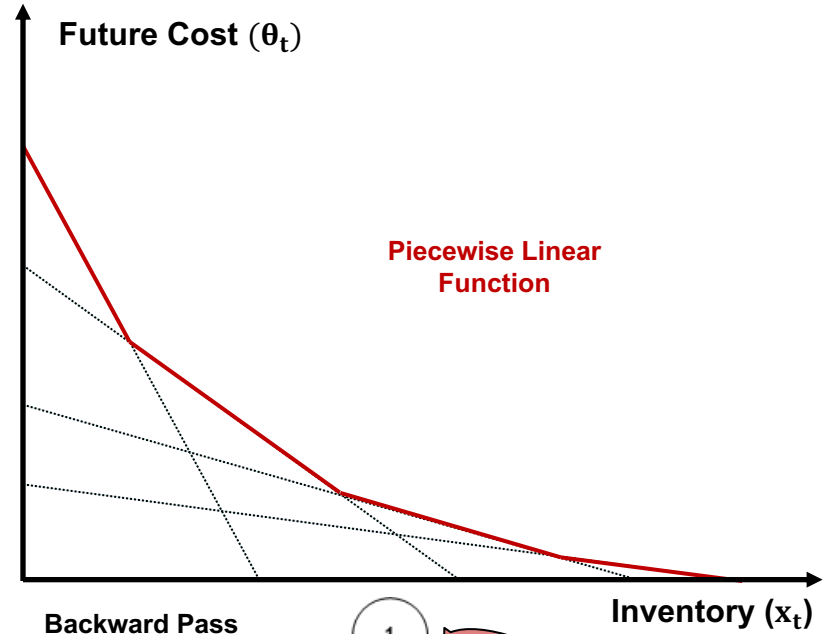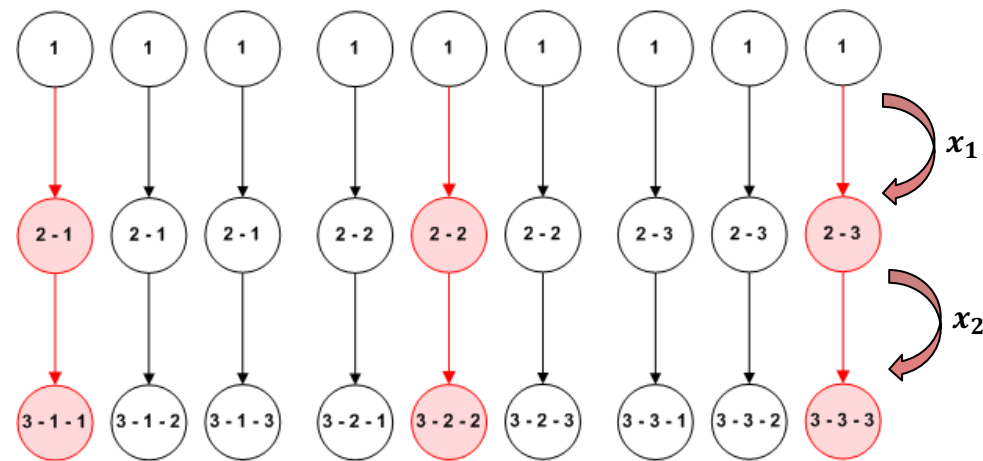ρ_t: matrix to transform water into energy inflows

$B_t x_{t-1}$: storage from last stage

k_t : deterministic demand
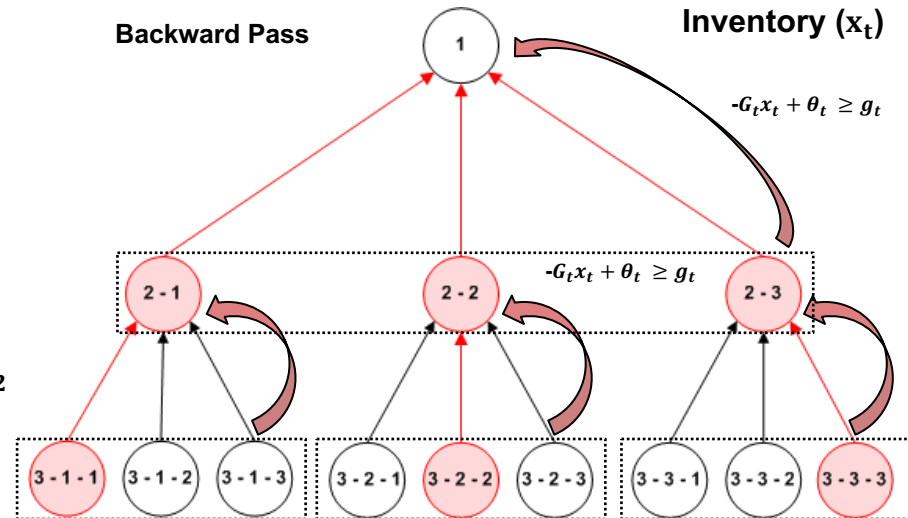
Sampling-based Decomposition Algorithm

Forward Pass

Backward Pass

Future Cost (θ_t)

Piecewise Linear Function

Inventory (x_t)

x_1

x_2

$-G_t x_t + \theta_t \geq g_t$
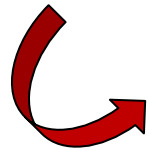
$-G_t x_t + \theta_t \geq g_t$
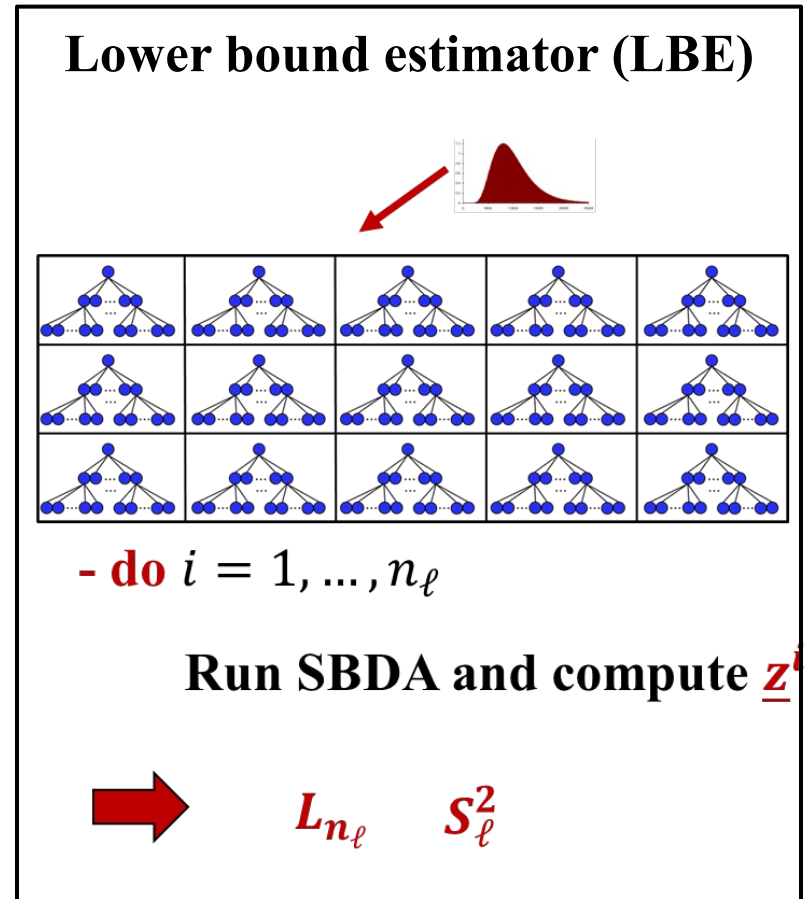
Solution Quality Assessment

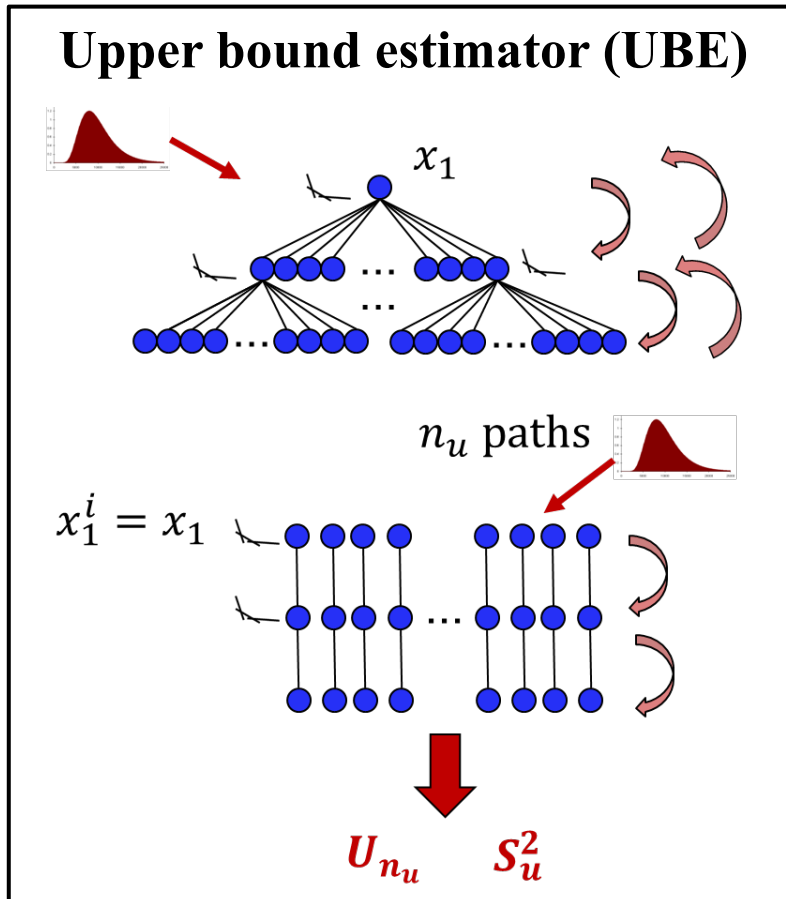- Monte Carlo simulation $\rightarrow$ is used to assess if a candidate solution (i.e., policy) is near optimal

  cannot solve the SP exactly

- When optimizing a sample-mean estimator we get an optimistic bound for z^*

- This implies a weak statement regarding quality of candidate solution $\longrightarrow$ **Estimate may have large bias**

- When bias is large it is not possible to be sure if a candidate solution is near optimal

Confidence Interval Construction

Upper bound estimator (UBE)

x_1

n_u paths

$x_1^i = x_1$

$U_{n_u} \quad S_u^2$

Lower bound estimator (LBE)

- **do** $i = 1, \dots, n_\ell$

Run SBDA and compute $\underline{z}^i$

$L_{n_\ell} \quad S_\ell^2$
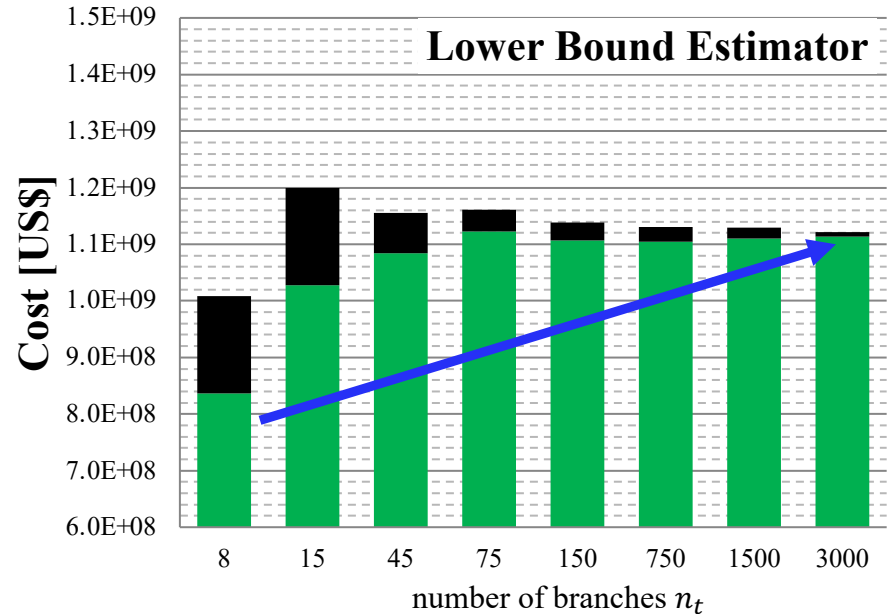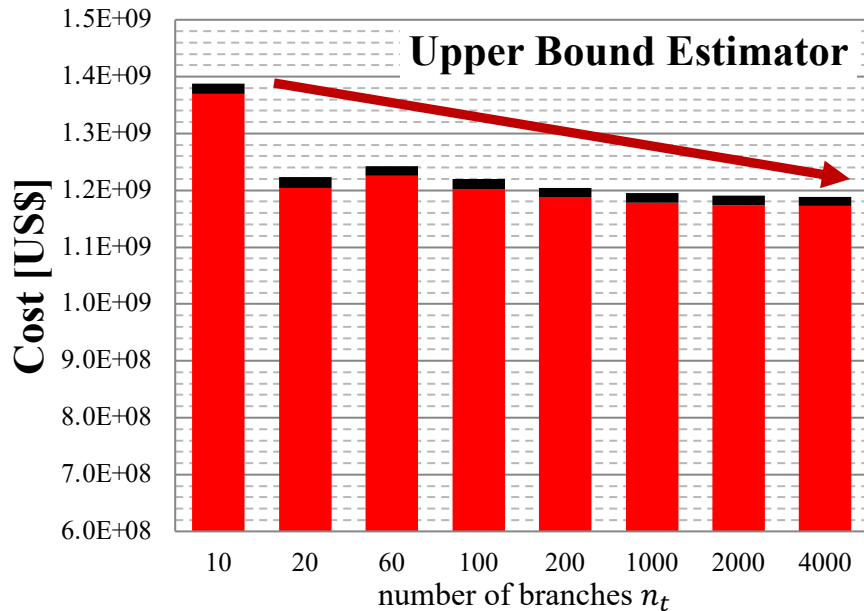
Let $\epsilon_\ell = t_{n_\ell - 1} S_\ell / \sqrt{n_\ell}$ and $\epsilon_u = z_\alpha S_u / \sqrt{n_u}$

Output one-sided **CI** on $\mathbb{E}\boldsymbol{U} - \boldsymbol{z}^*$, $\left[0 , \left(U_{n_u} - L_{n_\ell} \right)^+ + \epsilon_\ell + \epsilon_u \right]$

Previous Results
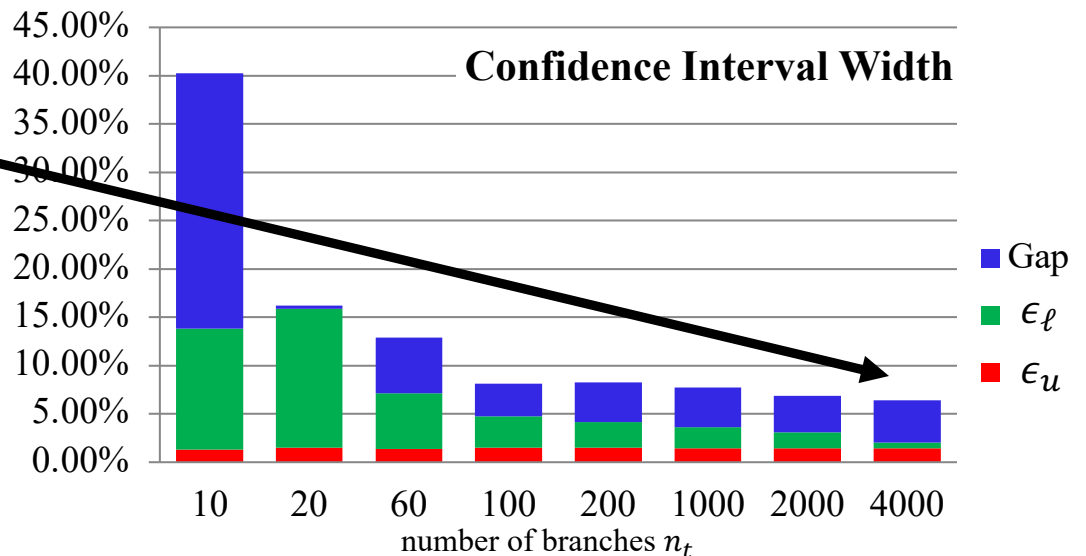
Jackknife Estimators

- Jackknife is a technique developed by Quenouille in 1949 to estimate the bias of $\hat{\theta}$ (estimator)

- It is also known as the "leave one out" procedure

- Resampling procedure (same class as Bootstrap)

- Reduce bias of $O\left(\frac{1}{n}\right)$

Adaptive Jackknife Estimators
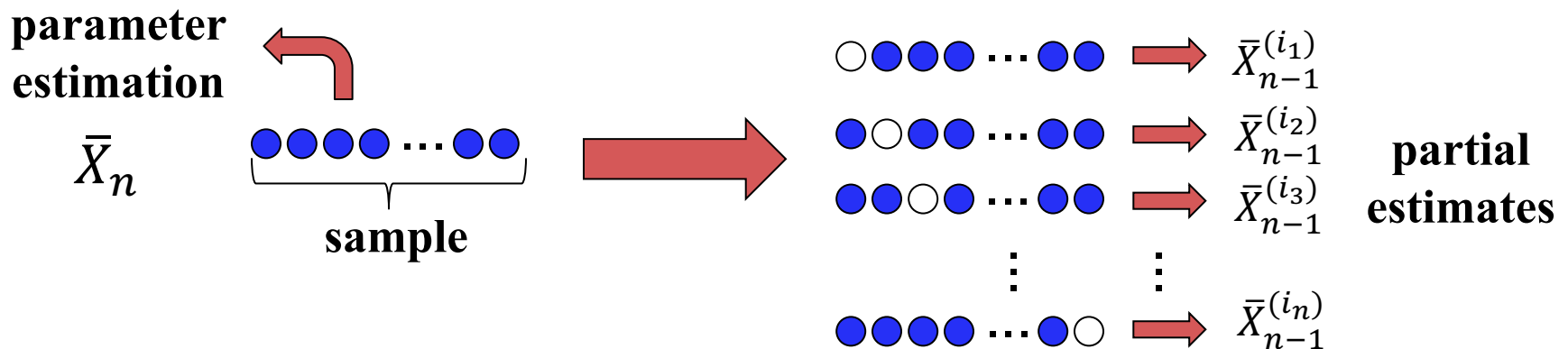
- In SAA the bias has $O\left(\frac{1}{n^p}\right)$ where p is unknown, $p \in [\frac{1}{2}, \infty)$ **(Bayraksan et al. 2006)**

- Following the idea of the generalized jackknife estimators → adaptive jackknife estimators (AJE) **(Partani et al. 2006)**

- In AJE the **order of the bias is not assumed to be known** when forming an estimator

  - Applied to reduce bias in static and two-stage models

AJE Procedure in Multi-stage Setting

Full tree

$\widehat{L}_{n_t}^i$ $\longrightarrow$ $\widehat{L}_{n_t}^i$

**Two trees
(1/2 scenarios)**

$\widehat{L}_{\frac{n_t}{2}}^{i_1}$ $\widehat{L}_{\frac{n_t}{2}}^{i_2}$ $\longrightarrow$ $\widehat{L}_{\frac{n_t}{2}}^{i}$

**Four trees
(1/4 scenarios)**

$\widehat{L}_{\frac{n_t}{4}}^{i_1}$ $\widehat{L}_{\frac{n_t}{4}}^{i_2}$ $\widehat{L}_{\frac{n_t}{4}}^{i_3}$ $\widehat{L}_{\frac{n_t}{4}}^{i_4}$ $\longrightarrow$ $\widehat{L}_{\frac{n_t}{4}}^{i}$
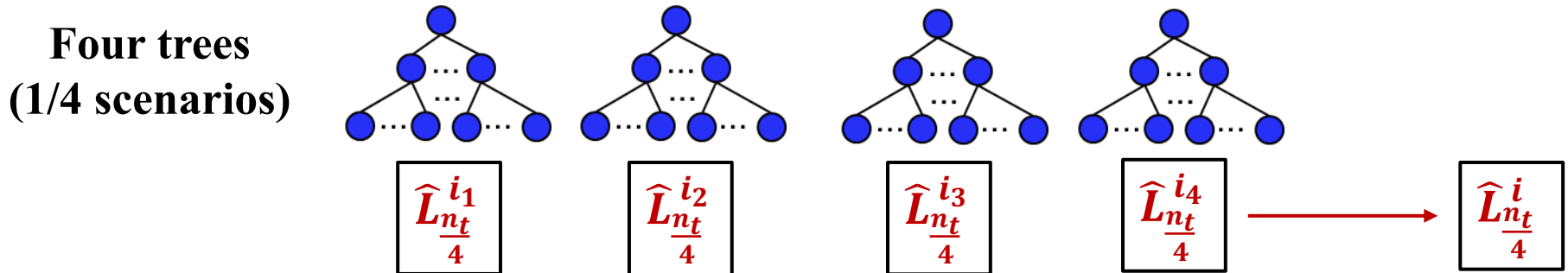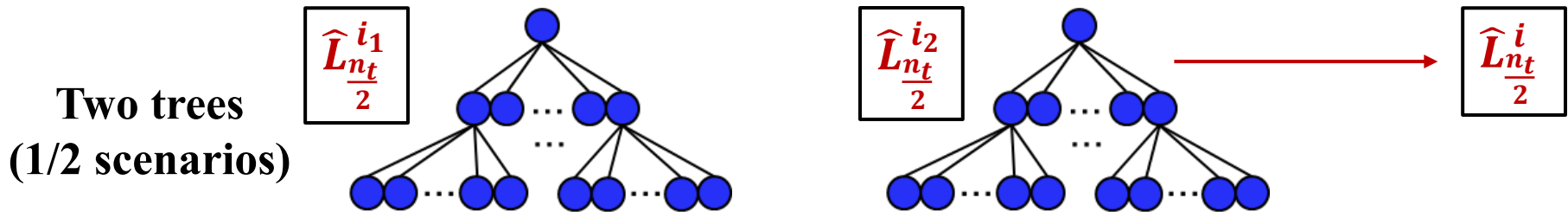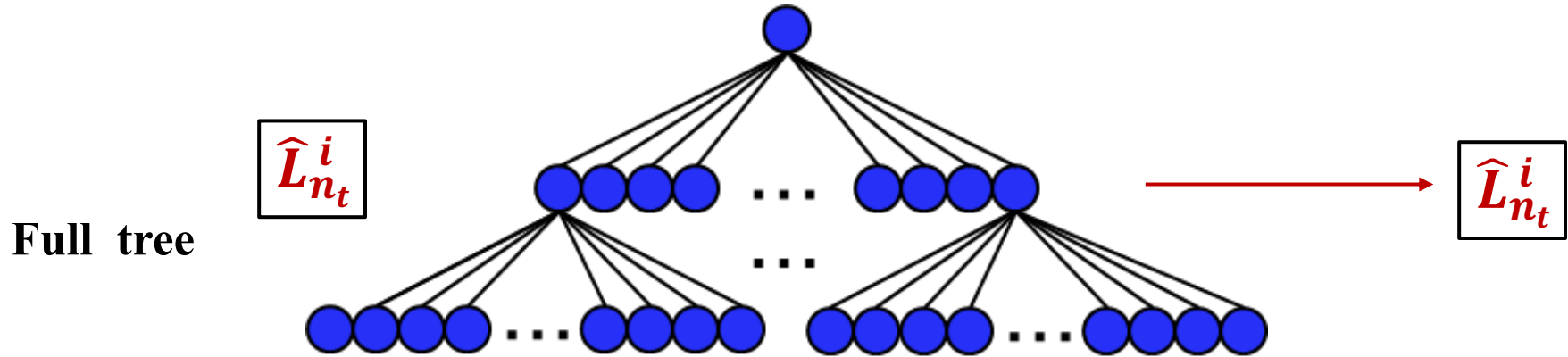
AJE and LBE in Multi-stage
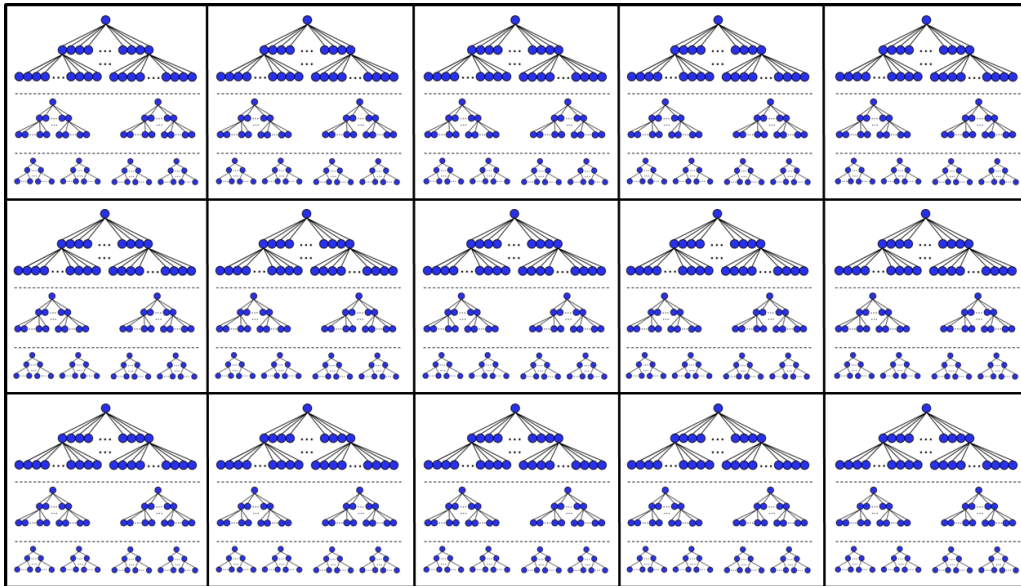
$$\bar{L}_{n_t} = \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \hat{L}_{n_t}^i$$

$$\bar{L}_{n_t/2} = \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \hat{L}_{\frac{n_t}{2}}^i$$

$$\bar{L}_{n_t/4} = \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \hat{L}_{\frac{n_t}{4}}^i$$

$$J^A = g_\gamma(\bar{L}_{n_t})$$

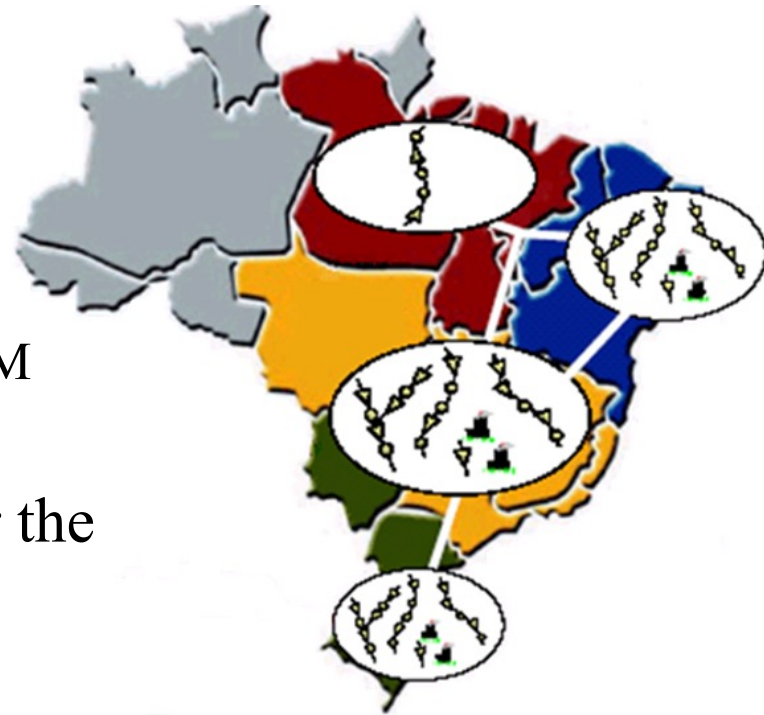$$J^A = \bar{L}_{n_t} - \frac{\hat{r}}{(1-\hat{r})^2}(\bar{L}_{n_t/2} - \bar{L}_{n_t})$$

$$\hat{r} = \frac{\bar{L}_{n_t/2} - \bar{L}_{n_t}}{\bar{L}_{n_t/4} - \bar{L}_{n_t}}$$

$$\hat{C} \rightarrow \text{covariance of } \left(\hat{L}_{\frac{n_t}{4}}^i, \hat{L}_{\frac{n_t}{2}}^i, \hat{L}_{n_t}^i\right)$$

$$s^2 = \nabla^T g_\gamma(\bar{L}_{n_t})\hat{C}\nabla g_\gamma(\bar{L}_{n_t})$$

Application to the Brazilian Electric Power System

- 80% of generation capacity $\rightarrow$ hydro
  - 150 hydro generators, 150 thermal generators

- Model Characteristics
  - **Optimization over 24 stages**
  - Aggregated reservoir scheme
  - Water inflow forecasts produced by a DLM **(Marangon Lima, 2011)**

- We consider different sample sizes for the same problem instance
  - n_u = **25600 for UBE**
  - n_ℓ = **15 for LBE**

AJE-M Results

Confidence Interval Width Comparison

	12	16	24	48	100	200	400	n_t
■ Regular	29.26%	14.67%	17.62%	10.08%	8.97%	7.29%	5.63%	
■ Jackknife	27.70%	13.19%	16.82%	8.89%	8.81%	7.18%	4.97%	
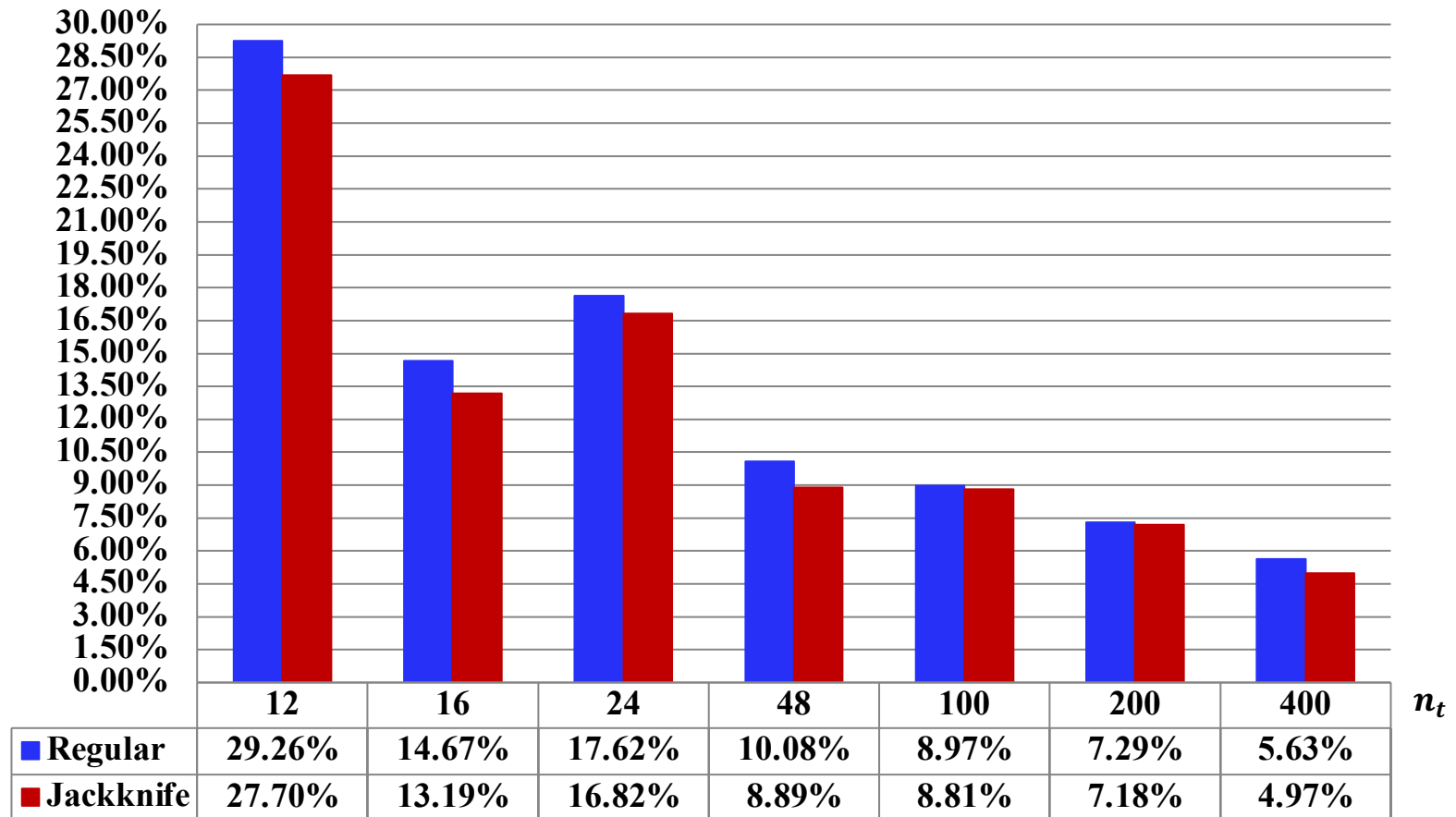
- Jackknife estimator reduced CI widths by a modest amount -

AJE-M Results

Confidence Interval Width with Jackknife

n_t	12	16	24	48	100	200	400
■ Gap	15.86%	3.31%	8.29%	2.43%	3.17%	3.44%	2.05%
■ El	10.83%	8.38%	7.10%	6.05%	4.55%	2.74%	1.88%
■ Eu	0.79%	1.08%	1.07%	1.05%	0.99%	1.01%	1.05%

before
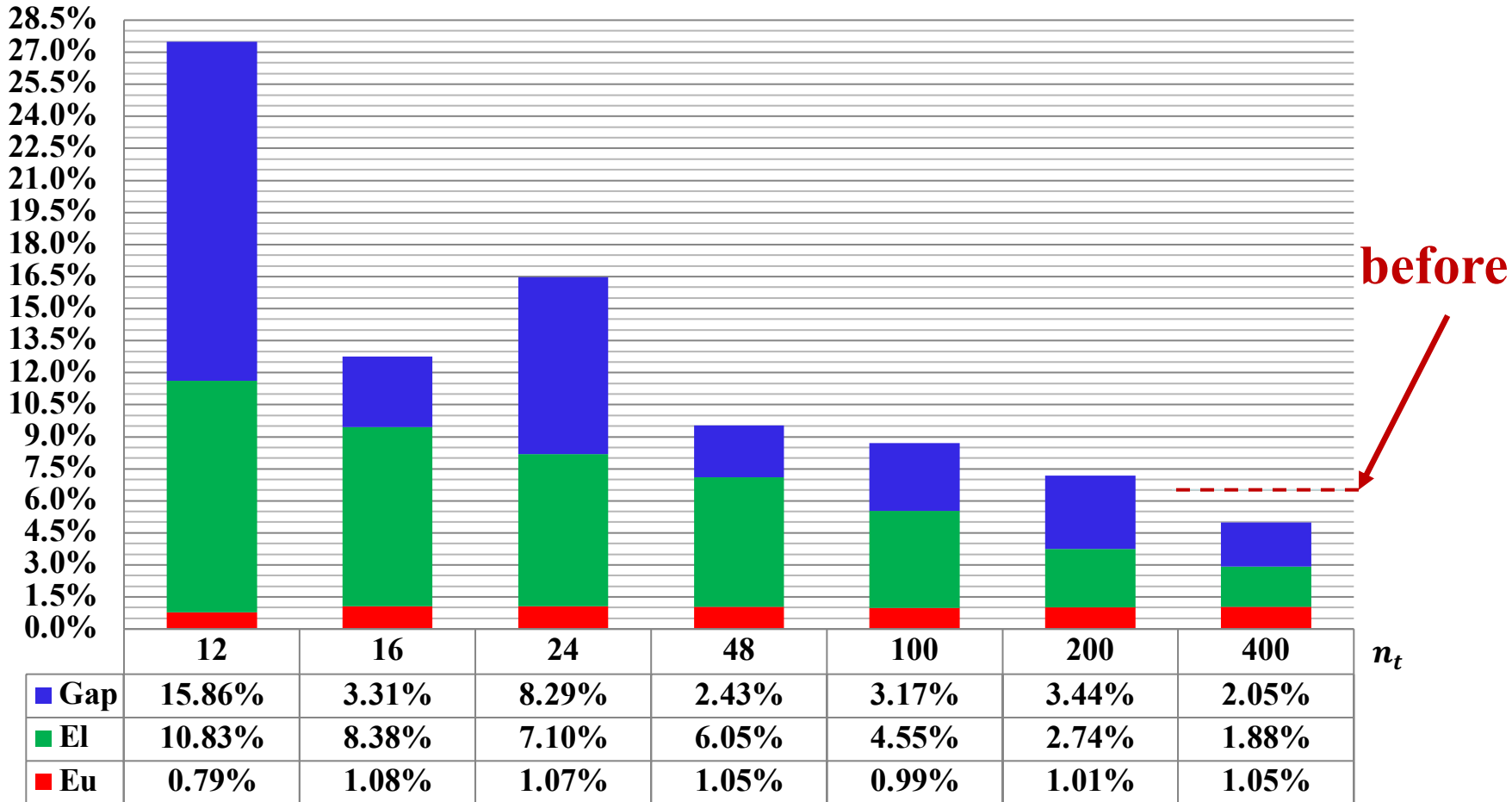
Final Remarks

- The hydro-scheduling problem is a challenging multi-stage stochastic optimization problem. SBDA handles the problem

- We study the **solution quality** with respect to the **true problem**

- We presented a procedure to assess the quality of the solution using jackknife estimators in the multi-stage setting

- The **improvement from** previous results are **1.44%** which means we are reducing the estimate of the optimality gap by **$29,526,897.24** over a period of 2 years

References

- A. Chiralaksanakul and D.P. Morton, "Assessing policy quality in multi-stage stochastic programming", Stochastic Programming E-Print Series, 2004

- A.R. Queiroz, "On a Sampling-based Decomposition Algorithm with Application to Hydrothermal Scheduling: Cut Formation and Solution Quality", PhD dissertation, The University of Texas at Austin, 2011

- A.R. Queiroz and D.P. Morton, "Sharing Cuts under Aggregated Forecasts when Decomposing Multi-stage Stochastic Programs", Operations Research Letters, Vol. 41(3): 311-316, 2013

- A. Partani, D.P. Morton and I. Popova, "Jackknife Estimators for Reducing Bias in Asset Allocation" Proceedings of the Winter Simulation Conference, 2006

- A. Partani, "Adaptive Jackknife Estimators for Stochastic Programming", PhD dissertation, The University of Texas at Austin, 2007

- G. Infanger and D.P. Morton, "Cut sharing for multistage stochastic linear programs with interstage dependency", Mathematical Programming 75 (1996) 241-256

- H. Abdi and L.J. Williams, "Jackknife", In Neil Salkind (Ed.), Encyclopedia of Research Design, Thousand Oaks, CA: Sage, 2010

- L.M. Marangon Lima, "Modeling and forecast of Brazilian reservoir inflows via dynamic linear models under climate change scenarios", PhD dissertation, The University of Texas at Austin, 2011

- L.M. Marangon Lima, E. Popova, and P. Damien, "Modeling and forecasting of Brazilian reservoir inflows via dynamic linear models", International Journal of Forecasting, v. 30, p. 464-476, 2014.

- M.V.F. Pereira and L.M.V.G. Pinto, "Multi-Stage Stochastic Optimization Applied to Energy Planning", Mathematical Programming, N. 52, 1991

- T. Homem de Mello, V. de Matos and E. Finardi, "Sampling strategies and stopping criteria for stochastic dual dynamic programming: a case study in long-term hydrothermal scheduling", Energy Systems, 2: 1–31, 2011

Thank you!

Comments & Suggestions

ar_queiroz@yahoo.com.br