Mathematical Decomposition and Solution Quality in Multi-stage Hydrothermal Scheduling

Anderson Rodrigo de Queiroz — **GESIS**
David P. Morton — **IEMS**

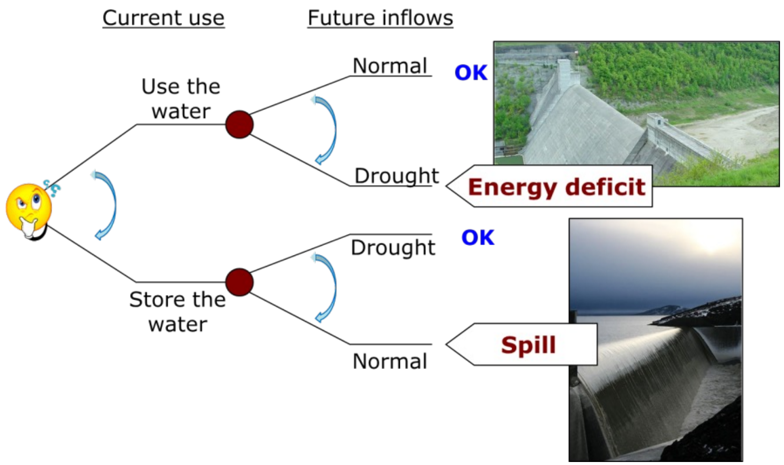
November 8th, 2014

Outline

- Hydrothemal Scheduling Problem
- Model Formulation
- SBDA Multi-stage Scheme
- Solution Quality Evaluation in Multistage Stochastic Programs

Hydrothermal Scheduling Problem

- Find the sequence of **hydro releases** and **thermal plant dispatches** for a planning horizon in order to match system demand
  - Resource management
  - Input variable forecasting
  - Operational aspects



- Basic economic criterion
  - **Minimize operational costs** (present + expected future)
- Multi-stage Stochastic Linear Program

Stage t Benders' Master Problem

- Suppose we are at stage t under ω_t and we have:

$vec(\eta_t, c_t, B_t, A_t), t = 2, \dots, T$ are $\perp\!\!\!\perp$

$$\begin{aligned} \min_{x_t, \theta_t} \quad & c_t x_t + \theta_t \\ s.t. \quad & A_t x_t = B_t x_{t-1} + \rho_t b_t + k_t \quad : \pi_t \longrightarrow \textbf{Structural constraints} \\ & -\vec{G}_t x_t + e\,\theta_t \geq \vec{g}_t \quad : \alpha_t \longrightarrow \textbf{Benders' cuts} \\ & x_t \geq 0 \end{aligned}$$

$$b_t = R_{t-1} b_{t-1} + \eta_t$$

x_t: stage t decision variables including: hydro generation, hydro storage, spillage, thermal generation, energy transfers, load curtailment

A_t: constraint matrix including energy balance, demand satisfaction

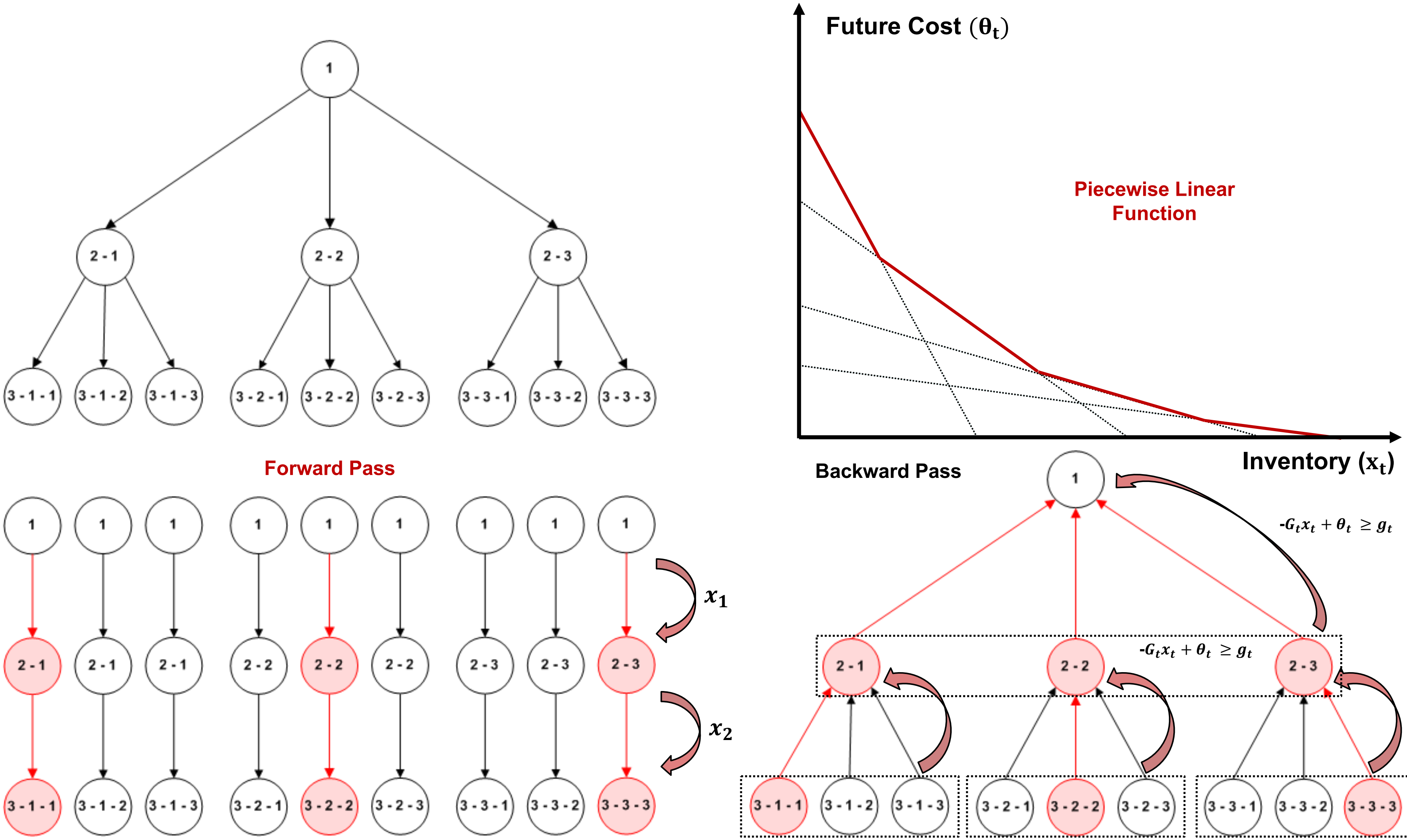
b_t: stochastic water inflow at each hydro plant

ρ_t: matrix to transform water into energy inflows

$B_t x_{t-1}$: storage from last stage

k_t : deterministic demand

Sampling-based Decomposition Algorithm



Solution Quality Assessment

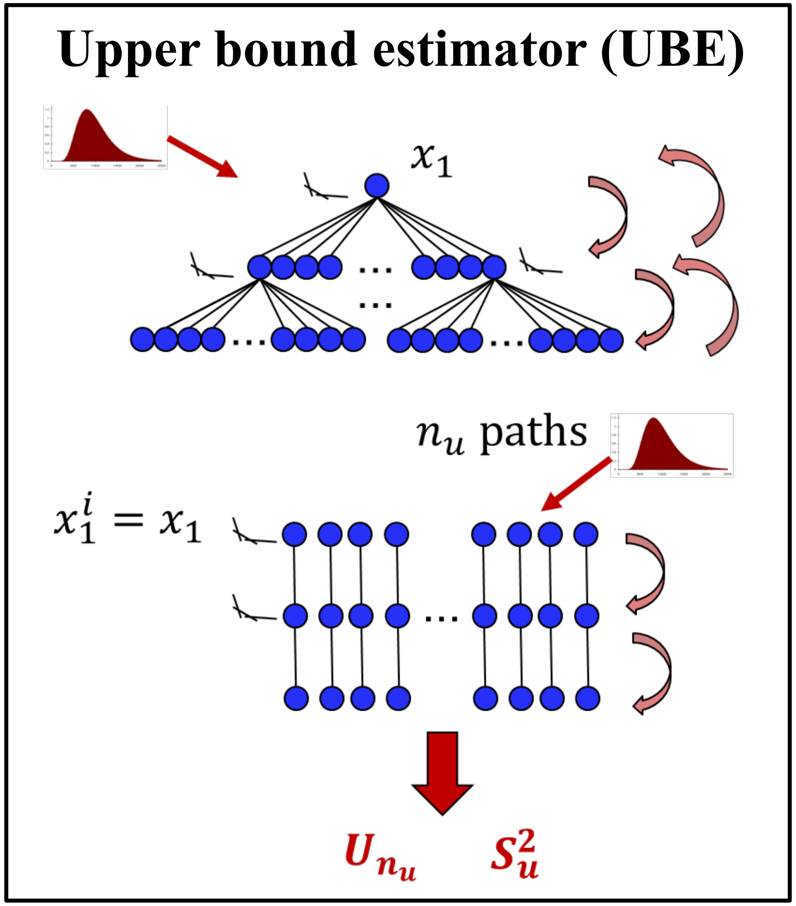
- Monte Carlo simulation → is used to assess if a candidate solution (i.e., policy) is near optimal

cannot solve the SP exactly

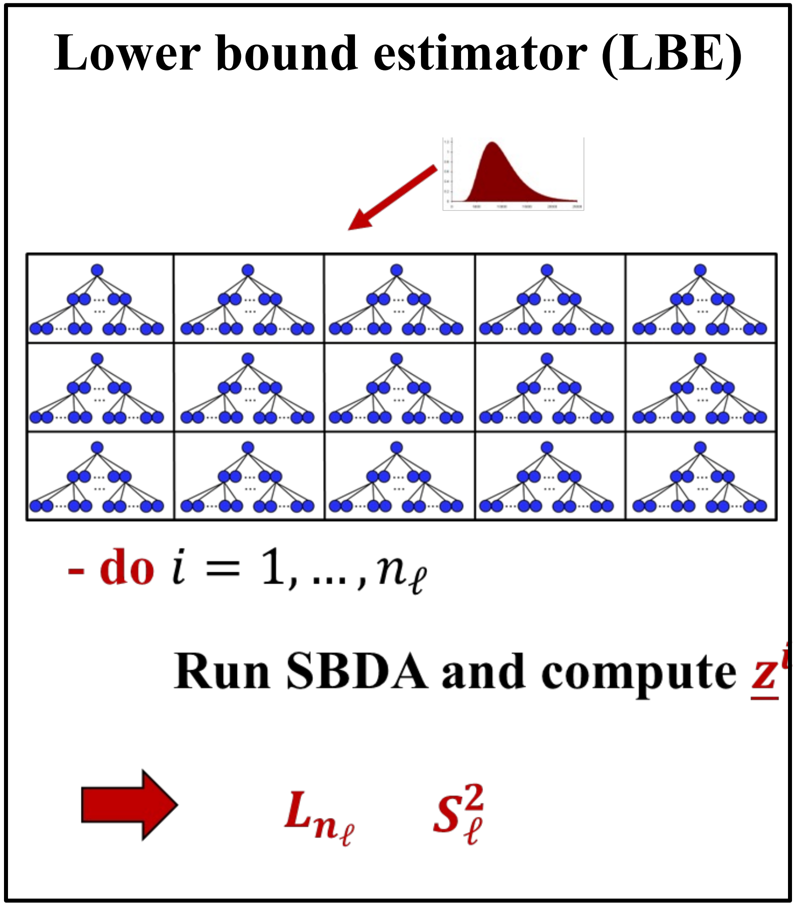
- When optimizing a sample-mean estimator we get an optimistic bound for z^*
- This implies a weak statement regarding quality of candidate solution → **Estimate may have large bias**
- When bias is large it is not possible to be sure if a candidate solution is near optimal

Confidence Interval Construction

Upper bound estimator (UBE)



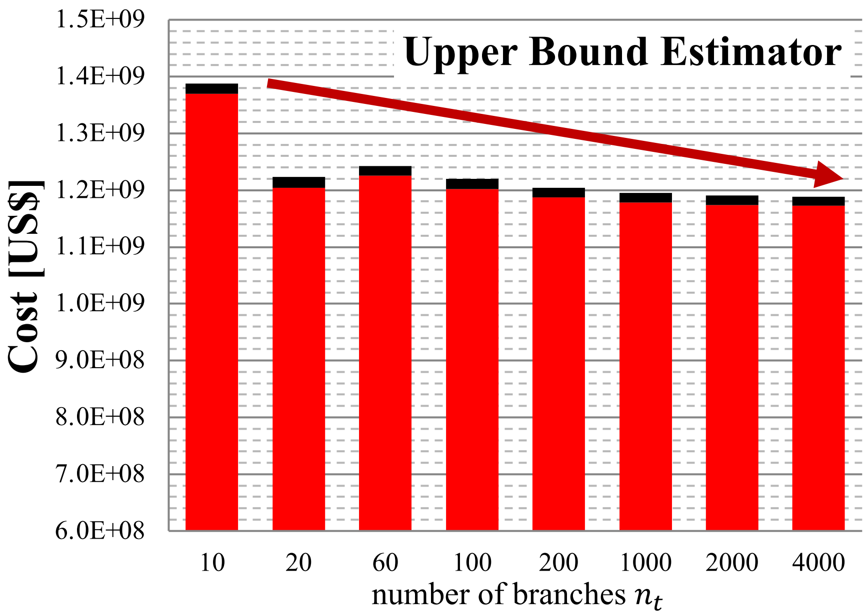
Lower bound estimator (LBE)

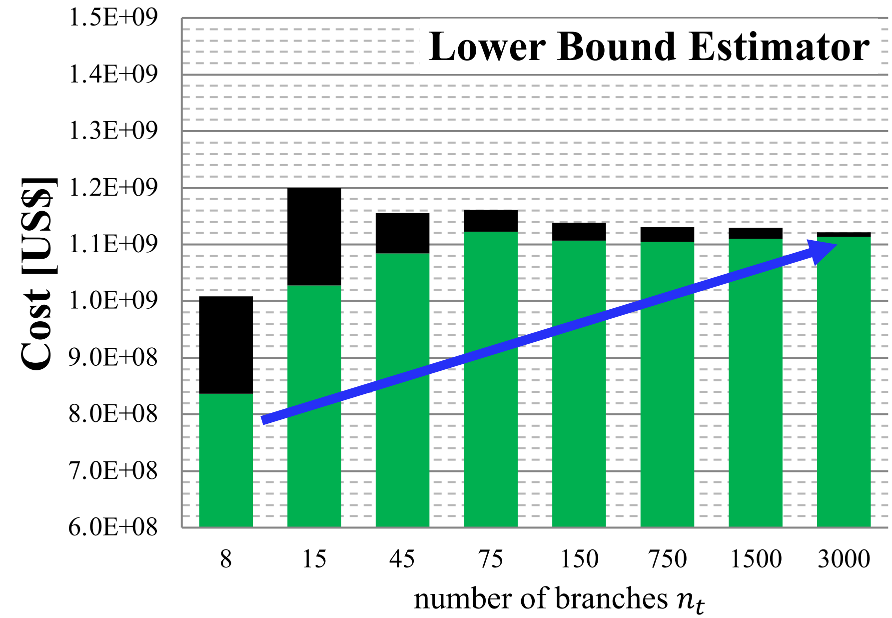


Let $\epsilon_\ell = t_{n_\ell - 1} S_\ell / \sqrt{n_\ell}$ and $\epsilon_u = z_\alpha S_u / \sqrt{n_u}$

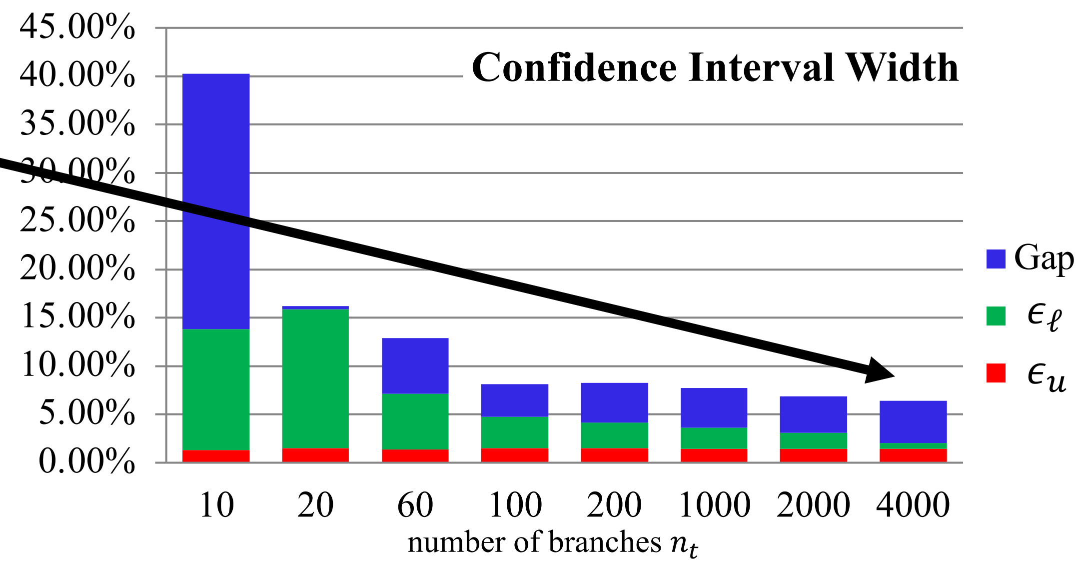
Output one-sided **CI** on $\mathbb{E}\boldsymbol{U} - \boldsymbol{z}^*$, $\left[0\,, \left(U_{n_u} - L_{n_\ell}\right)^+ + \epsilon_\ell + \epsilon_u\right]$

Previous Results



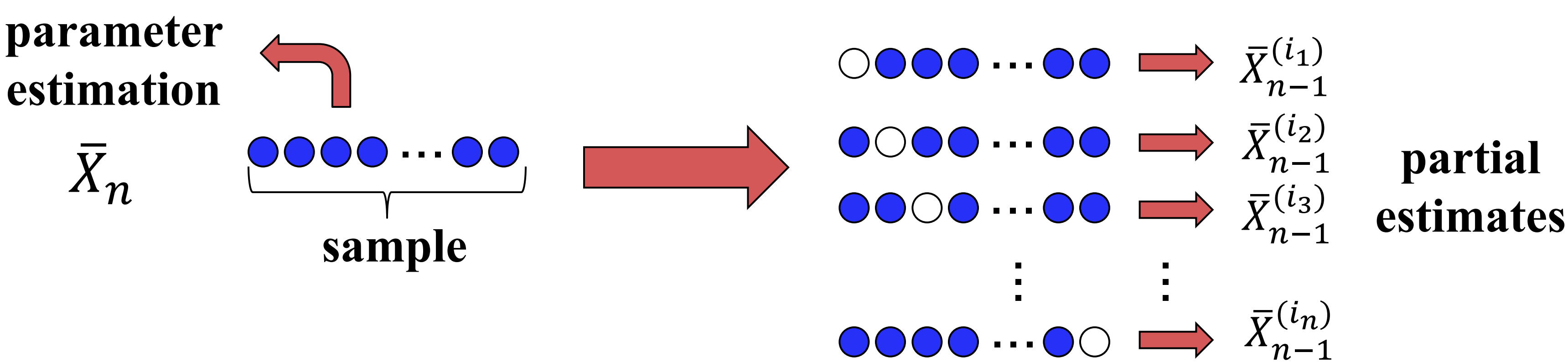


CI width was 6.41%



Jackknife Estimators

- Jackknife is a technique developed by Quenouille in 1949 to estimate the bias of $\hat{\theta}$ (estimator)
- It is also known as the "leave one out" procedure
- Resampling procedure (same class as Bootstrap)

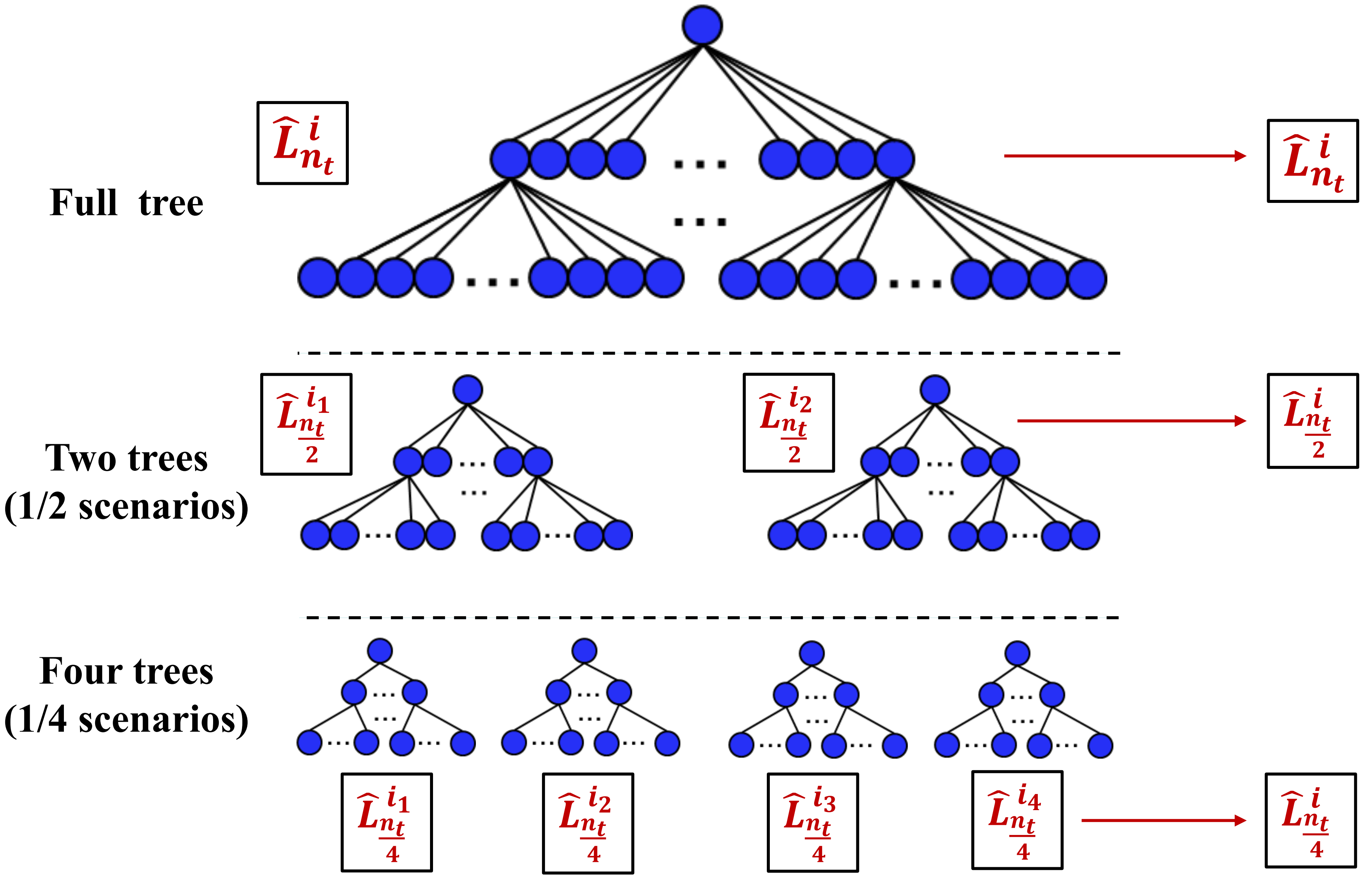


- Reduce bias of $O\left(\frac{1}{n}\right)$

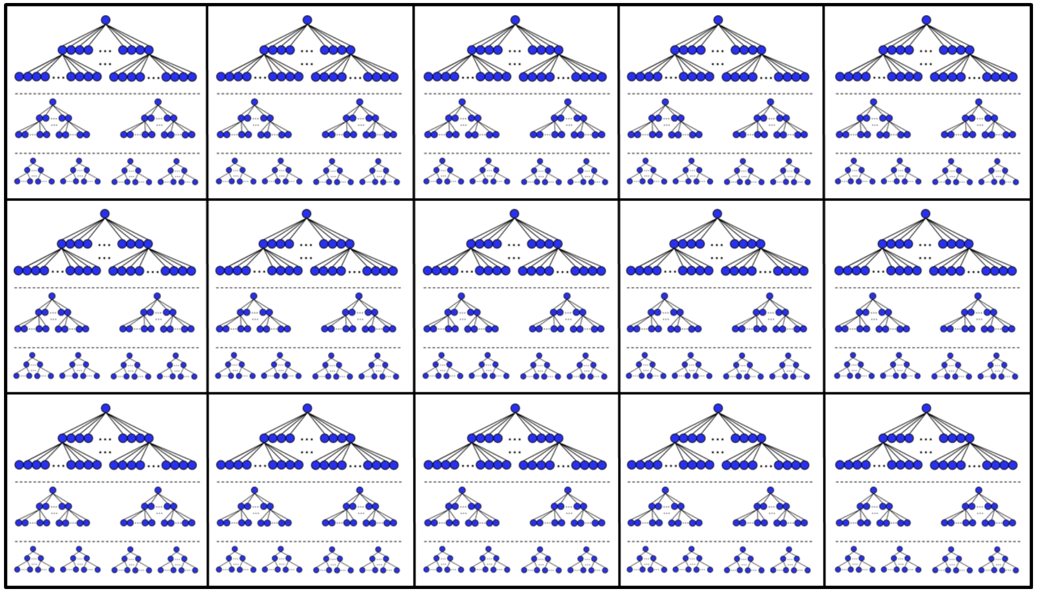
Adaptive Jackknife Estimators

- In SAA the bias has $O\left(\frac{1}{n^p}\right)$ where p is unknown, $p \in [\frac{1}{2}, \infty)$ **(Bayraksan et al. 2006)**
- Following the idea of the generalized jackknife estimators → adaptive jackknife estimators (AJE) **(Partani et al. 2006)**
- In AJE the **order of the bias is not assumed to be known** when forming an estimator
  - Applied to reduce bias in static and two-stage models

AJE Procedure in Multi-stage Setting



AJE and LBE in Multi-stage



$$\bar{L}_{n_t} = \frac{1}{n_\ell}\sum_{i=1}^{n_\ell} \hat{L}_{n_t}^{i}$$

$$\bar{L}_{n_t/2} = \frac{1}{n_\ell}\sum_{i=1}^{n_\ell} \hat{L}_{\frac{n_t}{2}}^{i}$$

$$\bar{L}_{n_t/4} = \frac{1}{n_\ell}\sum_{i=1}^{n_\ell} \hat{L}_{\frac{n_t}{4}}^{i}$$

$$J^A = g_\gamma(\bar{L}_{n_t})$$

$$J^A = \bar{L}_{n_t} - \frac{\hat{r}}{(1-\hat{r})^2}\left(\bar{L}_{n_t/2} - \bar{L}_{n_t}\right)$$

$$\hat{r} = \frac{\bar{L}_{n_t/2} - \bar{L}_{n_t}}{\bar{L}_{n_t/4} - \bar{L}_{n_t}}$$

$$\hat{C} \rightarrow \text{covariance of} \left(\hat{L}_{\frac{n_t}{4}}^{i}, \hat{L}_{\frac{n_t}{2}}^{i}, \hat{L}_{n_t}^{i}\right)$$

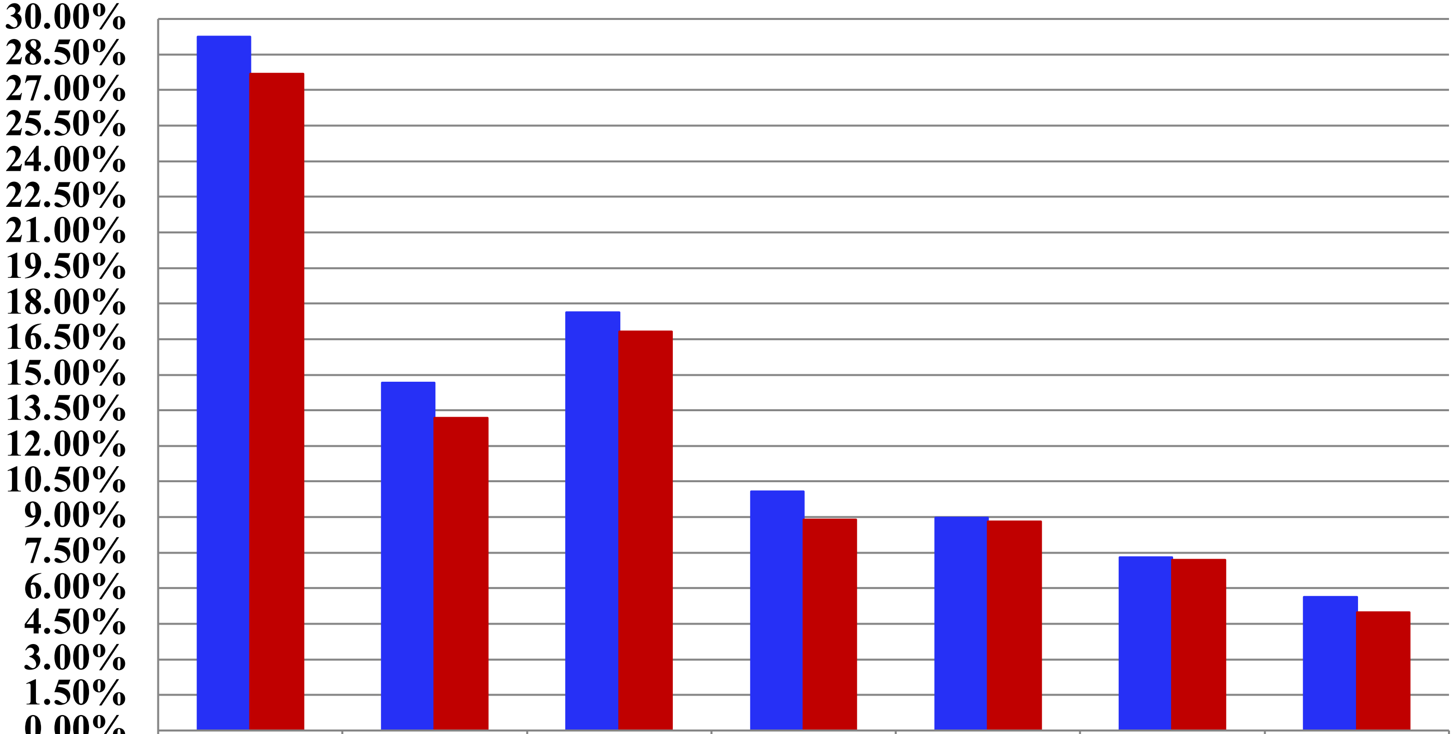
$$s^2 = \nabla^T g_\gamma(\bar{L}_{n_t})\hat{C}\nabla g_\gamma(\bar{L}_{n_t})$$

Application to the Brazilian Electric Power System

- 80% of generation capacity → hydro
  - 150 hydro generators, 150 thermal generators
- Model Characteristics
  - **Optimization over 24 stages**
  - Aggregated reservoir scheme
  - Water inflow forecasts produced by a DLM **(Marangon Lima, 2011)**
- We consider different sample sizes for the same problem instance
  - $\boldsymbol{n_u = 25600}$ **for UBE**
  - $\boldsymbol{n_\ell = 15}$ **for LBE**

AJE-M Results

Confidence Interval Width Comparison

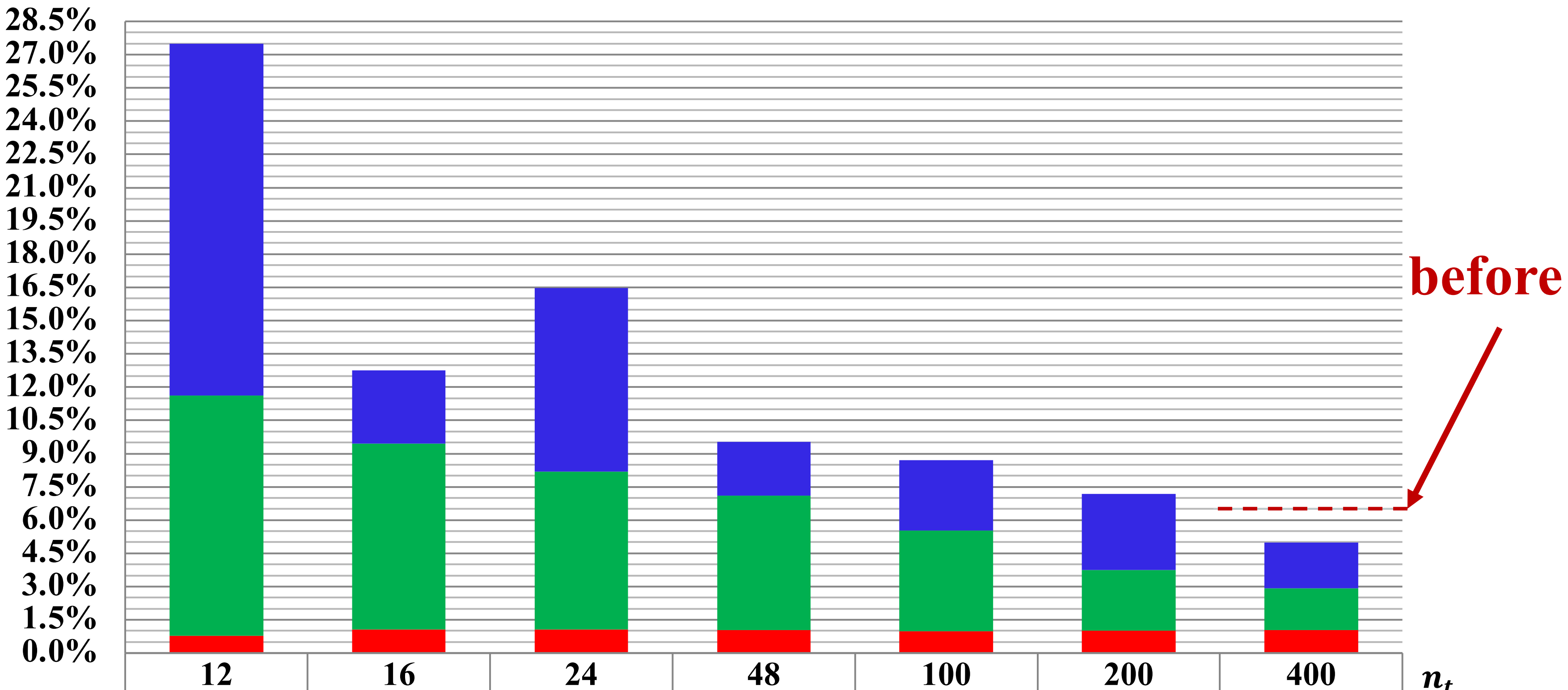


n_t	12	16	24	48	100	200	400
Regular	29.26%	14.67%	17.62%	10.08%	8.97%	7.29%	5.63%
Jackknife	27.70%	13.19%	16.82%	8.89%	8.81%	7.18%	4.97%

- Jackknife estimator reduced CI widths by a modest amount -

AJE-M Results

Confidence Interval Width with Jackknife



	12	16	24	48	100	200	400
Gap	15.86%	3.31%	8.29%	2.43%	3.17%	3.44%	2.05%
El	10.83%	8.38%	7.10%	6.05%	4.55%	2.74%	1.88%
Eu	0.79%	1.08%	1.07%	1.05%	0.99%	1.01%	1.05%

Final Remarks

- The hydro-scheduling problem is a challenging multi-stage stochastic optimization problem. SBDA handles the problem
- We study the **solution quality** with respect to the **true problem**
- We presented a procedure to assess the quality of the solution using jackknife estimators in the multi-stage setting
- The **improvement from** previous results are **1.44%** which means we are reducing the estimate of the optimality gap by **$29,526,897.24** over a period of 2 years

References

- A. Chiralaksanakul and D.P. Morton, "Assessing policy quality in multi-stage stochastic programming", Stochastic Programming E-Print Series, 2004
- A.R. Queiroz, "On a Sampling-based Decomposition Algorithm with Application to Hydrothermal Scheduling: Cut Formation and Solution Quality", PhD dissertation, The University of Texas at Austin, 2011
- A.R. Queiroz and D.P. Morton, "Sharing Cuts under Aggregated Forecasts when Decomposing Multi-stage Stochastic Programs", Operations Research Letters, Vol. 41(3): 311-316, 2013
- A. Partani, D.P. Morton and I. Popova, "Jackknife Estimators for Reducing Bias in Asset Allocation" Proceedings of the Winter Simulation Conference, 2006
- A. Partani, "Adaptive Jackknife Estimators for Stochastic Programming", PhD dissertation, The University of Texas at Austin, 2007
- G. Infanger and D.P. Morton, "Cut sharing for multistage stochastic linear programs with interstage dependency", Mathematical Programming 75 (1996) 241-256
- H. Abdi and L.J. Williams, "Jackknife", In Neil Salkind (Ed.), Encyclopedia of Research Design, Thousand Oaks, CA: Sage, 2010
- L.M. Marangon Lima, "Modeling and forecast of Brazilian reservoir inflows via dynamic linear models under climate change scenarios", PhD dissertation, The University of Texas at Austin, 2011
- L.M. Marangon Lima, E. Popova, and P. Damien, "Modeling and forecasting of Brazilian reservoir inflows via dynamic linear models", International Journal of Forecasting, v. 30, p. 464-476, 2014.
- M.V.F. Pereira and L.M.V.G. Pinto, "Multi-Stage Stochastic Optimization Applied to Energy Planning", Mathematical Programming, N. 52, 1991
- T. Homem de Mello, V. de Matos and E. Finardi, "Sampling strategies and stopping criteria for stochastic dual dynamic programming: a case study in long-term hydrothermal scheduling", Energy Systems, 2: 1–31, 2011

Thank you!

Comments & Suggestions

ar_queiroz@yahoo.com.br